Effigies Ionæ Moore Matheseos
Professoris Ætat: suæ 45 An°: Dni: 1660

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MOORE'S ARITHMETICK: IN Four Books.

Treating of *Vulgar Arithmetick* in all its Parts, with ſeveral new Inventions to eaſe the Memory, by *Logarithms*, *Decimals*, *&c.* fitted for the uſe of all Perſons.

Together with *Arithmetick* in *Species* or *Algebra*: whereby all difficult Queſtions receive their *Analytical Laws* and *Reſolutions*, made very plain and eaſie for the uſe of *Scholars*, and the more Curious.

The Third Edition with Additions.

To which are added two Mathematical Treatiſes:

1. *A new* Contemplation Geometrical *upon the* Oval *Figure called the* ELLIPSIS.
2. *The two firſt Books of* Mydorgius *his* Conical Sections *Analyzed by that* Reverend Divine *Mr.* W. Oughtred, *Engliſhed and compleated with Cuts.*

By Sir *JONAS MOORE*, Maſter Surveyor of His Majeſty's Ordnance.

London, Printed by *R. H.* for *Obadiah Blagrave*, at the *Bear* and *Star* in St. *Paul's* Church-yard, 1688.

To the King's Most Excellent Majesty, *JAMES* the Second, of Great *Britain*, *France* and *Ireland*, King, Defender of the Faith, *&c.*

Most Dread Sovereign,

SIR Jonas Moore *the Author of this following Arithmetick, having finished a Second Impression, with some Mathematical Tracts here annexed, in the Year* 1660. *when you were Duke of* York, *Dedicated it to Your Majesty in these very Words which follow.*

TO THE

ILLUSTRIOUS PRINCE

JAMES

Duke of YORK,

Lord High Admiral of the British Seas.

May it please Your Highness,

I Can rather hope for Your Pardon for this presumption of mine, in humbly tendring these my Labours to You, for Your *Protection*; than I can expect the *Approbation* from others for this my *Boldness*, who shall not know these *Reasons* following.

1. None are fit to *Patronize* the *Studies Mathematical* but Princes, and Men

of

of Quality: For, though they have in them the Grounds and Seeds of all Arts and Professions, yet as to the Professors of them they afford small profit, being *Studia inutilia*; the old Proverb as to them being truly verified, *They have the Labour for their Pains*: and therefore who more fit, *Great Prince*, than Your Self to be a *Patron*, who is known to be a Lover of these Studies?

2. Among the rest of the Arts, to which Arithmetick puts forth her helping hand, *Navigation* acknowledgeth a great assistance. No man can pretend to any moderate skill in it without *Arithmetick*, no account by the *Log-line*, no *Proportion* in *Triangles*, *&c.* without *Arithmetick*; and therefore who more fit, *High Admiral of the Seas*, than Your Self to Patronize this Piece?

3. Whilst Your *Father*, our late *Gracious Sovereign*, (of Glorious Memory) was at *Durham*, in His Expedition to

the Northern Parts, I was favoured so much by a Person of Quality, who had acquainted His Majesty of my Studies, that I gave an account of them to Him, and received this comfortable Expression from Him, *That I should follow that Study, and should receive Encouragement*; and afterwards from *Holmby* House, when your Highness was at St. *James's*, did direct, that I should be imployed as Your Highness Servant, for Your Instruction in *Arithmetick, uses of the Globes, and Geography*; but the malicious and cunning subtilty of Mr. *Ascham*, and Your Highness happy departure from thence, caused, that no great Progress was made therein: To You therefore, Illustrious SIR, (whose Word next to His Sacred Majesty, can only Patronize and Advance the *Mathematicks* and *Mathematicians*) I, (whose name cannot be found in the black book) with all Duty, Dedicate these

my

my Labours, and humbly offer them at Your Feet, praising God for the *Royal Restoration*, and praying to him for the *Prosperity of His Majesty* and the *Royal Progeny* here, and future Happiness in the World to come.

Jun. 10.
1660.

Your Highness most

Obedient Servant,

JONAS MOORE.

To

To the Honourable
Sir *EDWARD MOUNTAGUE* K[t].
VICE-ADMIRAL:
And one of the *Knights* of the Honourable Order of the *GARTER*.

My Lord,

ALL *Loyal Hearts are now satisfied: I see my* Liege Lord *and* Prince (the Glory of Monarchs) *restored: I see my* Master *the* DUKE, *Lord High* Admiral, *in full Splendour: And I see You, my* assured Mecænas, *escaped out of the ways of the Wicked, His* VICE-ADMIRAL: *And in You I perceive all* Ingenuity *and* Arts *ready to flourish: Who knows You, and doth not easily confess, That among the* Gentry *of* England *You are the great* Mathematician, *both for Theory and*

Practice:

Practice: *The* grand Master of Musick, *the great* Favourer *of Learning and Learned Men*: *Nay*, *indeed, who is it that is any way excellent in what he professeth, that doth not readily acknowledge You his Patron? Amongst whom, I, the most bounden of Your Servants, subscribe to be,*

Honourable SIR,

At Your Command,

JONAS MOORE.

To

To the READER.

Courteous Reader,

I Have endeavoured with what care I could to correct the miſtakes both of *Pen* and *Preſs*, and to alter all Paſſages and Rules in my former *Arithmetick*, which I could hear were miſliked, found too abſtruſe or difficult; ſo that in effect this is wholly another thing, though I retain the order and method in the Chapters. For the avoiding the Errors of the Preſs, (which Books of this kind are ſubject to) I have been much beholden to Mr. *John Leake*, (*an able Mathematician*, and good Friend of mine) who did Correct all the firſt Book, carefully amending the Errors of Tranſcription: the reſt was attended by my ſelf. In the ſecond Book of *Algebra* I have not proceeded to further Myſteries, but contented my ſelf to employ my whole ſtrength to explain what was formerly delivered, in the beſt manner I could imagine, and doubt not but have attained my aim, *To write nothing that might be ſuperfluous, nothing too difficult or obſcure:* That Book ſerving for an Introduction to Mr. *Oughtred*'s *Clavis Math. Vietta, Des-Chartes*, *Diaphantus*, and the reſt of thoſe

thoſe Learned Authors that have proceeded in this worthy Art.

Theſe Times have not encouraged me to finiſh or compleat many Pieces I formerly promiſed, and had by me: Indeed the abuſe in the birth in the Midwife's hand is none of the leaſt diſcouragements; a Piece of mine, of Aſtronomy and Aſtronomical Tables, which coſt me a years labour and above, was ſtifled in the Preſs, when one ſheet was brought forth, and a great part of the Copy loſt, though I know the method (not yet uſed by any) would have infinitely pleaſed the ingenious: However I have hereunto added two *Geometrical* Pieces, one of the *Ellipſis* my own, the other of the *Conical Sections* analyzed by that Famous and Reverend Divine Mr. *W. Oughtred*, tranſlated and compleated by me. All which (without any other aim or end but thy profit, content and pleaſure,) I offer to thy favourable acceptation, remaining

A Lover of Arts.

JONAS MOORE.

A Catalogue of the Chapters contained in the first Part of *Moore*'s Arithmetick.

The Contents.

A Catalogue of the Chapters contained in the Second Part.

A Catalogue of the Chapters contained in the Third Part.

A Catalogue of the Chapters contained in the Fourth Book of *Algebra.*

Chap. 8.

The Contents.

Two Mathematical Treatises here annexed.

1. E*Ntituled*, Contemplationes Geometricæ, *in two Treatises.* 1. *A new Contemplation Geometrical upon the Ellipsis, plainly setting out the Nature of that Figure.*

2. *Conical Sections, or the several Sections of a Cone, being an Analysis of the two first Books of* Mydorgius; *and whereby the Nature of the Parabola, Hyperbola and Ellipsis are very plainly laid down.*

Translated and drawn from the Papers of the Learned Mr. William Oughtred, *By Sir* Jonas Moore.

CHAP. I.

CHAP. I.

A ſhort *Introduction* into the Parts of *Arithmetick*.

ARITHMETICK is the Art of numbring well.

The *Subject* of *Arithmetick* is *Number*, as that unto which all the precepts, and doctrine of the Art of Computation (whether by numbers whole or broken) hath a particular Relation.

2. *Arithmetick* is either *ſimple* or *comparative.*

3. *Simple*, is that which conſidereth the ſimple nature of *Numbers*.

4. *Number*, is that, according to which any thing is numbred.

According to an unit any thing is ſaid to be one, as one God, one World, &c. [This is according to *Ramus*: the Ancients ſay, that unity is the beginning of *Number*, yet no *Number*.]

But our modern Numeriſts reſt themſelves not ſatisfied with this Definition of the beginning of Number, it being altogether diſagreeable to the principles of Geometry, Muſick, Time, &c. whoſe beginnings are called a Point, a Sound, a

Moment, &c. and are defined to be indivisible; but an Unite is divisible *ad infinitum*, and therefore can it not be the beginning of number; and consequently, if that which is divisible cannot be the Beginning of Number, it must of necessity be (o) a Cypher, or nothing, that is its Beginning; for that which is greater than (o) nothing may be divided and subdivided into parts *ad infinitum.* For as a Point in Geometry is defined to be the Beginning of Magnitude, and it self no Magnitude; so may a Cypher be said to be the beginning of Number, and it self no number; nor is the one any more divisible than the other. *Vide Cockers* Arithmetick *Chap.* 1. *Def.* 2, 3, 4 and 5.

According to the number two things are said to be two, as two Starrs, two men, *&c.*

5. *Number* is either *whole* or *broken.*

Whole we call *Integers* from the Latin word *Integrum*, and *broken* Numbers *Fractions.*

6. A *whole Number* is either of *unity* or of *multitude.*

7. An *unity* or *unite* is the beginning of *multitude*, though not the beginning of Number.

8. *Multitude* is the Collection of *unites*, as 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, are equal to 10. being collected together.

9. *Fractions* from the division of the Unite into parts; As one Yard being divided into four equal parts, the half of that Yard is less than one, and is set down as a Fraction thus, $\frac{1}{2}$.

10. In Numbers of any sort, two things are to be considered, *viz.* $\left\{\begin{array}{l}\text{Notation,}\\ \text{and}\\ \text{Numeration.}\end{array}\right.$

11. *Notation*

11. *Notation* teacheth how to deſcribe any number by certain Notes or Characters, and to declare the value thereof being ſo deſcribed.

This *Notation* is that which ſome call *Numeration.*

12. But *Numeration* (here) is that, which teacheth how from Numbers given to find out another required, and which (as is afterwards declared) conſiſts in the *Compoſition* and *Diſſolution* of Numbers.

13. Notation (as it is here conſidered) is, { 1. *Certain* and *Determinate.* 2. *Uncertain*, *Undeterminate*, and *Arbitrary.* }

14. *Notation* certain and determinate, I here call that moſt excellent Invention of expreſſing all Numbers whatſoever by theſe ten Characters, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. whereof the firſt nine are commonly called ſignificant Figures, and the laſt a Cypher, which ſignifies nothing, but only ſerves to augment a Number according to its place.

15. Theſe 10. *Characters* for the expreſſing of all *Numbers* are ordered into certain places, proceeding from the right hand towards the left, in a *decimal Progreſſion*; ſo that the firſt place is of *Unites*, the ſecond of *Tens*, the third of *Hundreds*, the fourth of *Thouſands*, &c. As here you may ſee in the Example following, where the Order of the Places is noted by the Letters of the Roman Alphabet, and the value of them by the Capital *Numeral* letters, *X* for the place of *Tens*, *C* for the place of *Hundreds*, and *M* for the place of *Thouſands*, &c.

Example.

	r	q	p	o	n	m	l	k	i	h	g	f	e	d	c	b	a	v
I	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
n.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	C	X	Unites.
m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m,	m.	m.	C	X				
m	m.	m.	m.	m.	m.	m,	m.	m.	m.	C	X							
m.	m.	m.	m.	m.	m.	m	C	X										
m	m.	m.	m.	C	X													
n.	C	X																
&c. Trillion.	the ſixth period is of thouſ. of thouſ. of thouſ. of thouſ. of thouſ. of thouſ. of mill. of mill.			the fifth period is of thouſ. of thouſ. of thouſ. of thouſ. or mill. of mill. or *Duil.*			the fourth period is of thouſ. of thouſ. of thouſ. or thouſands of millions			the third period is thouſan. of thouſ. or *Million.*			the 2d period of thouſands.			the firſt period of unites.		

16. From hence all *Quantities* take their *Denomination.*

17. All Quantities under the number of 10 are expreſſed by figures under (u) or in the Unites place; all above 10 and under 100 with Figures under (a) in the Tens place, and with Cyphers or Figures in the place (u) that is Tens and Unites; and ſo of the reſt.

18. If the Quantity conſiſts of two Figures, as of 57, it ſignifieth fifty ſeven, as if it ſhould be 50 and 7. if of three Figures as 789, it ſignifieth ſeven hundred eighty and nine, as if it ſhould be 700, 80, and 9. and after this manner are all Quantities valued that fall in the firſt period.

19. If the Quantity fall in the ſecond, third, *&c.* as conſiſting of more than three Figures, note this

this always, that after you have diſtinguiſhed the Figures into periods, as in the Table, the Figure laſt in each period gives diſtinction or name to the reſt in that period, as 75 : 221. ſignifieth 75 thouſ. 221. and 325 : 317. ſignifieth 325 thouſand 317. Likewiſe 1: 758: 327: 821. ſignifieth 1 thouſand of Mill. 758 Mill. 327 thouſand, 821. that ſo having learned the value of three Figures, the Values of the reſt are eaſily known.

20. Thus are all whole *Numbers* expreſſed in one rank of Figures. But *Fractions* are expreſt in two ranks with a little Line between them. The lower being called the *Denominator*, and the upper the *Numerator*; as if the 17 twentieth parts of any thing were to be expreſſed, it is thus done.

17 *Numerator.*

---

20 *Denominator.*

21. An *Unite*, or any whole thing, may be conceived in the mind as diviſible into any equal parts whatſoever, and theſe parts borrow their *Name* or *Denomination* from the number of parts, ſuppoſed to be contained in that Unite.

As if the Unite be conceived to be divided into two parts, the parts are called ſeconds or halves, and the *Denominaeor* or *Name* of the parts will be the figure 2. thus $\overline{2}$. So if the Unite be ſuppoſed to be divided into three parts, the parts are called Thirds, and the Denominator 3, thus $\overline{3}$. If into 20 parts (as in the Example before) then the parts are called twentieth parts, and the Denominator $\overline{20}$.

22. The Number of parts ſignified in the Fraction is declared by the Numerator. For that

part of the Fraction ſheweth always how many of thoſe Parts named are underſtood.

As if an Unite were ſuppoſed to be divided into 3 parts, and one of thoſe three parts were to be expreſt, it muſt thus be done; with the Numerator 1 over the Denominator 3, *viz.* $\frac{1}{3}$ one third part, which of one ſhilling taken as the Unite and divided into 3 parts is 4 pence.

If two of theſe third parts were to be expreſt, it muſt be thus; with the Numerator 2 over the Denominator 3, *viz.* $\frac{2}{3}$, 2 third parts, which of a Shilling taken as before is 8 *d.*

More Examples.

$\frac{1}{2}$ One half. $\frac{1}{5}$ one fifth part, $\frac{2}{5}$ parts, $\frac{3}{5}$ parts, $\frac{4}{5}$ four fifth parts, $\frac{5}{7}$ five ſeventh parts, $\frac{1}{4}$ one fourth part, or one quarter.

$\frac{3}{4}$ Three fourth parts, or 3 quarters, $\frac{1}{6}$ one ſixth part, which of a Crown is 10 pence, $\frac{5}{6}$ five ſixth parts, which of a Crown is 50 *d.* or 4 *ſh.* 2 *d.* $\frac{1}{7}$ one ſeventh part, $\frac{7}{10}$ ſeven tenth parts, $\frac{37}{100}$ thirty ſeven hundredth parts, $\frac{375}{4789}$ three hundred ſeventy and five, four thouſand, ſeven hundred eighty ninth parts.

Where in this laſt Example, the unite is ſuppoſed to be divided into 4789 parts, and of theſe parts 375 are ſignified by the Fraction.

23. Thus are *Fractions* expreſt at large, but we uſe a more brief Expreſſion of ſome kinds of them by omitting the *Denominators*, when the parts are commonly known, and have names either Artificial or Inartificial, and that there be ſome mark or ſign to diſtinguiſh them. As

As becauſe the twentieth parts of a Pound are famouſly known by the name of Shillings, we uſe to ſay and write 2 *s.* 3 *s.* 4 *s.* 5 *s.* 6 *s.* 7 *s.* 8 *s.* 17 *s.* briefly with the Numerators only, and not fully with the Denominators, thus $\frac{2}{20}$, $\frac{3}{20}$, $\frac{4}{20}$, $\frac{5}{20}$, $\frac{6}{20}$, $\frac{7}{20}$, $\frac{8}{20}$, $\frac{17}{20}$, of a Pound.

The ſame we do alſo in the twelve parts of a Shilling, ſaying, 1 *d.* 2 *d.* 3 *d.* 5 *d.* 10 *d.* briefly with the Numerators only, and not fully thus, $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{10}{12}$, of a Shilling with the Denominator.

The like is ſaid and expreſt by the famous parts of Weights, Meaſures, Time, *&c.* Weights are commonly Troy, or Averdupois, either little or great, as in the Tables following.

1. *Table of Engliſh Coyn.*

1 Farthing	makes	1 Farthing.
4 Farthings		1 Penny.
4 Pence		1 Groat.
12 Pence		1 Shilling.
20 Shillings		1 Pound.

2. *Table of Troy Weight.*

24 Graynes	make	1 Penny weight.
20 Penny Weight		1 Ounce.
12 Ounces		1 Pound Troy.
14 Ounces 12 penny weight		1 Pound Averdupois.

3. *Table of Averdupois.*

16 Drams	make	1 Ounce Averdupois
16 Ounces		1 Pound Averdupois
14 Pound		1 Stone
2 Stone 28 ℔		$\frac{1}{4}$ Of an Hundred
4 Stone 56 ℔		$\frac{1}{2}$ An Hundred
8 Stone 112 ℔		1 Hundred
5 Hundred		1 Hogſhead
10 Hundred		1 Butt or Pipe
20 Hundred		1 Tun or Load.

4. *Table of Apothecarys Weights.*

24 Grains of Wheat	make	1 Scruple
3 Scruples		1 Dram
8 Drams		1 Ounce
12 Ounces		1 Pound.

All which are expreſt in Account without Denominators by Characters thus.

A Pound	by	℔	A ſcruple	by	℈
A Shilling		s	A dram		ʒ
A Penny		d	A quarter		qr.
$\frac{1}{2}$ Penny		ob	A crown		△
An Ounce		℥	An Hundred weight		C
A Grain		gr.			

5. *Table of long Measure.*

3 Barly Corns	make	1 Inch
4 Inches		1 Palme
12 Inches or 3 Palmes		1 Foot
3 Feet		1 Yard
3 Feet and 9 Inches		1 Ell
5 Feet		1 Geo. Pace
6 Feet or 2 Yards		1 Fathom
$5\frac{1}{2}$ Yards		1 Perch or Pole
40 Perches or 132 Paces		1 Furlong
8 Furlongs or 320 Perches		1 Mile
3 Miles		1 League

6. *Table of dry Measures.*

2 Pints or Pounds	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel Land-Measure
5 Pecks		1 Bushel Water-Measure
4 Bushels		1 Coombe
2 Coombes		1 Quarter
4 Quarters		1 Chalder
5 Quarters		1 Tun or Wey

7. *Table*

7. *Table of liquid Meaſures.*

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon (Herring
8 Gallons		1 Firkin of Ale, Soap,
9 Gallons		1 Firkin of Beer
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrell 36 Gallons
42 Gallons		1 Teirce
63 Gallons		1 Hogſhead
2 Hogſheads		1 Pipe or Butt
2 Butts 252 Gall.		1 Tun

8. *Table of Time.*

60 Seconds	make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day natural
7 Days		1 Week
4 Weeks		1 Month
13 Months 1 day		1 Vulgar year, or 365 days.

The comparing of theſe Tables with Forrain accounts ſhall follow in its due place.

23. From this *Abreviation* of *Fractions* ariſeth that late moſt uſeful Invention of *Decimal*

Arithmetick,

Arithmetick, where the Denominators are always 1 with Cyphers, 10, 100, 1000, 10000, *&c.* and therefore the Denominators are quite omitted (as always certainly known being ſtill more by one place than the Figures of the Numerator) and the Numerators only ſet down, as ſhall be more fully ſhewed hereafter.

24. Hitherto hath been ſpoken of *Notation certain*, and *determinate.*

Now

Notation, uncertain, undeterminate, and *arbitrary*, is when any Quantity, either Magnitude or Number, whole or broken, for the time preſent (or during the working of a Queſtion) is noted by any Letter of the Alphabet, as by *A. B. C. D. E. a. b. c, d. e,* &c. And when the Queſtion is ended, and another Queſtion is in working, the ſame Letters may ſignifie other Magnitudes or Numbers, according to the will of the *Arithmetician.*

25. The Letters thus arbitrarily ſignifying magnitudes or numbers, are called their *Species*, and the ordering theſe in Arithmeticall form is called in Latine *Arithmetica ſpecioſa*, or *Logiſtica ſpecioſa*, and in Engliſh I call it, Arithmetick in *Species.*

26. The expreſſion of quantity or number in this way is moſt doctrinal and of great eaſe; As in the fourth part you ſhall find, that any Arithmetical queſtion propounded, though otherwiſe very difficult, may eaſily receive ſuch a Reſolution in *Species*, that thereby in numbers when you pleaſe

ic

it may be anſwered ; beſides the infinite help to Memory in the Invention of many Rules therein, concerning which more in its proper place.

Thus we have done with the Notation of Quantities in Numbers.

27. *Numeration* is that by which two Numbers being given, we do rightly find out the third, and is comprhended under *Compoſition* and *Diſſolution.*

28. *Compoſition* comprehends *Addition* and *Multiplication* ; *Diſſolution*, *Subtraction* and *Diviſion.*

Addition and *Subtraction* being accounted the prime and ſimple parts of *Numeration* ; *Multiplication*, and *Diviſion* the *Conjunct* : and now we are to proceed to the reſt of the work.

But for the better remembrance of that which hath been ſpoken, and the eaſier underſtanding of that which hereafter is to be taught, it will be neceſſary to proceed in ſuch an eaſie Method, as may ſooneſt be imprinted in the Memory of him that would deſire with ſpeed to be an underſtanding *Arithmetician.*

Thus having ſhewed (as it were) the Sum of all the Arithmetician's Task, we will proceed to declare the manner of Operation in all the particulars mentioned, obſerving this order in this firſt part, *viz.* Firſt of the four principal parts of Numeration, *viz.* Addition, Subſtraction, Multiplication and Diviſion, in Integers, with all ſuch eaſie Rules as may advantage the Memory herein ; then of Proportion, and other Rules

thereon

thereon depending, together with the ſame four parts again, in vulgar Fractions, and parts of Numbers; ſecondly, the Doctrine of decimal Fractions; thirdly, the uſe of the Logarithms, or Artificial Numbers; and then to the fourth part in Species, and ſo by Gods aſſiſtance, paſs through all the uſeful parts of Arithmetick in order.

CHAP. II.

Addition of Integers.

1. ADdition is that part of *Numbring* or *Numeration*, whereby two or more Numbers are added together, and ſo the total or ſum of them is found.

2. For the eaſie apprehending of Addition and Subſtraction, conſider theſe pricks ſtanding directly all in ſtraight lines, which we call Ranks, and every prick directly over another, which we call Fyles.

3. In Addition of Numbers the Numbers to be added are to be ſo orderly placed in Ranks and Fyles, as that all the Figures of the ſame place muſt ever

ever ſtand in the ſame Fyle, and then a line is to be drawn under them, remembring to ſet Unites under Unites, Tens under Tens, *&c.*

4. Then beginning at the right-hand Fyle all the Figures therein are to be added together, and their Sum (if it conſiſt but of one Figure) is to be ſubſcribed under the line, and in the ſame Fyle, the ſame muſt be done in every other of the Fyles; but if the Sum of the places or Fyle conſiſt of two Figures, that is, amount above 9; then the right-hand Figure is only to be ſet down, and the left-hand Figure to be reſerved, and added to the next place or Fyle.

5. In Addition all the Numbers taken together are equal to the Sum.

Examples.

42	9851	1646	981275
56	7850	345	112345
98	182	98	2981
	17883	1	32972
		2090	5498
			957
			82
			3981
			1140091

Explanation of the Examples.

In the firſt Example it is deſired, that 42 and 56 may be added together, ſetting them as in the Example in Rank and Fyle, ſo that always Unites may ſtand under Unites; we ſay in the laſt

Rank,

Rank, 6 and 2 makes 8 which is ſet under the line in that Fyle, then 5 and 4 makes 9, which is ſet under his own Rank. So is their Sum 98.

In the ſecond Example,it is deſired to add 9851, and 7850, and 182, in one Sum, ſetting Unites in one Fyle, and the reſt as in the Example. I ſay 2, and 0, and 1, makes 3, to be ſet in the ſame Fyle; then I ſay 8, and 5, and 5, makes 18. I ſet 8 in that Fyle, but carry the 1. to the next, (which is to be obſerved in all) then I ſay 1. that I carried, and 1, and 8, and 8, makes 18. I ſet 8. in his Fyle,and carry 1.Laſtly,1.that I carried, and 7. and 9. makes 17. both which are ſet down in their proper places, as in the Example, and their ſum is 17883.

The reſt of theſe Examples are wrought as is ſet down, in the foregoing Rule, *viz.*

In the third example, where it is required to add 1646 and 345 and 98 and 1 together, in order to find out their Sum; to perform which I firſt place the number 1646, and then the number 345 under it, in ſuch order that the Figure (5) which is in the Rank of Unites, may ſtand under (6) the place of Unites, in the firſt number; and likewiſe (4) the place of Tens in the ſecond number under (4) the place of Tens in the firſt; and (3) the place of hundreds in the ſecond Number under (6) the place of hundreds in the firſt;then I proceed to place down the other two Numbers *viz.* 98 and 1, ſtill obſerving the ſame order of placing Unites under Unites, Tens under Tens, *&c.* as before hath been taught; and then proceed to the Operation, ſaying, 1 and 8 makes 9, and 5 is

is 14, and 6 is 20; now becauſe 20 conſiſteth of two Tens and nothing over, therefore I ſet(o)under the line in the Rank of Unites, and carry 2 (for the two Tens) to the next Rank; ſaying 2 that I carry and 9 is 11, and 4 is 15, and 4 makes 19, which is 9 more than one 10, therefore I ſet down 9 under the Line in the place or Rank of Tens, and for the one Ten, I carry 1 to the next Rank; and proceed ſaying 1 that I carry and 3 is 4, and 6 is 10, then I ſet o under the Line in the Rank of Hundreds,(becauſe the ſum of that Fyle is even 10) and for the Ten I carry 1 to the next; and proceed ſaying, 1 that I carry and 1 is 2, which I ſet in the Rank of Hundreds under the Line, and ſo is the whole Work finiſhed; and I find the ſum of all the ſaid given numbers to be 2090, as appears in the Example. And here

Note, that when it is required to add ſeveral Numbers together, in placing them one under another in order to Addition, it matters not which of the Numbers you place firſt or uppermoſt; whether a number of one or two places, or one of five or ſix places, if any ſuch be given; for the Operation will ſtill be the ſame, and their Sum the ſame, if the third Rule of this Chapter be obſerved in placing them ; which indeed is all that is herein required, as you may ſee in the fourth Example foregoing; where the two firſt numbers that are placed conſiſt of 6 places, and the third conſiſteth but of 4 places, and the fourth again conſiſteth of 5 places, &c. I thought it neceſſary to give this Caution, becauſe I have known young Learners oftentimes puzled in the placing of their numbers,

for

for when they have found their Sum to be false according to their Proof, they have been apt to imagine their Error to have proceeded from a wrong placing of their given numbers. Observe the following Examples, where the Numbers to be added are placed promiscuously.

	34		9
	176		17
	9		348
	168		16741
	45		37648
Sum	432	*Sum*	39696

Here followeth Addition of Numbers having several Denominations, and first of

Addition of Money.

WHen it is required to add together several Sums of Money, that are of divers Denominations, that is to say, if they consist of Pounds and Shillings, or of Pounds, Shillings and Pence, or of Pounds, Shillings, Pence and Farthings, or in Weight of Pounds, Ounces, Penny weights, and Grains, *&c.* You are then (in the placing of the given Numbers in order to Addition) to observe the following

Rule.

Let every Denomination be placed under its

Correspondent Denomination, that is to say, let Pounds be placed under Pounds, Shillings under Shillings, Pence under Pence, and Farthings under Farthings, *&c.*

Then draw a Line underneath the Work, and begin at the lowest Denomination, *viz.* that on the Right hand, and add them all together, then consider how many Unites in the next superior Denomination are contained therein, and set down the Surplusage (over and above the Unites) underneath the said Line under its proper Denomination, and carry the Unites to the next, *&c.* proceed after the same manner till the whole Work is completed.

As in the following Example, where it is required to add 126 *l.* 13 *s.* 08 *d.* 3 *qrs.* and 240 *l.* 10 *s.* 11 *d.* and 64 *l.* 08 *s.* 07 *d.* 2 *qrs.* and 9 *l.* 11 *s.* 06 *d.* 1 *qr.* and to find out their Sum.

In order whereunto I first place the given Numbers one under another, in order as is before directed, and as you see in the following Work.

l.	*s.*	*d.*	*qrs.*
126	13	08	3
240	10	11	0
64	08	07	2
9	11	06	1
441	04	09	2

Then I begin at the Right hand to add them together, *viz.* at the Denomination of Farthings, saying

ſaying, 1, 2, and 3 Farthings are ſix Farthings, which is one Penny and two Farthings, wherefore I put down the two farthings under the Denomination of Farthings, and carry the Penny to the Denomination of Pence; and proceed, ſaying, 1 that I carry and 6 is 7, and 7 is 14, and 11 is 25, and 8 makes 33 Pence, which is 2 Shillings and 9 Pence, wherefore I put the 9 Pence down under the Denomination of Pence, and carry the 2 Shillings to the Denomination of Shillings, and proceed ſaying, 2 that I carry and 11 is 13, and 8 is 21, and 10 is 31, and 13 is 44 Shillings; which is 2 Pounds 4 Shillings, wherefore I put down 4 under the Denomination of Shillings, and carry 2 to the Denomination of Pounds; and proceed, ſaying, 2 that I carry and 9 is 11, and 4 is 15, and 6 is 21, wherefore I put down 1 under the Line, and carry 2 (for the two tens) to the next File, and proceed ſaying 2 that I carry and 6 is 8, and 4 is 12, and 2 is 14, wherefore I put down 4 in its proper place under the Line, and carry 1 (for the ten) to the next File; and proceed ſaying, 1 that I carry and 2 is 3, and 1 is 4, which I likewiſe place under the Line, and ſo the Work is finiſhed, and I find the Sum to be 441 *l.* 4 *s.* 9 *d.* 2 *qrs.* as you may ſee by the Example.

But it is a method uſed generally in the Writing-Schools, to make a Point at ſo many Unites in every inferior Denomination, as make an Integer in the next ſuperiour, and to carry ſo many Unites thereto as they make Points; and to proceed in the ſame manner, till they come to the greateſt, or higheſt Denomination, and then to carry an

Unite for every Ten, till the Work be finished; as in the following Example.

Let it be required to add 345 *l.* 16 *s.* 10 *d.* 3 *qrs.* and 126 *l.* 14 *s.* 07 *d.* 2 *qrs.* and 624 *l.* 07 *s.* 06 *d.* 2 *qrs.* and 240 *l.* 06 *s.* 08 *d.* 2 *qrs.* together, and to bring them to one Total Sum.

First they are to be placed one under another in order, as hath been before directed, which is as you see in the following Operation.

	l.	*s.*	*d.*	*qrs.*
Sums to be added.	345	16.	10.	3.
	126	14.	07	2
	624	07	06.	2.
	240	06	08	2
Total Sum	1337	05	09	1

The Example wrought.

First, beginning at the Denomination of Farthings, I say 2 and 2 make 4, and because 4 Farthings is a Penny, I put a Point at the second 2; then I proceed saying 2 and 3 are 5, wherefore I put a Poynt at the 3 for the Peny, and place the odd Farthing under the Line, and under the Denomination of Farthings; and because there is two points made in the Farthings, I carry 2 to the Pence, and proceed saying, 2 that I carry and 8 make 10, and 6 is 16, now because 16 Pence is 1 Shilling

1 Shilling 4 Pence, I put a Point at 6, and carry 4 to the next Figure ſaying, 4 and 7 is 11, and 10 is 21, and becauſe 21 Pence is 1 Shilling 9 Pence, I make a Point at 10, and put the 9 Pence under the Line, and under the Denomination of Pence, and for the 2 points made in the Pence, I carry 2 to the Denomination of Shillings, and proceed to the Shillings ſaying, 2 that I carry and 6 is 8, and 7 is 15, and 14 is 29; now becauſe 29 Shillings is 1 Pound 9 Shillings, I put a Point againſt 14, and carry the 9 to the next Number ſaying, 9 and 16 make 25, which is 1 Pound 5 Shillings, wherefore I put a Point at 16, and place the 5 Shillings under the Line, in the Denomination of Shillings, and for the two Points therein I carry 2 to the Denomination of Pounds, and proceed ſaying, 2 that I carry and 4 is 6, and 6 is 12, and 5 is 17, wherefore I put down 7 under the Line, and carry 1 (for the 10) to the next place ſaying, 1 (that I carry) and 4 is 5, and 2 is 7, &c. as hath before been directed; ſo that the Work being compleated, the Sum is found to be 1337 *l.* 05 *s.* 09 *d.* 1 *qr.* as appears by the Example.

More Examples wrought the ſame way are ſuch as follow.

Examples.

	l.	*s.*	*d.*	*qrs.*	*l.*	*s.*	*d.*	*qrs.*
	340	12.	05.	1	160	08	06	3.
	629	07	09.	3.	92	18.	08.	1
	75	15.	11	0	18	12.	10.	2.
	13	14.	08.	3.	64	07	09	0
	54	06	10.	2	13	13.	03.	1
	9	09	04	1	61	08	07	2
Sum	1123	07	01	2	411	09	09	1

This last way (of placing Farthings by themselves, under aparticular Denomination, and likewise of making pricks in the Farthings at every (4) Peny, and in the Pence at every (12) Shilling, and in the Shillings at every (20) Pound,) is generally taught in the Writing-Schools, but is not practised by Merchants, Tradesmen, Book-keepers, *&c.* But their Method is, to place the Farthings close to the Denomination of Pence, *viz.* for one Farthing they set $\frac{1}{4}$, for 2 Farthings, or a Half-Peny they put $\frac{1}{2}$, and for three Farthings they put $\frac{3}{4}$, as for Example, if I would set down six Pence Farthing, it is thus done $06\frac{1}{4}$ with the mark of Pence over the 6; for four Pence Half-Peny, I would put down $04\frac{1}{2}$, with the mark of Pence over the 4, and two Pence three Farthings I would put down thus, *viz.* $02\frac{3}{4}$ setting the mark of Pence over the Figure 2, *&c.* And when in any Addition of Money there are Farthings, they may very easi-

ly

ly be caſt up without putting a prick at every 4, and when caſt up, you may eaſily tell how many Pence they make (without breaking your Brain) which being known, ſet down the Farthings under the Line, and carry the Pence they make to the Denomination of Pence, and then proceed to caſt up the Pence, as is hereafter Taught; but firſt learn the following Table, to ſay it perfectly by heart, *viz.*

d.		*s.*	*d.*
20		01	08
30		02	06
40		03	04
50		04	02
60		05	00
70	is	05	10
80		06	08
90		07	06
100		08	04
110		09	02
120		10	00

In which Table the Numbers on the left Hand of the Line of Connection are all Pence, and of thoſe on the right hand of the connecting Line, the firſt are Shillings, and the ſecond are Pence, and therefore is the Table thus to be Read, *viz.* 20 Pence is 1 Shilling 8 Pence; 30 Pence is 2 Shillings 6 Pence, *&c.*

When the foregoing Table is learnt perfectly by heart, let it not trouble thee to have perfectly in the Mind the following Table, which every

Plough-man, and Shepherd hath at his Fingers Ends, *viz.*

s.		l.	s.
20	is	1	00
30		1	10
40		2	00
50		2	10
60		3	00
70		3	10
80		4	00
90		4	10
100		5	00
110		5	10
120		6	00
130		6	10
140		7	00 *&c.*

Which Table is thus explained, the Numbers on the left Hand of the Line of Connection, *viz.* 20, 30, 40, *&c.* are Shillings, and of those on the right Hand the first are Pounds, and the second are Shillings, and it is thus to be read, *viz.* 20 Shillings is 1 Pound, 30 Shillings is 1 Pound 10 Shillings, 40 Shillings is 2 Pounds, *&c.*

Which two Tables being perfectly learned by heart, I will now lead thee (as it were by the Hand) to the most practical way of working a Sum in Addition of Money for which this is

The

The Rule.

Having caſt up your Farthings (as before was taught, proceed to the Denomination of Pence, and firſt run up the place of Unites, (that is, the Figures on the Right hand) and then you may come back again down the place of Tens, as faſt as you can ſpeak; then by the help of the firſt of the foregoing Tables, you may eaſily tell how many Shillings are contained in the Pence; therefore put the odd Pence (if there be any) under the Line,and carry the Shillings to the Denomination of Shillings, and run up the place of Unites in the Shillings,and back again down the place of Tens; which being done, by the help of the ſecond of the foregoing Tables,you may tell how many Pounds are contained in the Shillings; then put the odd Shillings (if there be any) under the Line, and carry the Pounds to the Denomination of Pounds, and then finiſh the Work by the former Rules. This Rule may eaſily be underſtood,by the Directions given, in operating the two following Examples.

1. *Example*

1. *Example.*

l.	*s.*	*d.*
128	16	$11\frac{1}{4}$
346	13	09
274	09	$10\frac{1}{2}$
148	12	$06\frac{3}{4}$
279	08	08
430	14	$10\frac{1}{4}$
164	07	07
140	04	$03\frac{1}{2}$
1913	08	$06\frac{3}{4}$

2. *Example.*

l.	*s.*	*d.*
384	16	$04\frac{1}{2}$
194	07	10
468	15	$07\frac{3}{4}$
194	08	11
439	18	$06\frac{1}{4}$
79	06	05
84	14	$03\frac{3}{4}$
18	10	09
1864	18	$09\frac{1}{4}$

In the first Example, I begin with the Farthings thus, 2 and 3 make 5 and 3 make 8, and 2 make 10,

10, and 1 makes 11 Farthings in all, which is 2 Pence 3 Farthings, wherefore I put down the 3 Farthings under the Line, and carry the two Pence to the Pence, and proceed ſaying, 2 that I carry and 3 are 5, and 7 are 12, and 8 are 20, and 6 are 26, and 9 make 35, and 1 are 36, ſo I am now come to the Top or uppermoſt Figure; then I go again downwards, Reckoning for every Unite 10, in the place of Tens thus 36 and 10 make 46, and 10 are 56, and 10 are 66; ſo that in the Denomination of Pence there are 66, now (by the firſt of the foregoing Tables) 60 Pence is 5 Shillings, and 6 Pence is 5 Shillings ſix Pence; wherefore I put 6 under the Line, and under the Denomination of Pence, and carry the 5 Shillings to the place of Shillings, and proceed ſaying, 5 that I carry and 4 is 9, and 7 is 16, and 4 is 20, and 8 is 28, and 2 is 30, and 9 is 39, and 3 is 42, and 6 is 48; then I run down the place of Tens back again as before in the Pence, ſaying, 48 and 10 is 58, and 10 is 68, and 10 is 78, and 10 is 88 Shillings, which (by the ſecond of the foregoing Tables) is 4 Pounds 8 Shillings, wherefore I put 8 under the Line, and under the Denomination of Shillings, then I proceed to caſt up the Pounds, and ſay 4 (that I carry) and 4 is 8, and 9 is 17, *&c.* as hath been before directed; ſo that when the Work is finiſhed, the Sum will be found to be 1913 *l.* 08 *s.* $06\frac{3}{4}$.

Again, in the ſecond Example, caſting up the Farthings as before in the firſt, I find them to be 9, which is $2\frac{1}{4}$ *d.* wherefore I put down 1 Farthing under the Line, and carry 2 to the Pence, and having

having cast up the place of Unites in the Pence I find them to be 37, then run down the Tens saying 47, 57, now 57 Pence is 4 *s.* 9 *d.* wherefore I put 9 under the Denomination of Pence, and carry 4 to the Shillings, and having cast up the place of Unites in the Shillings I find them to be 48, then I run down the place of Tens, saying 58, 68, 78, 88, 98; now 98 Shillings is 4 *l.* 18 *s.* wherefore I put 18 under the Shillings and carry 4 to the Pounds, which being cast up, as is before directed, amount to 1864 *l.* so that the total Sum is 1864 *l.* 18 *s.* 09¼ *d.*

Other Examples for Practice.

l.	*s.*	*d.*		*l.*	*s.*	*d.*
186	17	03¼		440	14	06
438	19	11		39	07	10½
374	14	09½		46	12	05¼
94	07	10		34	15	11
47	08	05¾		8	05	04
12	12	04		3	07	06
1155	00	07½		573	03	06¾

Addition of Troy Weight.

In Addition of Troy Weight, the operation is the same as in Addition of Money, only observing in casting up the Grains to carry 1 for every 24, to the peny Weights, and for every 20 in the peny Weights to carry 1 to the Ounces, and for every

every 12 in the Ounces to carry 1 to the Pounds, *&c.* as in the following Examples.

	l.	*oz.*	*pw.*	*gr.*	*l.*	*oz.*	*pw.*	*gr.*
	136	10	15	11	36	04	13	09
	248	09	13	21	18	10	15	10
	364	04	10	14	34	07	06	08
	194	11	19	16	41	06	16	16
	268	07	05	08	15	11	12	13
	94	04	08	10	64	07	17	20
Sum	1308	00	13	08	212	01	02	04

Addition of Averdupois Weight.

In Averdupois Weight the particular Sums are to be added together by the former Rules, obſerving to carry 1 to the Quarters for every 28 in the Pounds,and to carry 1 to the Hundreds for every 4 in the Quarters, and 1 for every 20 in the Hundreds to the Tuns, *&c.* as in the following Examples.

Tun	*C.*	*qrs.*	*l.*	*C.*	*qrs.*	*l.*	*oz.*
126	13	1	20	17	1	24	15
347	15	3	25	47	3	18	11
165	10	2	16	36	1	17	07
225	09	0	10	25	0	15	14
865	09	0	15	126	3	20	15

I might proceed to give many more Examples, of Weight, Meaſure, Time, Motion, *&c.* But what hath been already done is ſufficient for the meaneſt Capacity; There being no more difficulty therein than in the former Examples, always noting thus much, to have a care to carry ſo many of the next Denomination, as are contained in that Row you add up, and ſet down the Remainder.

The Proof of Addition.

When you prove the Work of Addition, ſeperate the uppermoſt Line from the reſt by a Daſh of the Pen, and then caſt up the Sum again as before, leaving out the uppermoſt Line, and the Amount thereof add to the uppermoſt Line, and if that Sum be equal to the Sum firſt found, the operation is Right, but otherwiſe not.

l.	*s.*	*d.*
126	13	10$\frac{1}{2}$
347	14	07
424	09	09$\frac{3}{4}$
248	15	11
1147	14	02$\frac{1}{4}$
1021	00	03$\frac{3}{4}$
1147	14	02$\frac{1}{4}$

As in the Example in the Margent, the Sum is found to be 1147 *l.* 14 *s.* 02$\frac{1}{4}$ *d.* to prove which I cut off the uppermoſt Line 126 *l.* 13 *s.* 10$\frac{1}{2}$ *d.* and caſt up the ſame again leaving it out, and the Sum then is 1021 *l.* 00 *s.* 3$\frac{3}{4}$ *d.* which being added to the Number before left out, *viz.* 126 *l.* 13 *s.* 10$\frac{1}{2}$ *d.* that Sum is 1147 *l.* 14 *s.* 02$\frac{1}{4}$ *d.* equal to the

the Sum first found, wherefore I conclude the work to be true.

CHAP. III.

Substraction of Integers.

1. *Substraction* is that part of Numeration, where one Number is substracted or taken out of another, and so the Remainder is gotten, which is also called the Difference or Excess.

2. In Substraction of Numbers, the lesser is to be set down under the greater, as in Addition, orderly observing to set Unite under Unite in Rank and Fyle, and a Line is to be drawn under both.

3. Then beginning at the right Hand, the particular Difference of each place, are to be found by substracting the lower Figure from the higher, and to be subscribed in their proper places.

4. But if the higher Figure be less than the lower, 10 must be put to the higher Figure, that the Remainder may be taken and subscribed; and then in the next place toward the left Hand, either the higher Figure is to be accounted 1 less than it is; or else (which is more usual) the lower Figure is to be accounted 1 more than it is, and then the Substraction is to be made.

5. In Substraction, the Number to be substracted,

ſtracted, together with the Difference, are equal to the Number from which the Subſtraction is made.

Examples of Subſtraction.

58	456	4562	513217	1659
(1) 32	(2) 321	(3) 1368	(4) 159832	(5) 1588
Rem. 26	135	3194	353385	71

	1000000
(6)	325782
	674218

In the firſt Example it is deſired to ſubſtract 32 from 58, I ſet 32 under 58 in Rank and Fyle, then I ſay 2 from 8 reſts 6, and 3 from 5 reſts 2, which I ſet down, as in the Example; ſo that the Difference or Remainder is 26, and of the ſecond Example the Remainder is 135. In the third Example it is deſired to ſubſtract 1368 from 4562, the which after it is ſet down is done thus, 8 from 2 I cannot, and therefore I borrow 10 to make it 12, and ſay 8 from 12 and there remains 4, to be ſet under the Line in Fyle; then I ſay 1 that I borrowed, and 6 is 7, which I cannot take from 6, but out of 16 borrowing 10, and there remains 9, then 1 I borrowed and 3 is 4, which from 5 reſts 1. And laſtly, 1 from 4 reſts 3, and the Remainder is 3194, as in the Example.

If any Numbers, as in the ſixth Example, are to be ſubſtracted from 1, and Cyphers, as it ſometimes falls out in Decimals, but moſt commonly in the Logarithms; make the laſt figure equal to

10

10, ſetting down the Remain, and all the reſt make 9, as 1 make 2, 10, by ſetting 8 under, and all the other 9 by ſetting the Remain. View the Example.

Subſtraction of Money.

6. If the Number, out of which Subſtraction is to be made, be of diverſe Denominations, as of Money, Weight, Meaſure, Time, *&c.* The work is ſtill the ſame, as before (the given Numbers being placed as is directed in Addition) as in the firſt of the following Examples; except the Number, out of which you are to ſubſtract, be leſſer than the Number to be ſubſtracted, and then inſtead of borrowing 10, as by the former Rule, you muſt borrow ſo many as make an Unite of the next Denomination, and add to it, and then ſubſtract from that Sum, and place the Remainder under the Line, as before was taught. As if in Pence you muſt borrow 1 *s.* or 12 Pence, and in Shillings 1 *l.* or 20 Shillings.

	l.	*s.*	*d.*		*l.*	*s.*	*d.*
	375	11	5		1754	11	2
(1)	132	9	4	(2)	982	13	5
	243	2	1		771	17	9

As in the ſecond Example I am to take 5 *d.* from 14 *d.* becauſe I borrow 12 *d.* reſt 9, then 13 *s.* and 1 *s.* I borrowed makes 14, which from

31 rest 17 because I borrowed 1 *l.* or 20 *s.* then in the pounds; I say 1 that I borrowed and 2 is 3, which taken from 4 rest 1, *&c.* and so on as in whole Numbers.

And note here, that it is better to take the lower from that you borrow, and add the Remainder to the Number set down, and write the Sum in the Remain; thus to say, 5 from 12 rest 7, which with 2 makes 9 to be set down, then 14 from 20 rest 6, which with 11 makes 17 to be set down as above; And this is the Method used by the best Accomptants.

More Examples.

	l.	*s.*	*d.*
Received	7854	11	$9\frac{1}{2}$
Disbursed at several times	321	15	5
	781	19	$8\frac{1}{4}$
	983	00	1
	783	11	$10\frac{1}{4}$
Sum	2870	7	1
Remains	4984	4	$8\frac{1}{2}$
Proof	7854	11	$9\frac{1}{2}$

Of Troy Weight.

l.	*oz.*	*d.*	*gr.*
17	9	13	22
15	11	19	20
1	9	14	2

Of Time.

da.	*ho.*	*min.*
312	13	15
112	16	45
199	20	30

More Examples may be added, but these are sufficient.

7. The proof of Substraction is, that the lesser Number and Remain must always make up the higher Number, which is soon done by Addition, as in the last Example of Substraction of Money, the Number substracted is 2870 *l.* 07 *s.* 01 *d.* and the Remainder is found to be 4984 *l.* 04 *s.* $08\frac{1}{2}$ *d.*

which

which being added to the Number that was ſubſtracted, the Sum is 7854 *l.* 11 *s.* 09½ *d.* which is equal to the Number out of which the Subſtraction is made, and therefore I conclude the Work to be rightly performed.

8. And here the young Arithmetician may receive ſome comfort, for that he can now work ſome Queſtions of uſe by Addition, and Subſtraction only:

Examples.

One ſaith my Father was born in the firſt year of *Henry* the 8; how long, or how old would he have been in this year 1686? I find that *Henry* the eighth began his Reign in 1509, the which being ſubſtracted from the year preſent 1686, reſteth 177; which is the time ſince, and would have been the age of that Man if he had lived.

And he may ſum up the ſeveral Pages of an Account-Book in *l.* *s.* and *d.* he may likewiſe tell the difference of any two Numbers; as in the laſt Example of plain Numbers; how long ſince 1588, the year that the *Spaniard* thought to have invaded *England*, which being ſubſtracted from 1686 reſts 98.

In the third Example of Subſtraction of Money, if 7854 *l.* 11 *s.* 9½ *d.* were given in Stock, you have disburſed at ſeveral times, as appears 2870 *l.* 7 *s.* 1 *d.* which ſubſtracted from the Principal reſt 4984 *l.* 4 *s.* 8½ *d.* yet remaining to account for.

9. Of *Bypartition*; I muſt here a little digreſs from the order, and teach how to take the half

of any Number, or to divide a Number by two, which is thus: If the Number consist of even figures, take the half thereof, and set it under the Number; but if it consist of uneven Numbers, if you begin from the left hand taking the half of it, augment the Figure following by 10, if the Figure before were odd; and so go on.

Examples.

(1) $\frac{4862}{2431}$ (2) $\frac{47892}{23946}$ (3) $\frac{9011780}{4505890}$

(4) $\frac{2350}{1175}$

In the first Example set down only the half of each Number, in the second say the half of 4 is 2, of 7 is 3, of 18 is 9, of 9 is 4, of 12 is 6, and I termed 8 to be 18, because 7 was an odd Number.

10. The halfing of any Number from the left hand was shewed by the last, but it will be of more speed in the Works following, to do it from the right Hand, which is only by observing, that if the Figure standing in Rank next before be odd, you must account the Figure next after 10, more than it is, as in the second Example, beginning at the right Hand with 2: I say the the half of 12 (because 9 is odd) is 6, the half of 9 is 4, the half of 18 (because 7 is odd) is 9, the half of 7 is 3, the half of 4 is 2.

CHAP.

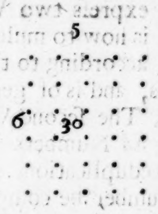
CHAP. IV.

Of Multiplication.

1. MUltiplication is a part of conjunct Numeration, or numbring, whereby the *Multiplicand* (which is the number to be multiplied) is so often added to it self, as an Unite is contained in the Multiplier (which is the Nnmber multiplying) and so the *Factus* (or Product) which is the Resnlt of the Work, is had.

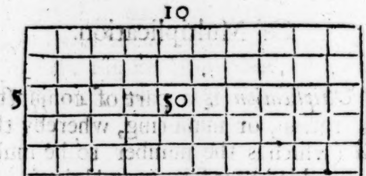
2. The Number thus found, is called the Product and *Factus*, that is the Number made, and then the Numbers to be Multiplied, are called the *Factores*, Makers. This Number, is also called the Rectangle, or the Plain, then the one of the Numbers is taken for the length, and the other for the breadth of a Rectangular Plain, as here,

In Unites 6 the Length, and 5 the Breadth the Plain being 30.



Or in little Squares made of the measure (such as is a Foot, or the like) 10 being the Length

in ſuch Meaſures, and 5 the Breadth, the Product being 50 of little Squares of the Rectangular Plain.



3. Multiplication and Diviſion, (being the only *Remora's* that perſwade thoſe who delight to take ſmall Journies, and little pains) from the knowledge of this ſweet and pleaſant Country of *Arithmetick*, conceiving it *Terra incognita*, I ſhall endeavour to render the eaſieſt ways that have yet been found out to perform the ſame, if the Logarithms could work Numbers to ſome few Periods, as to three, then I muſt confeſs great Labour might be ſaved; in the mean time, I ſhall expreſs two Ways: The firſt Way or Practice, is how to multiply any one Number by another, according to the Method uſed by all Accomptants, and is of general uſe, in all Caſes whatſoever; The ſecond Way or Practice is for multiplying all Numbers by Duplication, Triplication, Reduplication, and Bypartition.

4. If a Number be compounded of two Numbers, and that Number multiply another Number; the Product is equal to the Product of that Number multiplied by thoſe two Numbers,

Example.

Example.

6 is compounded of the Product of 2 and 3, let 6 multiply 9, the Product is 54, which is equal to 9 times 2, that is 18, and 18 times 3, which is 54.

5. The Multiplication of the single Figures by single Figures, or of any other Number, by single Figures, is first to be perfectly learned. The Multiplicand is to be written down, and the Multiplyer under it, which afterwards is taught.

I. Practice of Multiplication.

1. The ordinary way wherein consisteth the Multiplication of single Figures among themselves, is first perfectly to be learned by heart.

As 2 times 2 is 4, 2 times 3 is 6, 2 times 4 is 8, *&c.* 3 times 3 is 9, 3 times 4 is 12, *&c.* as in this Table is expressed, wherein you may enter with your two Figures, the one above the other on the side, and the Square answering to them both in the common Angle, contains the Product of these two Numbers.

Table of Multiplication.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

2. If two or more Figures be to be multiplied by one, the Multiplicand is to be written down, together with the Multiplier as before, and then every particular figure of the Multiplicand in order from the right Hand toward the left, is to be multiplied by that one Figure of the Multiplier, and the particular Products are to be set down in their places, if they be written with one figure, but if any be written with two figures, the right-hand figure only is to be set down, and the left-hand figure is to be reserved as in Addition, and added to the next particular Product; that is to say, 'tis to be added to the next Product for every 10 in the former. As for Example.

2132	31461
3	6
6396	188766

In the latter Example I say 6 times 1 is 6, which I set down, then 6 times 6 is 36, I set 6 down, and bear 3 in mind, then 6 times 4 is 24, and 3 that I carry is 27, I set 7 down and bear 2, and say 6 times 1 is 6, and 2 that I carry is 8, which is set down; and lastly say 6 times 3 is 18, which is also set down.

3. If the Multiplier consist of more figures than one, you must work as in the last Section by one figure, and set them orderly down, with this *Caution*, That the last figure of every *several Product* shall exactly be set down under that figure of the Multiplier which you work by: that being done, those *particular Products* are to be added, whose Sum will be the *Factus* or *total Product* of the whole Multiplication.

Example.

Multiplicand	32569
Multiplier	527
	227983
	65138
	162845
	17163863

Let 32569 be multiplied by 527, first, set them down as you were taught in Addition, and begin by 7, as before, and say 7 times 9 is 63: set down 3 and carry 6 in mind, 7 times 6 is 42, and 6 in mind makes 48. I set down 8 and carry 4 in mind, 7 times 5 is 35, and

and 4 is 39, I ſet down 9 and bear 3. then I ſay 7 times 2 is 14, and 3 is 17, I ſet down 7 and bear 1. Laſtly, 7 times 3 is 21, and 1 in mind is 22, which I ſet down as in the Example: ſo have I done with one of the Figures of the Multiplier, *viz.* 7.

Then in the ſame manner I multiply the Multiplicand by 2, and ſay 2 times 9 is 18, I ſet the 8 under the 2, and ſo proceed with the other figure 5, and laſtly add all together, which makes 17163863, as above, which is the general Product. So that I conclude 527 times 32569 is 17163863.

4. If there be Cyphers at the end of either or both, the *Multiplicand* or *Multiplier*, make the Multiplication in the whole Numbers, and at the laſt add ſo many Cyphers to the right Hand of the Product, as the number of thoſe were both in the Multiplicand and Multiplier. As if it were required to multiply 35000 by 7000 I only multiply 35 by 7, and to the Product (245) I annex 6 Cyphers, it makes 245000000 for the Product.

35000
7000
245000000

5. And if there be Cyphers in the middle part of the Multiplier, work as in this Example.

Example.

37825
7002
75650
26477500
264850650

6. The

6. The Memory will eaſily retain the Table for the firſt five parts, that is the ſingle Multiplication to 1,2,3,4,5, but for the eaſier remembring the Remainer, I uſe to teach my Scholars to do them by the Fingers, for let either Hand cloſed ſignifie 5, by raiſing the Thumb it will be 6, by raiſing the Thumb and one Finger 7, and ſo learn preſently to ſet 5, 6, 7, 8, and 9, on either Hand; which done, ſet the two Figures, whoſe Product you deſire, on both the Hands; all thoſe Fingers which are up are to be accounted ſo many Tens, and multiply thoſe which are down by themſelves, adding it to the Tens, it gives the Deſire.

Example.

If 7 times 9 be demanded, I ſet 9 on one Hand by cloſing only one Finger, and 7 by raiſing two; then on both Hands there are 6 raiſed, which makes 60, and multiply the Fingers down, *viz.* 1 by 3 makes 63, and ſo of any other

More Examples of Multiplication.

(1)

```
          0              74358
Proof   0 X 1             6472
          0            --------
                        148716
                       520506
                      297432
                     446148
                    ----------
Prod.               481244976
```

(2)

(2)

```
          2             3475864
 Proof  1 X 2            357482
          2            --------
                        6951728
                       27806912
                      13903456
                     24331048
                    17379320
                   10427592
                 --------------
          Product  1242558814448
```

Proof

```
     o          (3)              o           (4)
   o X 7       45000           o X 8        20007
     o          1600             o           3005
              -------                      --------
                 270                        100035
                45                         6002100
              ---------                   ----------
              72000000                    60:21035
```

The Proof of Multiplication.

First make a Croſs with your Pen, and then adding the Figures of the Multiplicand together (not regarding their places, but as if every Figure ſtood in the place of Unites) caſting away 9 as often as may be in the adding of them together, and the laſt Remainder put on the left ſide of the Croſs, then add together the Figures in the Multiplier,

Multiplyer, casting away the Nines as before, and place the last Remainder on the right side of the Cross over against the other; then multiply those two remainders together, and cast the Nines out of their Product, placing the Remainder on the top of the Cross, then (as before) cast the Nines out of the Product, and place the Remainder at the bottom of the Cross, and if the Remainders aforesaid at the top and bottom of the Cross, be equal, then you may conclude the work to be truly wrought, but if unequal 'tis certainly erroneous.

As in the second of the foregoing Examples, the Unites being cast out of (3475864) the Multiplicand there Remaineth 1, which I place on the left side of the Cross, then I cast the Nines out of (35748²) the Multiplier, and there remains 2, which is placed against the 1 found as before; then I multiply those two Remainders 1 and 2, and their Product is likewise 2, which is placed on the top of the Cross; then I cast the Nines out of (124255881448) the Product and there remaineth 2, which I put at the bottom of the Cross, and because there is 2 at the top of the Cross and 2 at the bottom, I may from thence reasonably conclude the operation to be true.

But Multiplication is most exactly proved by Division, as shall be shewn in the next Chapter.

II. *Practice.*

This, as it may be of great ease, so it is likewise

of great curiosity, and will be of great use to such as are daily concerned in accompts.

Duplication, or Multiplying by 2.

1. If any Figure or Number be to be multiplied by 2, beginning at the right Hand, call every Figure by a name double to it self, and set it down; if the Figure exceed 4, you must account the next on the left hand one more than the double.

Examples.

34231	3987217	590019
2	2	2
68462	7974434	1180038

In the first (in my mind) I double each Figure, saying, 2, 6, 4, 8, 6, and set them down, and note this, that you must not use to say 2 times 1 is 2, and 2 times 3 is 6, *&c.* nor to say 1 and 1 is 2, and 3 and 3 is 6, but (for the reasons following) double each Figure in your mind.

In the second I say 14 or 4, going one in mind, and 3 (because I carried 1) and 4 and 4 and 7, (because I borrowed 1) *&c.* the same is to be observed in the third Example, and hereof (as also of Bypartition) the Reader must be ready, there being no difficulty at all in doing it, and much use, as follows.

Triplication, or Multiplying by 3.

2. Add the double to the Number given, the Sum is the Product.

Examples.

```
 24321     793256
     3          3
 -----    -------
 72963    2379768
```

Here you still carry the double in mind (it being perfectly learned) as in the first, I say 2 and 1 is 3, 4 and 2 is 6, 6 and 3 is 9, 8 and 4 is 12, 4 and 2 is 6, and 1 is 7, all which are set down.

In the second, I say 12 and 6 is 18, 10 and 5 and 1 that I carry is 16, 4 and 2 and 1 is 7, 6 and 3 is 9, 18 and 9 is 27, 14 and 7 and 2 is 23, all to be set down.

Reduplication, or Multiplying by 4.

4. This is doubling the Duplication in your mind.

Examples.

```
 2345     549276     5001210
    4          4           4
 ----    -------    --------
 9380    2197104    20004840
```

In

In the first, I say 10 and 10 makes 20, I set down a Cypher and carry 2 in my mind, then 8 and 8 and 2 makes 18, setting down 8 and carry 1, then 1 and 6 and 6 makes 13, and 1 and 4 and 4 makes 9, and so of the rest, as you may see in the other Examples, which will be very easie, the greatest Number (to keep in memory) being but 3.

To Multiply by 5, 6, *or* 7.

5. First, you must know, that to multiply any Number by 10, is but to put a Cypher to the end thereof; therefore to multiply any Number by 5 (having put, or conceived to be put, a Cypher to it) take half that Number, and you have multiplied it by 5, as to multiply 57831 by 5, having put a Cypher to it, thus, 57831|0, take half thus, the half of 10 is 5, of 11 is 5, of 3 is 1, *&c.* 289155 is the Product.

	312513|0	But it is best to conceive the Cypher annexed thus	5298001
So	5		5
	1562565		26490005

To multiply by 6, is to take half adding a Cypher, and to add to the half the Figure standing next before.

2132|0	34578|0	9825458|0
6	6	6
12792	207468	58952748

In

In the first, I say the half of 0 is 0 and 2 is 2, the half of 12 is 6 and 3 is 9, the half of 13 is 6, and 1 is 7, the half of 1 is 0 and 2 is 2, the half of 2 is 1, all which are to be set down in the Product, and so of the rest.

To multiply by 7, is to take half and add it to the double of the former Figure, still conceiving a Cypher to be added as before.

2132|0	34578|0	9125458|0
7	7	7
14924	242046	63878206

In the first, I say the half of 0 is 0 and 4 is 4, the half of 12 is 6 and 6 is 12, I set down 2 and bear 1, the half of 13 is 6, and 1 is 7, and 2 is 9, the half of 1 is 0, and 4 is 4, the half of 2 is 1, and of so the rest.

To multiply any Number by 9 or 8.

6. First conceive the Number multiplied by 10, by putting to a Cypher, then substract each former figure from the following, beginning with that next before the Cypher, and the Remain is the Product of that Number multiplied by 9.

Example.

3785|0	39801210|0	525291|0
9	9	9
34065	358210890	4727619

In the first, I substract 5 from 10, remains 5, 9 from 15 remains 6, 8 from 8 remains 0, 3 from 7 remains 4, and 0 from 3 remains 3.

To multiply by 8, is to double each former Figure, and substract it from the following, as before.

Examples.

3785|0	39801210|0	5123012|0
8	8	8
30280	318409680	40984096

In the first I double the former, and I say 0 from 0 remains 0, then 17, or 7 from 15, remains 8, then 15 or 5 and 1 (I borrowed) from 8 remains 2, then 6 and 1 (I borrowed) from 7 remains 0, and 0 from 3 remains 3, and so of the rest.

Thus hath been shewed Multiplication by one Figure, with no trouble at all, only by multiplying by 8 and 7 there is a little, which by a little Practice, and the due observation of the Multiplier, may divers times be cleared.

7. There may be many times a total Product given

given at once of two or more figures; for if you add 2 Cyphers to any Sum, and take the fourth part, it is multiplied by 25 *&c.*

Example.

Let it be required to multiply 274 by 25, first to 274 I annex 2 Cyphers, and it makes 27400; of which I take the fourth part, and it is 6850, which is the same with the Product of 274 multiplied by 25, as you may prove at your Leisure.

Thus have I, with as much Plainness and Brevity as possibly I could, run over the Practice of Multiplication, whose Effects are so many, that of necessity, he that intends to understand any thing in Arithmetick, must be cunning and well practised therein; for by it is the content of any *Superficies* known, whose length and breadth is given; by the Content and Sum of any, thing the Values of many of that kind, are found out, and it is generally useful in Planimetry, the Golden Rule, and in all the parts of Arithmetick: so now we will proceed to the practice of Division.

CHAP. V.

Of Division.

1. DIvision is that part of Conjunct *Numeration*, whereby one Number is substracted from another, as often as it is contained in it; and by that means it is found, how many of the one is contained in the other.

2. The Number to be divided is called the *Dividend*, and the Number dividing is called the *Divisor*, but the Number found is called the *Quotient*, because it sheweth *Quoties*, how often the Divisor is contained in the Dividend. The Quotient is also called *Parabola*, because it ariseth from the comparing or application of a plain Number to the length given, that the breadth thereof may be found: As if the plain Number 30 (described in plain by his Unites, 2 *Sect. Chap.* 4.) be applied to his length 6, his breadth will be found by application or Measure (as it were) to be 5; and if a Number be applied to another Number, with a little Line between them, as $\frac{20}{5}$ or $\frac{5}{12}$, it is thereby understood, that the higher Number is to be divided by the lower, to which it is applied, which is the Form and Nature of all *Fractions*.

3. And although the Work of Division will seem at first somewhat difficult, yet we will give as plain and easie Rules, as possible may be, and Examples suitable thereto; so that the Learner with

with little careful pains may unravel this Knot. First then, in Division we begin at the left Hand, and after that we have distinguished out of the Figures of the Dividend, a Dividual sufficient for the Divisor, we try how often some of the first Figures of the Divisor, may be had in the first Figures of the Dividual; and it being known, we set in the place of the Quotient, a Figure signifying how often as 1 for once, 2 for twice, &c. then the whole Divisor must be multiplied by the particular Quotient found, and the *Factus* (or Product) must be substracted from the Dividual, and the Remainder set under a Line: Then the next Figure of the Dividend is to be annexed to the Remainder, and another particular Quotient to be found, as the first was found; and the Multiplication and Substraction to be made as before, the same work must still be repeated, till we come to the end of all the Dividend. But every particular Quotient thus found, ought to be of the same degree or place, with that Figure of the Dividend, which standeth (or is supposed to stand) above the Unites place of the Divisor, in every particular Operation: As suppose in any Division, the place of Unites in the Divisor, stands under the place of Thousands in the Dividend; at the beginning of the Work, then, when the Work is ended, will the first Figure in the Quotient stand in the place of Thousands likewise; and the second Figure in the Quotient, will be of the same place with the Figure of the Dividend, that stood over the place of Unites in the Divisor, at the second Operation, &c.

4. If after the joyning of a Figure of the Dividend, with the former Remainder for a new Dividual, and the next Figure of the Dividend to be put to the next Remainder for another new Dividual, and this to be repeated, as oft as need shall require, and the Remainder at last (if there be any) is usually set over the Divisor, with a little Line between them, in form of a Fraction, and annexed close to the Quotient on the right Hand.

5. If the Divisor hath Cyphers at the Right Hand, they may be omitted, and so many of the last Figures of the Dividend cut off, and the Division made in the rest of the Figures; but after the Division is made, the Cyphers are to be restored to the Divisor, (and the Figures cut off) to the Remainder, and the Remainder and Divisor set in form of a Fraction.

6. But if instead of the Fraction, so made with the Divisor and Remainder, you desire to have decimal Parts annexed to the End of the Integers of the Quotient, that may express that Fraction, you must continue on the Division, supplying the void places of the Dividend with Cyphers, cut off with the *Separatrix*, in form of a decimal Fraction; or where Figures are cut off from the Dividend for the Cyphers of the Divisor, you may continue on the Division through them, being cut off as decimal parts; and if these be not sufficient, you may supply the defects with Cyphers, as shall be hereafter directed.

7. In Division, as the Dividend is in Proportion to the Divisor, so the Quotient is to an Unite: as if 24 be divided by 6, the Quotient will be 4, therefore as 24. 6: :4. 1.

8. If

8. If a Quantity, either Magnitude or Number, be made of two other Quantities by their Multiplication, the one of them being the one *Factor* will measure (that is, evenly divide) the same Quantity made, by the other (being the other *Factor*:) As the Quantity 20 being made of 5 by 4, the one of them 5 will measure (or evenly divide) 20 by the other 4: so likewise 4 will measure the same 20 by 5. For the Measure of 5 is 4 times, and the measure of 4 is 5 times in 20.

9. If a Quantity, either Magnitude or Number, be made of two other Quantities by their Multiplication, it is all one whether you divide any other Number by that one Quantity, or by those other two Quantities that make up that Quantity by Multiplication: as if 24 be to be divided by 6, the Quotient is 4: so if you divide the said 24 first by 2, the Quotient is 12, and again that 12 by 3, the Quotient is 4, equal to the Quotient when the Divisor was 6, as before; for 3 multiplied in 2 produceth 6; in the same manner it is all one, whether you divide by 12, or by 4 and 3. by 16, or 4 and 4. by 24, or and 4. by 24, or by 3 and 2, and 4. for 3 times 2 times 4 is 24.

10. *Bypartition* was shewed before; *Tripartition*, or dividing any Number by 3, is done easily without any Substraction: for beginning at the left hand, take the third part of each figure, and what remains, accounting 10, if 1 remain, or 20 if 2, to the next Work, as in this Example: Take the third part of 965427, the Quotient is 321809.

ſaying the 3d. part of 9 is 3. of 6 is 2. of 5 is 1 (reſts 2) of 24 is 8. of 2 is 0 (reſts 2) of 27 is 9. this with a little practice will be found eaſie.

Likewiſe to divide any Number by 4, is to take the fourth part thus, if 1789012 were to be divided by 4, the Quotient will be 447253. ſaying the fourth part of 17 is 4. of 18 is 4. of 29 is 7. of 10 is 2. of 21 is 5. of 12 is 3. likewiſe the fourth part of 32598763 is 8149690$\frac{3}{4}$.

Likewiſe to divide any Number by 5, is to cut off the laſt Figure with the *Separatrix*, and double all the former Figures; only if the laſt Figure be either 5, or a Figure above it, you muſt ſet the Exceſs of it above 5, with 5 in the manner of a Fraction, and add one to the double of the Figure going before; as if 5 divide 35785|4, the Quotient is 71570$\frac{4}{5}$, in a Decimal, which is always better to avoid Fractions, 71570'8, by doubling the Remainder: ſo if 5 divide 3278902|7, the Quotient is 6557805'4, or 6557805$\frac{2}{5}$.

To divide any Number by 6, is to divide that Number by 3 and 2, as to divide 3568 by 6 thus,

```
6) 3568 (594⅔     according to the Rules
   30                  aforegoing.
   ——
    56
    54
    ——
     28        4|0
     24        40
```

So 8 may be divided by 2 and 4.
9 by 3 and 3. } 16 by 4 and 4.
12 by 3 and 4. } 24 by 6 and 4, or 2, 3 and 4.

But to the Business of Division.

Ex. 1. Let it be required to divide 748656 by 8.

First I set down the Dividend 748656, and on the Right and Left hand thereof I draw two Crooked Lines, and before that on the left Hand I place the Divisor, and behind that on the right Hand is to be placed the Quotient, as followeth.

8) 748656 (

First (because 8 is not contained in 7, the first Figure of the Dividend) I put a point under 4 (the second Figure) and seek how often 8 is contained in the Dividual 74, which I find to be 9 times, wherefore I put 9 in the Quotient, and thereby multiply the Divisor (8) and the Product is 72, which I place in order under (74) the Dividual, and substract it therefrom, and there remaineth 2; then I put a Poynt under the next Figure (8) and annex it to the Remainder 2, and it makes 28, for a new Dividual; so far according to the third Section of this Chapter, as it followeth.

8)

8) 748656 (9
72
28

Secondly, I repeat the same Work as is directed in the third Section, and seek how often (8) the Divisor is contained in 28, which I find to be 3 times, therefore I put three in the Quotient, and thereby multiplying the Divisor, it makes 24, which I put under 28, and substract it therefrom there remaineth 4, to which I annex the next Figure 6, putting a Poynt under it, so have I 46 for for a new Dividual. See the Work.

8) 748656 (93
72
28
24
46 *Dividual.*

Thirdly, I seek how often 8 is contained in 46, and I find it to be 5 times, therefore I put 5 in the Quotient, and thereby multiply the Divisor (8) whose Product (40) I put under the Dividual 46, and substract it therefrom, and there remaineth 6, to which I annex (5) the next Figure of the Dividend, and it makes 65 for a new Dividual. See the following Work.

8)

8) 748656 (935

72

28
24

46
40

65 *Dividual.*

Fourthly, I ſeek how often 8 is contained in 65, which is 8 times, therefore I put 8 in the Quotient, and thereby multiply the Diviſor 8, and place the Product (64) under the Dividual 65, and ſubſtract it therefrom, and the Remainder is 1, to which I annex (6) the laſt Figure of the Dividend, and it makes 16 for a new Dividual. See the following Work.

8)

```
8) 748656 (9358
   .....
   72
   ----
    28
    24
    ----
     46
     40
     ----
      65
      64
      ----
      16 Dividual.
```

Lastly I seek how often 8 is contained in 16, which is 2 times, therefore I put 2 in the Quotient, and thereby multiply the Divisor (8) whose Product (16) I put in order under the Dividual 16, and substract it therefrom, and there remaineth (0) nothing; so is the Operation ended, and I find that 748656 being divided by 8, the Quotient is 93582, that is to say, 8 is contained in 748656, 93582 times, or if you would divide 748656 into 8 equal parts, one of those Parts will be 93582. The intire Work followeth.

8)

8) 748656 (93582
   .....
   72
   ——
   28
   24
   ——
   46
   40
   ——
   65
   64
   ——
   16
   16
   ——
   (0)

2. *Example.*

Let it be required to divide 14996 by 46.

The firſt Dividual is 149, (for note that the firſt Dividual muſt always be greater than the Diviſor) therefore I put a Prick under 9, and ſeek how often (4) the firſt Figure of the Diviſor, is contained in (14) the two firſt Figures of the Dividend, which I find to be 3; therefore I put 3 in the Quotient, and thereby multiply the whole Diviſor 46, and the Product (138) I put under (149) the Dividual, and ſubſtract it therefrom, and the Remainder is 11, to which I annex (9) the next figure in the Dividend, and it makes

makes 119 for a new Dividual. See the Operation.

```
46) 14996 (3
    ..
    138
    ---
    119 Dividual.
```

Secondly, I seek how often 4 (the first Figure of the Divisor) is contained in 11, the Answer is 2 times, therefore I put 2 in the Quotient, and thereby I multiply (the whole Divisor) 46, and the Product is 92, which I place in order under the Dividual 119, and substract it therefrom, and the Remainder is 27, to which I annex (the last Figure of the Dividend) 6, and it makes 276 for a new Dividual, as followeth.

```
46) 14996 (32
     ...
    138
    ---
     119
      92
     ---
     276 Dividual.
```

Thirdly, I seek how often 4 (the first Figure of the Divisor) is contained in 27 (the two first Figures of the Dividual) and the Answer is 6 times, therefore I put 6 in the Quotient, and thereby I multiply the whole Divisor 46, and the

the Product is 276, which I substract from 276, (the Dividual) and there remaineth (0) nothing, and so the work is ended, and I find the Quotient to be 326, as appears by the following Operation.

```
46) 14996 (326
    138
    ---
     119
      92
     ---
      276
      276
      ---
      (0)
```

11. If when (according to the third Section of this Chapter) you have multiplied the Divisor, by any Figure placed in the Quotient, the Product doth exceed the Dividual, then are you to cancel the said Figure in the Quotient, and place another in its Room lesser by an Unite, and then multiply the Divisor thereby; and if the Product still exceed the Dividual, then place yet a lesser Figure in the Quotient, continuing so to do till the Product of the said Figure and the Divisor be lesser than the Dividual, or at least equal to it; and then make Substraction, remembring always, that if after Substraction is made, there remaineth more than the Divisor, then the Figure you last placed in the Quotient is too little, and therefore must be cancelled

cancelled and a bigger placed in it's Room, as you may see in the following Example.

3. *Example.*

Let it be required to divide 743587 by 364.

Having placed the given Dividend and Divisor in order to the Work, as before hath been directed, I find the 3 first Figures of the Dividend to be my first Dividual, *viz.* 743, therefore I put a Point under the third figure (3) and seek how often 3 is contained in 7, and the Answer is, twice; therefore I put 2 in the Quotient, and thereby I multiply the Divisor (364) and the Product is 728, which is lesser than the Dividual 743, therefore I conclude that I have placed a true Figure in the Quotient; then I proceed to substract the said Product (728) out of the Dividual 743, and the Remainder is 15, as appears by the following Work.

364) 743587 (2

728

———

15

This being done, I put a Point under the next Figure of the Dividend, and annex it to the said Remainder (15) and it makes 155 for a new Dividual; so I seek how often 3 is contained in 1, and I find it not to be contained therein, therefore I put a (0) Cypher in the Quotient; *Now I should multiply*

multiply and Substract, but because a Cypher is placed in the Quotient, there is no need of it; but I proceed, and put a Point under the next Figure in the Dividend, *viz.* under 8, and annex it to the former Dividual (155) and it makes 1558 for a new Dividual; then I seek how often 3 is contained in 15 (the two first Figures of the Dividual) and the Answer is 5 times; therefore I put 5 in the Quotient, and thereby multiply the Divisor, and the Product is 1820, which is greater than the Dividual (1558) therefore I conclude that the Number last placed in the Quotient (*viz.* 5) is too much, wherefore I cancel it, and in it's Room put a 4, and thereby multiply the Divisor, and the Product is 1456, which is lesser than the Dividual, therefore I make Substraction, and there remaineth 102. See the Work.

```
364) 743587 (204
      • • •
     728
     ---
      1558
      1456
      ----
       102
```

Then I put a Point under the last Figure of the Dividend, and annex it to the said Remainder (102) and it makes 1027 for a new Dividual; then I seek how often I can have 3 in 10 (the two first Figures of the Dividual) and the Answer is 3 times, wherefore I put 3 in the

 Quotient

Quotient and thereby multiply the Divifor, and the Product is 1092, which is greater than the Dividual, therefore I conclude that I have put a Figure too big in the Quotient, and fo I cancel the 3, and put a 2 in it's Room, and thereby I multiply the Divifor, and the Product is 728, which I fubftract from the Dividual, and there remaineth 299, which I place over the Divifor 364, with a Line between them thus $\frac{299}{364}$ and annex it to the Quotient, according as is directed in the 4*th*. Section of this Chapter. View the whole Operation.

```
364) 743587 (2042 299/364
     728
     ----
      1558
      1456
      ----
       1027
        728
       ----
        299
```

So that I conclude that 743587 being divided by 364, the Quotient is $2042\frac{299}{364}$.

More

More Examples of Divifion in Integers.

1. *Example.*

Dividend *Quotient*

Divifor 8) 54789 ($6848\frac{5}{8}$

48

67
64

38
32

69
64

Remainder (5)

2. *Example.*

Divifor 6000) 340|970 ($56\frac{4970}{6000}$

30

40
36

Remainder 4970

 3. *Example.*

3. *Example.*

```
                    Dividend     Quotient
Divisor   385)  7892345   (20499 230/385
                770
                ----
                 1923
                 1540
                 -----
                  3834
                  3465
                  -----
                   3695
                   3465
                   -----
      Remainder   (230)
```

4. *Example.*

```
47586)  226747290  (4765
        190344
        ------
         364032
         333102
         ------
          309309
          285516
          ------
           237930
           237930
           ------
            (0)
```

In

In the firſt Example, according to the Rule, I enquire how many times 8 in 54, the which I find 6 times, 6 I ſet in my Quotient and multiply it by 8, my Diviſor, which makes 48, the which I take from 54 remains 6, the which I ſet under the Line, and place the next Figure in my Dividend 7 by it, and work till the end after the ſame manner.

In the ſecond I am to divide 340972 by 6000, I place my Cyphers to the latter end of my Dividend and divide by 6, according to the 5*th*. Section of this Chapter, and the Quotient is $56\frac{4970}{6000}$.

In the third, having placed the Dividend and Diviſor, I enquire how many times 3 is contained in 7, (having always regard to the quality of the Figure following, being multiplied by the laſt Figure in the Quotient, that the Product may be ſubſtracted) I find it twice, (ſaying in my mind twice 3 is 0, which taken from 7 reſts 1, which with the next Figure following makes 18, and twice 8 is 16, which will come out of 18) and ſet down 2 in the Quotient, then multiply the Diviſor by 2, the Product is 770, which I ſubſtract from 789 reſt 19, to which I annex the next Figure of the Dividend 2, and find it leſs than 385 my Diviſor; therefore I put a Cypher in my Quotient, and fetch another Figure 3 from the Dividend, to make my Number bigger than the Diviſor, and then work as in the Example.

But becauſe in chooſing out of due Quotients there is ſome trouble, the danger either to take

it too great, or too little, whosoever shall make use of a Table made of the *Multiplees* of the Divisor, as is taught in the 11. Sect. following.

11. You may likewise (which will be a great ease, especially to those that are not well practised in Division) multiply the Divisor (by doubling, trebling, redoubling, halfing, *&c.*) instantly to 9, which will serve you instead of Multiplees.

Example.

I am to divide 71234568 by 487. first, I set down my Divisor, and under it it's Double, and under that it's Treble, and under that it's Quadruple, and so on to 9, and against each Number I put the Numbers 1,2,3,4,*&c.* in order as you see in the following Work.

487	1	71234568	$(146272\frac{104}{487}$ *&c.*
974	2	2253	
1461	3	3054	
1948	4	1325	
2435	5	3516	
2922	6	1078	
3409	7		
3896	8		
4383	9		

Rem. (104) *&c.*

The

The Diviſor may be prepared of ſome of the Multiplees, and yet you may have the reſt as you pleaſe thus, and the intermediate places may be ſupplied by Memory, if need be.

487	1
974	2
1948	4
3896	8

Then conſidering my Dividend, I find it will be had once out of 712: and here having the Multiplees already ſet down, I ſubſtract the Diviſor from 712, and there reſts 225, to which I annex 3 the fourth figure of the Dividend, and then look the next leſs number which is 1948 and anſwereth the Index 4, therefore I ſet 4 in the Quotient, and ſubſtract 1948 from 2253, and there reſts 305, I work on *&c.* and this will be found as ſhort a Work as needs be, for though I multiply my Diſor, yet I ſave ſubſcribing all the ſeveral Products, under each part of the Dividend.

12. And here I might ſhew you, how to divide by many Numbers of two places all at once; as to divide any Number by 25, is to cut off two of the laſt places (remembring to carry as many Unites to the next place, as 25 is contained in theſe two Figures, which can never be above 3,) and redouble all the reſt of the Figures for the Quotient, and from hence alſo you may divide any Number by 24 (which will be of good uſe, 240 *d.* being in a pound) all at once if you firſt divide by 25, then ſee how many times 24 may be had in the Quotient, which in ſmall Numbers is ſoon perceived, which muſt be added to the former Quotient; the like might be done of many others, which for brevity ſake (deſiring however to be plain) I leave to the practice of the Ingenious Arithmetician.

The Proof of Multiplication and Division.

13. Although the usual Way in Schools, is to prove Multiplication by casting the Nines out of the *Multiplicand*, the *Multiplier*, and the *Product*, as is taught in the 44*th*. Page of this Book, yet ought not that Way of proving a Sum to be best approved of, since it is not infallible; for many times a Sum that is falsly wrought, may by that Rule prove right; but because a Learner is not qualified for a better way of proof, whilst he is in Multiplication, it is therefore thought necessary, that he should there learn that way of proving his Sums, which he is then only capable of, till he hath learned Division, which I hope by this time he is well acquainted with.

Now let him know therefore, that *Multiplication* and *Division*, do interchangeably prove each other, for, if (after you have wrought a Sum in Multiplication) you divide the *Product* by the *Multiplicand*, and you find the *Multiplier* in the *Quotient* without any Remainder, then you may be sure you have multiplied right.

Or if you divide the *Product* by the *Multiplier*, and the *Quotient* is equal to the *Multiplicand*, without any Remainder, then you may be sure your Multiplication is truly wrought.

Example.

Let us prove the Example at the bottom of the 43*d*. Page, where 74358 is multiplied by 6472, and

and the *Product* is there found to be 481244976; to prove which, I divide the ſaid *Product* 481244976 by the *Multiplicand* 74358, and the *Quotient* is 6472, which is equal to the *Multiplier* without any Remainder, as by the following Operation.

```
74358) 481244976 (6472
          ....
       446148
       ------
        350969
        297432
        ------
         535377
         520506
         ------
          148716
          148716
          ------
           (0)
```

And if you ſhould divide the ſaid *Product* by 6472 (which is the *Multiplier*) the Quotient would be 74358, equal to the *Multiplicand* without any Remainder, as you may try at your Leiſure.

When you have performed a Sum in Diviſion, and you are deſirous to prove the ſame, Multiply the *Quotient* by the Diviſor, and if the *Product* is equal to the *Dividend*, then you may conclude your Diviſion to be truly wrought, otherwiſe not.

Let

Let us prove the 4*th.* Example in the 86 page, where 226747290 is divided by 47586 and the *Quotient* is found to be 4765; now if you multiply 4765 by 47586 (the *Divisor*) the *Product* will be 226747290, which is equal to the *Dividend*, and therefore the Division is right. See the Work.

	47586
	4765
	237930
	285516
	333102
	190344
Product	226747290

But if, after your *Division* is ended, any thing remain, then when you come to prove your Work, you must add the said Remainder to the Product, and if the Sum be equal to the Dividend, then is your Operation true.

Let us prove the third Example in the 68*th.* Page, where 7892345 is divided by 385, and the *Quotient* is 20499 with a Remainder of 230; now if you multiply the said *Quotient* 20499 by the *Divisor* 385, the *Product* will be 7892115, to which if you add the said Remainder 230, the Sum will be 7892345, which is equal to the Dividend. See the Work.

20499

$$
\begin{array}{r}
20499 \\
385 \\
\hline
102495 \\
163992 \\
61497 \\
\hline
7892115 \\
230 \\
\hline
7892345
\end{array}
$$

You may likewiſe prove Diviſion by Diviſion, as I have ſhewed at large in the 7. Chap. Page 100, 101, 102. of Mr. *Cocker*'s Arithmetick, printed in the year 1685; for if you divide the *Dividend* by the *Quotient*, and the Quotient thence ariſing be equal to the *Diviſor*, then is the Diviſion right, but if after your Diviſion is ended any thing remains, then (before you go this Way to prove your work) ſubſtract the ſaid Remainder from the *Dividend*, and what remaineth after Subſtraction, divide as before.

I might proceed to ſhew ſeveral other Ways of Diviſion, by cancelling the Figures as you proceed in the Work, *&c.* but it being tedious I ſhall deſiſt, thoſe that deſire to know the ſame, let them conſult ſome able Arithmetician, if they have any near them, but if not (as too many places are wanting in able Artiſts) let them read *Baker*'s, *Johnſon*'s, or *Record*'s *Arithmetick*, where they may meet with large Directions for the ſame.

14. The

14. The general Uſes of Diviſion are infinite, as to know the ſide of any *Superficies*, the one ſide and Content being given; to know the Rate, Price, or Value of any thing, if the Rate, Price, or Value of many of theſe things in the ſame kind be given; it is of great Uſe in the forming and working of any other Rates, as of *Proportion*, *&c.* And if the diligent Reader be but perfect in *Multiplication* and *Diviſion*, he may then rejoyce and truly ſay that he hath paſſed the hardeſt; the reſt being all wrought by theſe four *Species* of *Numeration*, viz. *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion.*

15. Laſtly, Note that (if in any Sum of Diviſion) the Quotient be equal to the Diviſor, and that nothing remain after *Diviſion* is ended, then is the *Dividend* a Square Number, and the *Diviſor* is it's Root. And note, that if the *Dividend* be leſſer than the *Diviſor*, that then you ſet it in the manner of a vulgar Fraction; the *Dividend* being above the *Diviſor* with a Line between them, otherwiſe you may convert it into a Decimal Fraction, putting Cyphers to it, and then divide it, as hereafter ſhall be taught in its proper place.

And thus much for *Diviſion*, now we will proceed to *Reduction.*

CHAP.

CHAP. VI.

Of Reduction.

1. REduction may be divided into two Parts, *viz.* *Reduction* Aſcending, and *Reduction* Deſcending.

2. Reduction Aſcending, is when it is required to reduce a given Number of Integers of a leſſer Denomination into an Integer, or Integers of a greater Denomination equivalent to the Number given in a leſſer Denomination, and this is always performed by Diviſion. As if it were required to reduce 74580 Shillings into Pounds; foraſmuch as 20 Shilings are contained in a Pound, therefore I divide 74580 by 20, and the Quotient is 3729 Pounds equal in Value to 74580 Shillings.

Reduction deſcending, is when it is required to reduce a given Integer; or Integers of a greater Denomination into a leſſer, which Number in the leſſer Denomination ſhall be equivalent to the given Number of the greater Denomination; and this is always performed by *Multiplication*, as ſuppoſe it were required to reduce 3729 Pounds into Shillings, here I conſider that a Pound containeth 20 Shillings, and conſequently the Shillings will be 20 times as many as the Pounds, wherefore I multiply 3729 by 20, and the Product is 74580 Shillings equal to 3729 Pounds, from what is here ſaid you may eaſily underſtand that.

3. To

3. To reduce any Number or Fraction undenominate into it's least Terms, you must consider how many of the next lesser Denomination, are contained in the next greater before, and by that Number multiply the greater, and so work till you come to the least.

1. *Example.*

To reduce 178 *l.* into pence, according to the Rule, I consider 20 *s.* is contained in one Pound; therefore I double 178 *l.* and thereto adding a Cypher, it makes 3560 Shillings, then because 12 *d.* is contained in one Shilling, I multiply 3560 by 12, and it gives 42720, which are the Pence in 178 *l.* the like of any other.

If there be divers Denominations expressed of the same kind, you must add up the smaller Denominations, as followeth.

2. *Example.*

Let it be required to reduce 27 *l.* 19 *s.* 05 *d.* into Pence.

First, I multiply 27 by 20, and the Product is 540, to which I add 19, and it makes 559 Shillings, then I multiply 559 by 12, because in a Shilling there are 12 Pence, and the Product is 6708, to which I add the 5 Pence, and it makes 6713 Pence, equivalent to 27 *l.* 19 *s.* 05 *d.* See the Operation as followeth,

l.	*s.*	*d.*
27	19	05
20		
540		
19 *add*		
559 *Shillings.*		
12		
1118		
559		
6708		
5 *add*		
6713 *Pence.*		

Or ifyou add the ſmaller Denomination as you multiply, the work will be much more Compendious: As, when I multiply (in the foregoing Example) 27by 20, I ſay 0 times 7 is 0, but 9 is 9, (*viz.* the 9 in 19 *s.*) which I put down, then I ſay 2 times 7 is 14, and 1 is 15 (*viz.* the 1 in 19 *s.*) ſo I put down 5 and carry 1, then 2 times 2 is 4, and 1 that I carry is 5, which I put down, ſo have I 559 Shillings in 27 *l.* 19 *s.* then I multiply 559 by 12, and as I multiply I take in the 5 Pence thus, I ſay 2 times 9 is 18, and 5 is 23. *&c.* So I have 6713 Pence as before. See the Operation, according to the latter way, as followeth.

27 *l.*

l.	*s.*	*d.*
27	19	05
20		
559	*Shillings*	
12		
1123		
559		
6713	*Pence.*	

3. *Example.*

In 3455 *l.* 11 *s.* 09½ *d.* I demand how many Farthings?

To anſwer this Queſtion, firſt I multiply 3455 by 20, and take in the 11 *s.* as I multiply, and the Product is 69111 Shillings, and that I multiply by 12, and take in the 09 *d.* and the Product is 829341 Pence, then I multiply the Pence by 4, and take in the 2 Farthings, ſo I have 3317366 Farthings. Behold the following Work.

3455 *l.*

	l.	*s.*	*d.*
	3455	11	09$\frac{1}{2}$
	20		
	69111	*Shillings*	
	12		
	138231		
	69111		
	829341	*Pence*	
	4		
facit	3317366	*Farthings.*	

Note, that in reducing of a Number from one Denomination to another, if any thing remains after *Division* is ended, the ſaid Remainder is ever more of the ſame Denomination with the *Dividend*, as in the following Example.

4. *Example.*

In 574386 Farthings I demand how many Pounds?

Firſt I divide 574386 Farthings by 4 to bring them into Pence, and the *Quotient* is 143596 Pence, and there remaineth 2, which is 2 Farthings, becauſe the Dividend is Farthings; then I divide 143596 Pence by 12, to bring them into Shillings and the *Quotient* is 11966 Shillings, and there is a Remainder of 4, which is 4 Pence, becauſe the

Dividend is Pence, then I divide 11966 Shillings by 20, and the Quotient is 598 Pounds, and there is a Remainder of 6, which is 6 Shillings, becauſe the *Dividend* is Shillings; Now the Queſtion is anſwered, and I find that in 574386 Farthings, there are contained 598 *l.* 06 *s.* 04½ *d.* as appears by the following Operation.

```
4)  574386  (143596
    ......
    4
    ---
    17
    16
    ---
     14
     12
     ---
      23
      20
      ---
       38
       36
       ---
        26
        24
        ---
   Remains (2) Farthings.
         l.    s.    d.
Facit   598    06    04½
```

12)

```
12)  143596  (11966
       ....
     12
     ----
      23
      12
     ----
      115
      108
     ----
        79
        72
       ----
         76
         72
        ----
```

Remains (4) *Pence.*

```
                  l.
20)  11966  (598
       ...
     100
     ----
      196
      180
     ----
       166
       160
      ----
```

Remains (6) *Shillings.*

And here note, that although in Reducing the

 Shillings

Shillings to Pounds in the foregoing Example, we have divided 11966 by 20, (after the methodical way of Division) yet may the ſame be more conciſely done thus, *viz.* Cut off the Figure in the place of Unites from the reſt with a Daſh of a Pen, to ſignifie Shillings, and by the 5*th.* Rule of the 4*th.* Chapter, which is in the 48*th.* Page Bypart the Remaining Figures for Pounds, and if the laſt of thoſe Figures remaining (as aforeſaid) be odd, then increaſe the Figure before cut off for Shillings by 10, and the Work is done.

So to reduce 11966 Shillings (in the laſt Example) to Pounds, firſt I cut off the 6 in the place of Unites with a Daſh of the Pen thus, (1196|6) to ſignifie 6 Shillings, and take the half of 1196 (the Remaining Figures) which is 598, for Pounds, ſo is 11966 Shillings equal to 598 *l.* 6 *s.* See the Work.

1196|6

l. *s.*
598 06

In like manner, if it were required to reduce 6478 Shillings to Pounds, I firſt cut off the 8 with a daſh of the Pen for Shillings, as before and Bypart 647 for Pounds, and it makes 323 *l.* and becauſe 7 (the laſt Figure) is odd, I make the ſaid 8 to be 18 Shillings, behold the following Work.

647|8

$$\begin{array}{r|l} 647 & 8 \\ \hline \end{array}$$

l. *s.*
323 18

And hereafter in reducing Shillings to Pounds throughout this Treatiſe, we ſhall uſe this Method.

5. Example.

In 47538 Pieces of 3 *d.* I demand how many Pounds Sterling?

To reſolve this Queſtion I conſider, that 4 three Pences make a Shilling, therefore I divide the given Number by 4, and the Quotient is 11884 Shillings, and there remaineth 2 after Diviſion, which by the Note in the 81*ſt.* Page foregoing, is 2 three pences, or 6 Pence; And by the Note in the 83*d.* Page 11884 Shillings are reduced to 594 *l.* 04 *s.* ſo that in 47538 three pences there are contained 594 *l.* 04 *s.* 06 *d.* View the Reduction.

```
4) 47538 (1188|4
   .....
   4
   --
   07
    4
   --
    35
    32
    --
     33
     32
     --
      18
      16
      --
```

Remains (2) *three Pences, or 6 d.*

```
1188|4
----|
  l.   s.
 594   04
```

	l.	*s.*	*d.*
facit.	594	04	06

6. Example.

Suppose 48 *l.* were to be distributed amongst poor People, each Person to have 18 Pence, how many would be partakers of the said Sum?

Reduce the given Sum (48 *l.*) into Pence by the

the firſt Example,and it makes 11620 Pence, then divide 11620 pence by 18, and there ariſeth 640 in the Quotient, and ſo many will the ſaid Sum ſatisfie. Prove the work at your Leiſure.

4. But there may yet be a ſhorter way to reduce the greateſt into the leaſt Denomination, at one Operation, without taking notice of the intermediate Denominations, and that is by multiplying the greateſt, by as many as an Unite thereof contains of the leaſt.

Likewiſe to reduce the leaſt into the greateſt Denomination, and that is by dividing the leaſt by as many thereof as are contained in an Unite of the greateſt.

This Rule will be made plain by one or two Examples.

7. *Example.*

In 375 *l.* how many Pence?

Now becauſe in one Pound there are 240 Pence, therefore I multiply 375 by 240, and the Product is 90000, and ſo many Pence there are contain'd in 375 *l.* as by the Work in the Margent is manifeſt.

```
  375
  240
-------
 15000
 750
-------
 90000 Pence.
```

So in 35781 *l.* there are contained 8587440 Pence; Compare this with the Way before taught, by multiplying by 20, and by 12, and uſe thy Diſcretion.

8. Example.

In 7584 Pence, I demand how many Pounds? Here if you divide the given Number (7584) by 240 (the Pence in a Pound) the Quotient will be 31 *l.* and there is a Remainder of 144 Pence, which being divided by 12, the Quotient is 12 Shillings, ſo that in 7584 Pence are contained 31 *l.* 12 *s.*

Foreign Coyn may be reduced to Engliſh Coyn, *and the Converſe*, when there is given the Value (in Engliſh Coyn) of an Unite in the Foreign; as in the following Examples.

9. Example.

What is the Value of 223 Scots Marks in Engliſh Coyn, the Scots Mark being valued at $13\frac{1}{2}$ *d.*?

Firſt, I conſider that in $13\frac{1}{2}$ *d.* there are 27 half Pence, therefore I multiply (223) the Number of Scots Marks by 27, and the Product is 621 half Pence, which (by Reduction aſcending) is found to be 12 *l.* 10 *s.* $10\frac{1}{2}$ *d.*

You will find the Anſwer to be the ſame, if you multiply 223 by 54 (the Number of Farthings in $13\frac{1}{2}$ *d.* (or a Scots Mark) for then the Product will be 12042 Farthings, which will be reduced to 12 *l.* 10 *s.* $10\frac{1}{2}$ *d.* as before.

Example.

10. *Example.*

In 209 *l.* 06 *s.* 08 *d.* Sterling, how many *Dollars*, at 5 *s.* 04 *d. per Dollar*?

Firſt, reduce 209 *l.* 06 *s.* 8 *d.* into Pence, and you will find it to be 50240 Pence, then divide that by (64) the Pence in one Dollar (or 5 *s.* 04 *d.*) and the Quotient is 785, and ſo many Dollars at 5 *s.* 04 *d.* are contained in 209 *l.* 06 *s.* 08 *d.*

Queſt. 11. A certain Merchant delivereth at *London* 468 *l.* Sterling, which he is to receive at *Bourdeaux* in French Money called Solx, the Exchange at 5 Solx for 6 Pence. Now I demand how many Solx he ought to receive for the ſaid Sum? Anſwer, 93600 Solx.

To Anſwer this Queſtion, firſt bring the given Sum into 6 Pences, which you may do by multiplying by 20 to bring them into Shillings, and then multiplyby 2 to bring the Shillings into 6 Pences, and it makes 18720. Or if you multiply 468 *l.* by 40 (becauſe 40 ſix Pences make one Pound) the Product will be 18720 ſix Pences, as before, then (becauſe 6 Pence is 5 Solx) I multiply 18720 by 5, and the Product is 93600, and ſo many Solx muſt he receive at *Bourdeaux*.

Or you might conſider that if 5 Solx is 6 Pence, then 10 is a Shilling; wherefore if you multiply your Pounds by 20, and that Product by 10, the Anſwer will be found to be 93600 Solx as before.

5. In Reduction Aſcending, remember always that your *Dividend* and *Diviſor* be of one and the ſame

ſame denomination, or ſo reduced (which muſt be to the leaſt Denomination mentioned in either of the given Numbers) before you make Diviſion. The following Example will make this Rule eaſie.

12. *Example.*

In 435 Marks at 13 *s.* 04 *d.* *per* Mark, how many Scots Marks, at $13\frac{1}{2}$ *d.* *per* Mark?

The Numbers given in this Queſtion are 435 Marks, and $13\frac{1}{2}$*d.* the leaſt Name mentioned in either of which is $\frac{1}{2}$*d.* or 2 Farthings, and therefore I reduce $13\frac{1}{2}$ *d.* into 27 half pence, which muſt be my Diviſor; and becauſe my Diviſor is in half pence, therefore muſt my Dividend be in half-pence, wherefore I firſt find how many half pence there are in (13 *s.* 04 *d.*) a Mark, and I find there are 320; therefore I multiply 435 Marks by 320, and the Product is 139200 half pence (equal to 435 Marks) for my Dividend, wherefore I divide 139200 by 27, and the Quotient is $5155\frac{15}{27}$ Scots Marks, or 5155 Scots Marks, and 15 half pence, or $7\frac{1}{2}$*d.* over. View the Working of the next Example, and well conſider thereof.

$13\frac{1}{2}$*d.*

l.		
$13\frac{1}{2}$	435 *Marks*	
2	320	
27 *Divisor.*	8700	
	1305	
27)	139200	(5155
	135	
	42	
	27	
	150	*facit* $5155\frac{15}{27}$ *Sc. Mar.*
	135	
	150	
	135	

Remains (15) *half Pence.*

s.	*d.*
13	04
12	
160 *Pence*	
2	
320 *half Pence in a Mark.*	

Except

Except you can discern that the one is an *aliquot* or even part of the other, for then the Work (though it cannot be more truly) may be more expeditiously wrought, than by the foregoing Rule, as by the following Example is manifest.

13. *Example.*

In 374 *l.* how many Nobles at 6 *s.* 8 *d. per* Noble?

Here I may consider, that one Pound is 3 Nobles, wherefore by multiplying 374 by 3, the Answer to the Question is gained, *viz.* 1122 Nobles: Whereas if the Question had been solved by the foregoing Rule, I should have reduced 374 *l.* into Pence which is 89760, and divided it by the Pence in a Noble, *viz.* 80 Pence, and then the Quotient would have been (as before) 1122 Nobles, but this way is something tedious.

14 *Example.*

Let it be required to divide 315 *l.* 16 *s.* into pieces of $13\frac{1}{2}$ *d.* 12 *d.* of 9 *d.* of 6 *d.* of $4\frac{1}{2}$ *d.* of 3 *d.* of 2 *d.* of 1 *d.* so that there may be an equal Number of each sort of pieces: Now I desire to know, how many pieces of each sort there must be?

First

First add the pieces(into which the given Sum is to be reduced) together, and their Sum is 51 *d.* (for a Divisor) as you see in the Margent, then reduce 315 *l.* 16 *s.* into pence, (because the Divisor is pence) and they make 75792 pence for a Dividend, which being divided by 51, the Quotient will be 1486, and so many pieces of each sort will there be; and after Division is ended, there is a Remainder of 6, which by the *Note* in the 81*st*. page foregoing is 6 pence.

l.
$13\frac{1}{2}$
12
9
6
$4\frac{1}{2}$
3
2
1
51

I have been the larger upon *Reduction of Money* because Learners are generally more acquainted with the Nature of Coyn, than with Weights and Measures, *&c.* and the Learner being well acquainted with this kind of Reduction, the rest will be altogether as easie; as will appear by the Solution of the following Questions.

Reduction of Troy Weight.

15. *Example.*

In 574 *l.* Troy Weight, I demand how many Grains?

Multiply the given Number by 12, and the Product is 6888 Ounces, and that Multiplied by 20, the product is 137760 Peny Weights, and that Product again multiplyed by 24, the Product is 3306240 Grains, and so many are contained in 574 Pounds Troy Weight; as appears by the following Work. 574

$$\begin{array}{rl} 574 & \\ 12 & \\ \hline 1148 & \\ 574 & \\ \hline 6888 & oz. \\ 20 & \\ \hline 137760 & p.w. \\ 24 & \\ \hline 551040 & \\ 275520 & \\ \hline 3306240 & gr. \end{array}$$

This Question might have been wrought at one Operation by the 4*th*. Sect. of this Chap. page 87. if you multiply (the given Number) 574 by 5760 (the Grains contained in a Pound Troy Weight) as you may prove at your Leisure.

16. *Example.*

In 4758643 Grains, I demand how many Pounds Troy Weight?

First, divide 4758643 by 24, (the Grains in a peny Weight) and the Quotient is 198726 peny Weights, and there remaineth 19 Grains; then I divide 198276 by 20, (the peny Weights in an Ounce) and the Quotient is 9913 Ounces, and there

there is a Remainder of 16 peny Weights; then do I divide 9913 Ounces by 12 (the Ounces in a Pound) and the Quotient is 826 Pounds, and 1 Ounce remaineth; so the Work is finished, and I find that in 4758643 Grains there are 826 *l.* 01 *oz.* 16 *p.w.* 19 *gr.* as you will find if you actually perform the Work.

This Example might be wrought at one Operation, if you divide (4758643) the given Number of Grains by (5760) the Number of Grains in a Pound; but then there will be a Remainder of 883 Grains, which being reduced will be found equivalent to 01 *oz.* 16 *p.w.* 19 *gr.* as you may prove at your Leisure.

Reduction of Averdupois Weight.

17. *Example.*

In 4758692 Pounds I demand how many Tuns Weight?

To solve this Question, first I divide by 28 *l.* to reduce the given Pounds into Quarters of Hundreds, and the Quotient is 169953 Quarters, and there is a Remainder of 8 Pounds; then I divide the Quarters by 4, and the Quotient is 42488 Hundred, and there is a Remainder of 1 Quarter of a Hundred, then I divide the Hundreds aforesaid by 20 to bring them into Tuns, and the Quotient is 2124 Tuns, and 8 Hundred Weight remains; so that by the Work I find that in 4758692 *l.* there are 2124 *Tuns*, 08 *C.* 1 *qr.* 08 *l.*

18. *Exam-*

18. *Example.*

In 2124 *Tuns*, 08 *C.* 1 *qr.* 08 *l.* I demand how many Pounds Weight? *facit* as by the following Operation appeareth.

Tun.		*C.*	*qr.*	*l.*
2124		08	1	08
20				
42488	*C.*			
4				
169953	*qrs.*			
28				
1359632				
339906				
4758692	*l.*			

The leſſer Averdupois Weight (which conſiſteth of Pounds, Ounces, and Drams) is reduced by 16 and 16.

19. *Example.*

In 376 *l.* how many Drams?

	l.	
	376	
	16	
	2256	
	376	
	6016	*Ounces.*
	16	
	36096	
	6016	
facit	96256	*Drams.*

So of any other Weight or Meaſure the Operation is ſtill the ſame, obſerving the 3*d.* Sect. of this Chap. in the 78*th.* page; I ſhall give one more Example of *Long Meaſure.*

20. *Example.*

The Diſtance between *London* and *York* is accounted 150 Miles, I demand how many Barly Corns will reach the ſame Length?

Firſt I multiply 150 by 8, the Product is 1200 Furlongs, then multiply 1200 Furlongs by 40, and the product is 48000 Perches, or Poles, then

H 48000

48000 Poles being Multiplied by 11, produceth 528000 half Yards, and that again multiplied by 18 produceth 9504000 Inches, which being multiplied by 3, the *Product* is 28512000 Barly Corns, and ſo many will reach 150 Miles, which is the Diſtance from *London* to *York*.

The Proof of Reduction.

Reduction aſcending and deſcending do interchangeably prove each other, by inverting the Queſtion; As in the 17*th*. and 18*th*. Queſtions foregoing: In the 17*th*. Example, it is demanded how many Tuns Weight are contained in 4758692 Pounds, and the Anſwer thereto is 2124 *Tuns*, 08 *C*. 1 *qr*. 08 *l*.

And in the 18*th*. Example the Queſtion is ſtated back again, *viz*. In 2124 *Tuns*, 08 *C*. 1 *qr*. 08 *l*. how many Pounds? And the Anſwer is 4758692 Pounds, which proves the Work of the 17*th*. Example to be right.

I ſhall not give any more Examples in this Rule, but if any thing at any time ſeem difficult in Reduction; let the Learner have Recourſe to the following Tables, taken out of *Wing*. Arith. Chap. 7.

And

And first in Reduction Descending.

1. *Of Sterling Money.*

Pounds	multiplied by	20	produce	Shillings
Shillings		12		Pence
Pence		4		Farthings

2. *Of Troy Weight.*

Pounds	multipl. by	12	produce	Ounces
Ounces		20		Peny Weights
Peny Weights		24		Grains

Also in Apothecaries Weights.

Ounces Troy	multipl. by	8	produce	Drams
Drams		3		Scruples
Scruples		20		Grains

3. *Of Averdupois Weights.*

Tuns	multiplied by	20	produce	Hundreds
Hund. Weights		4		Quarters
Quarters		28		Pounds
Pounds		16		Ounces
Ounces		16		Drams

4. *Of Liquid Measure.*

Hogsheads	multiplied by	63	produce	Gallons
Gallons		2		Pottles
Pottles		2		Quarts
Quarts		2		Pints

5. *Of dry Measure*

Quarters	multiplied by	8	produce	Bushels
Bushels		4		Pecks
Pecks		2		Gallons
Gallons		2		Pottles
Pottles		2		Quarts
Quarts		2		Pints

6. *Of Long Measure.*

English Miles	multiplied by	8	produce	Furlongs
Furlongs		220		Yards
Yards		3		Feet
Feet		12		Inches
Inches		3		Barly Corns

Also

Yards or Ells	mult. by	4	produce	Quarters
Quarters		4		Nails

7. *Of superficial Land-Measure.*

Acres	mult. by	4	produce	Roods
Roods		40		Perches, or Poles.

8. *Of Time.*

Weeks	multipl. by	7	produce	Days
Days		24		Hours
Hours		60		Minutes

Secondly for Reduction Ascending.

1. *Of Sterling Money.*

Farthings	divided by	4	give	Pence
Pence		12		Shillings
Shillings		20		Pounds

2. *Of Troy Weight.*

Grains	divided by	24	give	Peny Weights
Peny Weights	divided by	20	give	Ounces
Ounces	divided by	12	give	Pounds

Also in Apothecaries Weights.

Grains	divided by	20	give	Scruples
Scruples	divided by	3	give	Drams
Drams	divided by	8	give	Ounces Troy

3. *Of Averdupois Weight.*

Drams	divided by	16	give	Ounces
Ounces	divided by	16	give	Pounds
Pounds	divided by	28	give	Quarters of C.
Quarters	divided by	4	give	Hundreds
Hundreds	divided by	20	give	Tuns Weight

4. *Of Liquid Measures.*

Pints	divided by	2	give	Quarts
Quarts		2		Pottles
Pottles		2		Gallons
Gallons		63		Hogſheads

5. *Of Dry Measure.*

Pints	divided by	2	give	Quarts
Quarts		2		Pottles
Pottles		2		Gallons
Gallons		2		Pecks
Pecks		4		Buſhels
Buſhels		8		Quarters

6. *Of Long Measure.*

	divided by	give
Barly Corns	3	Inches
Inches	12	Feet
Feet	3	Yards
Yards	220	Furlongs
Furlongs	8	English Miles

Also

	divid. by	give
Nails	4	Quarters of Yards, also of Ells
Quarters	4	Yards, also Ells

7. *Of Superficial Land Measure.*

	divid. by	give
Perches or Poles	40	Roods or Quarters of Acres
Roods	4	Acres

8. *Of*

8. *Of Time.*

Minutes	divided by	60	give	Hours
Hours		24		Days
Days		7		Weeks

Hitherto hath been ſpoken of the Simple parts of Arithmetick; now we ſhould come to treat of the Comparative part, *viz. of Arithmetical and Geometrical Proportion continued*, it being the next thing in Courſe; but becauſe the true Notion thereof will require ſomewhat more Knowledge in Arithmetick, than the Learner is yet acquainted with; and partly becauſe the Knowledge thereof, is not abſolutely neceſſary for the Merchant, or Trading Man; therefore I ſhall omit it in this place (and diſcourſe of it amongſt thoſe Rules which more nearly concern the Mathematical Student) and immediately come to Geometrical Proportion diſcontinued, wherein you will find the Exerciſe of all the Rules before laid down in this Book.

CHAP. VII.

Of Geometrical Proportion discontinued, or of the Golden Rule.

1. IF of four Numbers the first be to the second as the third is to the fourth, that is, if the Quotient of the former two divided be equal to the Quotient of the latter two divided, those four Numbers are said to be proportional.

2. Therefore if four Numbers be proportional, the Product of the Means is equal to the Product of the Extremes. As if 2. 4 : : 8. 16. 2 times 16 is equal to 8 times 4, *viz.* 32.

3. Therefore if the Product of the two Means be divided by the first, the Quotient is equal to the last Number or Term, as 4 times 8 is 32, which being divided by 2 the Quotient is 16 the last Term.

4. From hence ariseth the Knowledge of a fourth Number, which shall have such proportion to one Number given, as other two Numbers given have the one to the other, and this is called the *Rule of Three*, or, for the Excellency thereof, the *Golden Rule*, and herein we are to consider the Division, Disposition, Operation, and Use.

5. The *Rule of Three*, or *Golden Rule*, is either Simple or Compound, the Simple is either direct or indirect, and the Compound Rule is likewise direct

direct or indirect, aſcending or deſcending.

6. The *direct Rule of Three* is, if the ſecond Term be greater than the firſt, the fourth Term ſhall likewiſe be greater than the third, if leſſer, then leſſer: Or in the Queſtion, if more require more, or leſs, leſs, the Queſtion is to be anſwered by the *Golden Rule direct*, as if 4 Yards coſt 12 *s.* then 8 Yards muſt needs require more, or a greater number of Shillings, therefore that Queſtion is ſaid to be of the *Rule of Three direct*.

7. The manner of placing the Terms for Work is after the firſt Term to ſubſcribe two points, after the ſecond four, after the third two, as here 1 : 2 : : 3 : 4.

And by this means the Rule may be expreſſed in one or two lines. In this Rule that which you are mainly to conſider is, the 1*ſt.* and 3*d.* Terms are always of one Denomination or Name, and ſo are the ſecond and fourth, as if the firſt be Money, Weight, Meaſure, Time, *&c.* of the ſame Name is the third, and ſo muſt the fourth (which is ſtill ſought) be of the ſame Name with the ſecond, and the Terms of the Rule are to be read or underſtood ſo accordingly.

Example.

3 *l.* Pepper : 18 *s* : : 5*l.* : 30*s.* to be read thus, if 3 *l.* Pepper coſt or will give 18 *s.* then 5 pound of Pepper will give or coſt 30 *s.*

8. Omit in ſetting down the Queſtion that which is common to both ſides of the Figure, as if 20 Tun of Wine coſt 4 *d.* for the carriage of it 6 Miles,

Miles, then what will the Carriage of 20 Tun for 12 Miles coſt? Here 20 Tun is to be omitted, becauſe it bears no ſtreſs in the Queſtion, for the ſame Quantity that is carried 6 Miles is ſuppoſed to be carried 12 Miles, therefore the Queſtion will ſtand thus, 6 : 4 : : 12 :

8. The *three numbers given* are ſo to be diſpoſed, that the number whereof the value or price is ſought, or the Number demanding is to be placed in the 3*d*. place, or the firſt Term in the ſecond part of the Figure, the Term that is of the ſame denomination which is ſet in the firſt place, and the Number remaining ſet in the ſecond place, of which Nature the Number ſought muſt ever be: As in this Example, what will 9 Yards coſt, if 2 Yards be bought for 5*s*.? thus to be ſet down, 2 : 5 : : 9.

If 12 Yards of Damask will line 16 Yards of Velvet, how much Damask will line 24 Yards of Velvet? Thus to be ſet down, 16 : 12 : : 24 : 18.

9. After you have thus orderly diſpoſed the places for finding out the fourth Term, multiply the ſecond and third Terms together, and divide the Product by the firſt Term, the Quotient is the fourth Term or Number required, which is of the ſame Name with the ſecond Number.

And therefore if an Unite be in the firſt place, the fourth term is gotten by *Multiplication.*

Therefore if an Unite be in the ſecond or third places of the Rule, the fourth is gotten only by *Diviſion.*

1. *Example.*

1. Example.

If 24 Yards of Cloth cost 72 Shillings, how much will 564 Yards of the same cost at that Rate? Answer, 1692 *s.* or 84 *l.* 12 *s.*

The Operation.

```
Yards        Shill.            Yards       s.
 24     :     72     : :        564   :   1692
             564
            -----
             288
            432
           360
          -------                    s.       l.   s.
24)       40608                (1692  or  84   12
          ....
           24
          -----
           166
           144
          -----
            220
            144
           -----
             48
             48
            ----
            (0)
```

In

In order to anſwer this Queſtion, according to the Direction in Sect. 7. of this Chapter 564 Yards muſt be my third Number, becauſe the Demand is affixed to it, and that which is of the ſame Denomination or Name with it muſt be the firſt Number, *viz.* 24 Yards, and the Number remaining, which is 72 *s.* muſt be the ſecond Number in ſtating the Queſtion; and then the Numbers given will ſtand in order, fit for Operation, as you ſee before.

Then according to the 9. Sect. I multiply the ſecond Number by the third, *viz.* 72 by 564, and the Product is 40608, which I divide by the firſt Number (24) and the Quotient is 1692 and is the Anſwer to the Queſtion, which is 1692 Shillings or 84 *l.* 12 *s.* So that I conclude 84 *l.* 12 *s.* is the price of 564 Yards.

10. If when the given Numbers are placed in order (according to the Directions laid down in the eighth Sect. of this Chap.) the firſt and third Numbers be not of one Denomination they muſt be ſo reduced, or if one or both of them be compoſed of divers Names or Denominations, they muſt both of them be reduced to the leaſt Name mentioned in either of them, by the Rules in the laſt Chapter, and then proceed to multiply the ſecond by the third, *&c.*

2. *Example.*

If 1 *C.* of Tobacco be worth 56 Shillings, what will 36 *C.* 1 *qr.* 14 *l.* coſt at that Rate?

The

The Numbers being placed in order will ſtand as followeth, *viz.*

C.		*s.*		*C.*	*qr.*	*l.*
1	:	56	: :	36	1	14

Then becauſe the loweſt Name mentioned in the third Number is Pounds, I reduce the 36 *C.* 1 *qr.* 14 *l.* all into Pounds, and they make 4074 Pound, then becauſe the third Number is Pounds, therefore muſt the firſt Number be reduced likewiſe to Pounds, *viz.* 112 *l.* then multiply the third Number (4074) by the ſecond (56) and let the Product (which is 228144) be divided by the firſt Number (*viz.* 112) and the Quotient is 2037, which is Shillings, becauſe the ſecond Number is Shillings, which being reduced will be 101 *l.* 17 *s.* 00 *d.* See the whole Work as followeth.

C.	s.	C.	qr.	l.	
1 :	56 :	: 36	1	14	
4		4			
4		145			
28		28			
112		1164			
		291			
		4074			
		56			
		24444			
		20370			
	112)	228144	(203 / 7		
		224		*l.*	*s.*
				101	17
		414			
		336			
		784			
		784			
		(0)			

10. And if the second Number consist of divers Denominations, it must be reduced to the least Name mentioned, and lower if you please, and then proceed to multiply and divide as before is directed, and the fourth Number found will be of the same Name that you reduced your second Number to.

3. *Example.*

If 28 *l.* of Tobacco cost 3 *l.* 05 *s.* 04 *d.* what will 24 C. Weight cost at that Rate?

The Question stated and wrought.

```
 l.     l.  s.  d.          C.        l.   s.
 28  :  3  05  04  ::      24    :   313  12
       20                   4
       --                 ----
       65                  96
       12                  28
      ---                 ----
      134                 768
       65                 192
      ---                ------
      784                2688
                          784
                        -------
                         10752
                        21504
                       18816
                     ---------
               28)    2107392  (75264
                      196
                      ---
                       147
                       140
                       ---
                         73
                         56
                        ---
                        179
                        168
                        ---
                         112
                         112
                         ---
12)  75264  (6272|2      (0)

           l.     s.
 facit    313    12
```

In the foregoing Example the firſt Number is 28 *l.* and the third Number is 24 *C.* therefore is the 24 *C.* reduced to Pounds to make it of the ſame Name, with the firſt, and the ſecond Number, *viz.* 3 *l.* 05 *s.* 04 *d.* is reduced to Pence, becauſe the leaſt Name mentioned therein is Pence, ſo then are the given Numbers prepared for Operation, and then the Queſtion may be altered to this, *viz.* If 28 *l.* of Tobacco coſt 784 Pence, what will 2688 *l.* coſt at that Rate? Multiply and divide according to the Directions given in Sect. 9. p. 109. and you will find the Anſwer to be 75264 Pence, which being reduced is 313 *l.* 12 *s.*

4. *Example.*

Bought 3 Hogſheads of Sugar, each weighing $4\frac{1}{4}$ *C.* 21 *l.* at 2 *l.* 08 *s. per C.* I demand the price of the 3 Hogſheads at that Rate? Anſ. 31 *l.* 19 *s.*

Firſt, find the Content of the 3 Hogſheads, which is thus done, *viz.* reduce $4\frac{1}{4}$ *C.* 21 *l.* into Pounds, it makes 497 *l.* for the Content of one Hogſhead in Pounds, and that being multiplyed by 3 produceth 1491 *l.* for the Content of the 3 Hogſheads; Then may the Queſtion be ſtated as followeth, *viz.*

```
       l.      s.          l.
      112  :   48   : :   1491
                            48
                         ------
                         11928
                         5964
                         ------
               112)      71568  (63|9
                         ...     --
                         672      l.   s.
                         ------   31   19
                         436
        l.     s.        336
facit   31     19        ------
                         1008
                         1008
                         ------
                          (0)
```

5. Example.

Bought 126 Ells English, of Holland Cloth, to give for it after the Rate of 6 *s.* 6 *d. per* Ell Flemish. I demand what it all amounts to at that Rate? Answer 68 *l.* 05 *s.*

The Numbers being orderly placed according to the Directions given in the 8*th.* Sect. of this Chap. will be as followeth, *viz.*

Ell Fl.		*s.*	*d.*		*Ells Eng.*
1	:	6	06	: :	126

In

In this Queſtion the firſt Number and the third are both Ells, and yet they are of a different Denomination; for the firſt Number is 1 Ell Flemiſh, and the third Number is 126 Ells Engliſh, but they may be both reduced to quarters of Yards, by multiplying the Ells Flemiſh by 3, and the Ells Engliſh by 5, and the ſecond Number muſt be reduced to (78) Pence, then will the Queſtion be ſtated, and wrought as followeth, *viz.*

	Ell Fl.		*s.*	*d.*		*Ells Eng.*		*l.*	*s.*
	1	:	6	06	: :	126	:	68	5
	3		12			5			
qr.	3		78 *d.*			630 *qr.*			
			630						
			2340						
			468						
	3)		49140	(16380					

	d.		*l.*	*s.*	*d.*
facit	16380	*or*	68	05	00

6. *Example.*

What is a Wedge of Gold worth that weigheth 4 *oz.* 06 *p.w.* 15 *gr.* at 37 *l.* 17 *s.* 10 *d. per* Pound? Anſwer, 13 *l.* 13 *s.* 06$\frac{2106}{5760}$*d.* as appears by the following Operation.

	l.		*l.*	*s.*	*d.*		*oz.*	*p.w.*	*gr.*	
If	1	:	37	17	10	: :	04	06	15	
	12		20				20			
	12	:	757	:			86			
	20		12				24			
	240		1514				349			
	24		758				173			
	960		9094				2079	*Grains*		
	48						9094			
	5760						8316			
							18711			
							187110			
			5760)				18906426	(3282$\frac{2106}{5760}$ *d.*		

Facit 3282$\frac{2106}{5760}$ Pence, which being reduced will be found to be 13 *l.* 13 *s.* 06$\frac{2106}{5760}$ *d.*

Obſerve that in order to anſwer the foregoing Queſtion, I firſt reduce the third Number, (which is 04 *oz.* 06 *p.w.* 15 *gr.*) into Grains, and it makes 2079; then I reduce the firſt Number (which is 1 Pound)

1 Pound) to the ſame Denomination with the third, and it makes 5760 Grains; the ſecond Number (which is 37 *l.* 17 *s.* 10 *d.*) I likewiſe reduce to the leaſt Name mentioned, and it makes 9094 Pence, and then I proceed to finiſh the Work according to the Directions in Sect. 9. Page 109. *&c.*

11. If after you have divided the Product of the ſecond and third Numbers by the firſt, any thing remain, you may find out the value thereof by the following Rule, *viz.*

Multiply the ſaid Remainder by the Number of the known parts of the next inferiour Denomination, which are equal to an Unite in the Quotient, and divide the Product by the firſt Number, and the Quotient will be the Value deſired, in the ſaid next inferiour Denomination; and if any thing yet remain after that Diviſion is over, multiply it by the parts of the next inferiour Denomination, equal to an Unite of the laſt Quotient, and divide as before, *&c.* Proceed thus till you have reduced the Value aforeſaid, as low and as near as you deſire; An Example or two will ſufficiently explain this Rule.

7. *Example.*

If 13 Yards of Broad-cloth coſt 29 *l.* what will 6 Pieces, each Piece containing 24 Yards, coſt at that Rate? Anſw.

Multiply the 6 Pieces by 24, or 24 by 6, and the Product is 144, which is the Number of Yards in the 6 Pieces, then the given Numbers being

orderly placed as formerly hath been directed, will be 13 Yards, 29 *l.* and 144 Yards, the second and third being multiplied produce 4176, which being divided by the first Number (13) the Quotient is 321 *l* and there is a Remainder of 3 after the Division is ended, so that I conclude the Answer to be 321 *l.* and something more, because there is such a Remainder: Now to find the Value thereof, I consider the Quotient is Pounds, therefore I multiply the said Remainder (3) by 20, and the Product (which is 60) I divide by 13 (the first Number) and the Quotient is 4 Shillings, and there is still a Remainder of 8, which I again multiply by 12 (the Pence in a Shilling) and the Product is 96, which I likewise divide by 13, and the Quotient is 7 Pence, and there is yet a Remainder of 5, which I multiply by 4 (the Farthings in a Peny) and the Quotient (20) I again divide by 13, and the Quotient is 1 farthing, and there remaineth 7; so that I conclude the Price of 144 Yards of Cloth at the Rate propounded is 321 *l.* 04 *s.* 07 *d.* $1\frac{7}{13}$ *f.* View the whole Operation as followeth.

If 13

```
                                   24
                                    6
                                 ------
       yds.       l.             yds.        l.  s.  d.
   If   13   :    29    : :      144   :    321  4  7 1/4
                 144
               -------
                 116
                116
                29
               -------
       13)     4176   (321 l.
               ...
                39
              -------
                 27
                 26
              -------                  l.   s.   d.   f.
                 16    facit    321   04   07   1 7/13
                 13
               -------
     Rem.         3
                 20
               -------
         13)     60   (4  Shillings
                 52
     Rem.         8
                 12
               -------
         13)     96   (7  Pence
                 91
               -------
     Rem.         5
                  4
               -------
         13)     20   (1  Farthing
                 13
               -------
     Rem.        (7)
```

8. *Exam-*

8. *Example.*

If 05 *C.* Weight of Tobacco is worth 10 *l.* how much may be bought for 84 *l.* 16 *s*, at that Rate? Anſwer 42 *C.* 1 *qr.* $16\frac{160}{200}$ *l.*

The given Numbers being placed in order will be as followeth, *viz.*

l.		*C.*		*l.*	*s.*
10	:	5	: :	84	16

Then reduce the third Number (84 *l.* 16 *s.*) into Shillings, and it is 1696, reduce likewiſe the firſt Number (10 *l.*) into Shillings, and it is 200; then multiply and divide, and you will find the Anſwer to be 42 *C.* and there is a Remainder of 80, which I multiply by 4 (the Quarters in a hundred Weight) and the Product is 320, which I divide by 200 (the firſt Number) and the Quotient is 1 Quarter of a hundred, and there remaineth 120, which I multiply by 28 (the Pounds in a Quarter of a hundred) and the Quotient is 3360, which I divide by 200, and the Quotient is 16 *l.* and there remaineth 160, ſo that I conclude the Anſwer to the Queſtion to be 42 *C.* 1 *qr.* $16\frac{160}{200}$ *l.* View the whole Work as followeth.

10 *l.*

l.		*C.*		*l.*	*s.*		*C.*	*qr.*	*l.*
10	:	5	: :	84	16	;	42	1	$16\frac{160}{200}$
20				20					
200				1696					

```
           1696
              5
         ------        C.
2|00)   84|80      (42
         ------
  Rem.     80
            4
         ------
2|00)    3|20   (1 Quarter
          200
         ------
  Rem.    120
           28
         ------
          960
         240
         ------
200)   33|60   (16 Pounds
         ------
  Rem.   160
```

9. Exam-

9. *Example.*

If a *Philips* Dollar be worth 4 *s.* $11\frac{1}{2}$ *d.* what are 781 of the same worth? Answer 193 *l.* 12 *s.* $05\frac{1}{2}$ *d.*

	Doll.		*s.*	*d.*		*Doll.*		*l.*	*s.*	*d.*
	1	;	4	$11\frac{1}{2}$	: :	781	;	193	12	$05\frac{1}{2}$
			12			238				
			59			6248				
			4			2343				
						1562				
	l.		238					*l.*	*s.*	*d.*
Far. in	1			960)		185878	(	193	12	$05\frac{1}{2}$

10. *Example.*

What is a Grain of Gold worth at 37 *l.* 17 *s.* 10 *d.* the Pound? Answer, 1 *d.* $2\frac{1776}{5760}$ *qrs.*

l.		*l.*	*s.*	*d.*		*gr.*		*d.*	*f.*
1	:	37	17	10	: :	1	:	1	$2\frac{1776}{5760}$
12		20							
12		757							
20		12							
240		1514							
24		758							
960		9094							
480		4	*f.*						
5760	5760)	36366	($6\frac{1806}{5760}$.						
		(1806)							

11. *Exam*

¶ 11. *Example.*

If a *C.* of Sugar cost 6 *l.* 13 *s.* what shall 1 ℔ cost? Answer 1 *s.* 2 *d.* 1 *f.*

```
 l.        l.   s.        l.    s.   d.
112   :    6   13   ::    1  :  1   02¼
          20
        -----
         133
          12
        -----
         266
        133
       ------
        1596
           4
       ------
112)  6384  (57                     d.
        ··           4)  57   (14¼
       560                ·
      -----               4
        784             -----
        784              17
       -----             16
        (0)             -----
                         (1) f.
```

12. *Exam-*

12. *Example.*

If one spend in a year 397 *l.* 18 *s.* and 11 *d.* what is that a day? Answer, 1 *l.* 1 *s.* 9 *d.* 2 *f.*

days		*l.*	*s.*	*d.*		*day*		*l.*	*s.*	*d.*	*qrs.*
365	:	397	18	11	: :	1	:	1	01	09	$2\frac{238}{365}$

```
   20
 ----
 7958
   12
 ----
15917
7959
-----
95507
    4
-----
365)382028 (1046
```

	l.	*s.*	*d.*	*qrs.*
960) 1046	(1	01	09	$2\frac{238}{365}$

13. *Example.*

If a piece of 48 yards cost 3 *l.* 16 *s.* what is that a yard? Answer 1 *s.* 07 *d.*

yds.		*l.*	*s.*		*yd.*		*s.*	*d.*
48	:	3	16	: :	1	:	1	07

```
        20
       ---   s.   d.
48)    76   (1   07
       48
      ----
       28
       12
      ----
48)  336 (7
     336
     ----
     (0)
```

14. *Example.*

14. *Example.*

If I buy 3 ℔ 5 Ounces of Gold Plate for 136 *l.* 13 *s.* 4 *d.* what may I sell one pound for after that Rate? Answer 40 *l.*

```
 l.  oz.        l.   s.  d.        l.       l.
 3   05   :   136   13   04   : :   1   :   40
12              20                 12
----          ------              -----
41 oz.        2733                12 oz.
                12
              ------
              5470
              2733
              ------
              32800
                 12
              ------
              65600
              32800
              ------                   l.
   41) 393600 (9600      240) 9600 (40
```

15. *Example.*

If an ounce of Silver be worth 5 *s.* 6 *d.* what is a pound of the same worth? Answer, 3 *l.* 6 *s.*

```
 oz.      s.  d.          l.        l.  s.
  1   :   5   06   : :    1    :    3   06
          12              12
        ------          ------
          66              12
          12
        ------
         132
         66
        ------    l.  s.
240)     792   (3   06
```

16. *Exam-*

16. *Example.*

If a Pound of any Commodity or Grocery cost 1 *s.* $1\frac{1}{2}$ *d.* what will one *C.* Weight of the same cost? Answer, 6 *l.* 6 *s.*

l.		*s.*	*d.*		*l.*		*l.*	*s.*
1	:	1	$01\frac{1}{2}$	: :	112	:	6	06
		12			54			
		13			448			
		4			560			
		54		960)	6048		(6	06

17. *Example.*

One paid for his Diet in 3 weeks and 4 days 22 *s.* 4 *d.* what is that a year? Answer, 16 *l.* 6 s. $00\frac{20}{25}$ *d.*

w.	*da.*		*s.*	*d.*		*days*		*l.*	*s.*	*d.*
3	4	:	22	4	: :	365	:	16	06	$00\frac{20}{25}$
7			12							
25			268							
			365							
			1340							
			1608							
			804							
		25)	97820	(3912						

12) 3912 (32 | 6 *s.*

l. *s.*

16 06

18. *Exam-*

18. *Example.*

A Soldier receives for his Pay in 3 Months and 2 days, 6 *l.* 5 *s.* what comes his pay to for a year? Answer, 26 *l.* 10 *s.* $6\frac{24}{86}$ *d.*

```
    da.
 28 2
  3
 ---         l.  s.      days       l.  s.  d.
If 86 days : 6   5  ::  365   :   26  10  06 24/86
                20
              ----
               125
               365
              ----
               625
              750
             375
            -----
86) 45625 (53|0        20) 53|0
    ...                ----
    430                    l.   s.   d.
   ----                    26   10   06 24/86
     262
     258
    ----
     (45)
      12
    ----   d.
86)  540  (6
     516
    ----
     (24)
```

Note that in ſolving the laſt Queſtion 4 Weeks is accounted 1 Month.

19. *Example.*

How many Florins ought I to receive for 70 *l.* 15 *s.* 6 *d.* each Florin being eſtimated at 3 *s.* 2 *d.*? Anſwer 447 Florins.

s.	*d.*		*Fl.*		*l.*	*s.*	*d.*		*Fl.*
3	2	:	1	::	70	15	06	:	447
12					20				
38					1415				
					12				
					2836				
					1415				

38) 16986 (447
152
178
152
266
266
(0)

20. *Example.*

How many Yards of Satten will 155 *l.* 18 *s.* buy at 13 *s.* 4 *d.* the Yard? Anſwer, 233 Yards $\frac{3}{4}$ and $1\frac{96}{160}$ Nail.

s.	*d.*		*yd.*		*l.*	*s.*		*yds.*	*qrs.*	*na.*
13	4	:	1	: :	155	18	:	233	3	$1\frac{96}{160}$
12					20					
30					3118					
13					12					
160					6236					
					3118					
			160)		37416		(233	3	$1\frac{96}{160}$	

21. *Example.*

If a piece of Velvet being 32 Yards cost 32 *l.* 5 *s.* 6 *d.* what will 5 Yards of that Velvet cost? Answer, 5 *l.* 00 *s.* $10\frac{10}{32}$ *d.*

yds.		*l.*	*s.*	*d.*		*yds.*		*l.*	*s.*	*d.*
32	:	32	05	06	: :	5	:	5	00	$10\frac{10}{32}$

```
       32 : 32 05 06 : : 5 : 5 00 10 10/32
            20
          -----
            645
             12
          -----
           1296
           645
          -----
           7746
              5
          -----                    l.  s.  d.
    32) 38730 (1210     240) 1210 (5  00  10 10/32
        ....                 1200
         32                 ------
        -----                (10)
          67
          64
        -----
           33
           32
        -----
           10
```

22. *Exam-*

22. *Example.*

If 23 ℔ of Pepper cost 30 *l.* 7 *s.* 9 *d.* what shall I pay for 50 ℔? Answer, 66 *l.* 01 *s.* $02\frac{8}{23}$ *d.*

l.		*l.*	*s.*	*d.*		*l.*		*l.*	*s.*	*d.*
23	:	30	07	09	: :	50	:	66	01	$02\frac{8}{23}$

```
        20
       ----
       607
        12
       ----
      1223
      607
      ----
      7293
        50
      ----
23) 364650 (15854    240)15854(66 01 02 8/23
     ------
      (8)
```


23. *Exam-*

23. *Example.*

If 100 Yards coſt 3 *l.* 6 *s.* 8 *d.* what ſhall 55 Yards coſt? Anſwer 1 *l.* 16 *s.* 8 *d.*

```
yds.    l. s. d.      yds.    l.  s.  d.
100  :  3  6  8  ::  55   :   1  16  08
        20
       ----
        66
        12
       ----
       140
        66
       ----
       800
        55
       ----
      4000
     4000
     -----
100) 440|00 (440       240) 440 (1  16  08
                            240
                            ---
                        12) 200 (16
                             ..
                            12
                            ---
                             80
                             72
                            ---
                            (8)
```

24. *Exam-*

24. *Example.*

One desires me to take as much Hops as comes to 11 *l.* 10 *s.* out of a bag of 2 *C.* 14 lb, that cost 27 *l.* 19 *s.* 6 *d.* what quantity must I have? Ans. $\frac{3}{4}$ *C.* $13\frac{5622}{6714}$ lb.

l. *s.* *d.*	*C.* *qr.* *l.*	*l.* *s.*	*l.*
27 19 06 :	2 0 14 : :	11 10 :	$97\frac{5622}{6714}$
20	112	20	
559	238	230	
12	2760	12	
1124	14280	2760	
559	1666		
	476		
6714			

l. *qrs.* *l.*

6714) 656880 ($97\frac{5622}{6714}$ *or* 3 $13\frac{5512}{6714}$

60426

52620

46998

5622

25. *Example.*

If an Ounce of Gold be worth 3 *l.* 14 *s.* 6 *d.* what are 5 ℔ 11 *oz.* worth? Anſwer, 264 *l.* 09 *s.* 6 *d.*

oz.		*l.*	*s.*	*d.*		*l.*	*oz.*		*l.*	*s.*	*d.*
1	:	3	14	06	: :	5	11	:	264	09	06

```
      20            12
     ----          ----
      74            71
      12
     ----
     154
      74
     ----
     894
      71
     ----
     894
    6258
   -----       l.  s.  d.
240) 63474 (264 09 06
```

26. *Example.*

26. *Example.*

What are 3 lb 4 *oz.* 6 *p.w.* 12 *gr.* of Gold worth at $1\frac{1}{2}$ *d.* the Grain? Answer, 120 *l.* 19 *s.* 06 *d.*

gr		*d.*		*l.*	*oz.*	*p.w.*	*gr.*		*l.*	*s.*	*d.*
1	:	$1\frac{1}{2}$	::	3	04	06	12	:	120	19	06

```
  2      12
  —      ——
  3      40
         20
        ————
        806
         24
        ————
       3226
       1613
       —————
      19356
          3
      —————  l.  s.  d.
 480) 58068 (120 19 06
        ..
      480
      ————
      1006
       960
      ————    s.
  24)  468  (19
      ————
```

Rem. (12) *half Pence, or* 6 *Pence.*

27. *Exam-*

27. *Example.*

A Goldsmith buys 9 Ounces and 12 p. Weight of Gold for 36 *l.* 12 *s.* what is that an Ounce? Answer, 3 *l.* 16 *s.* 3 *d.*

```
oz. p.w.     l.  s.       oz.     l.  s.  d.
 9  12   :   36  12  : :   1   :  3   16  03
20           20           20
--          ----          --
192          732          20
              20
        ---------
  192)  14640  (7|6
         1344   ---
        -------      l.  s.  d.
         1200        3   16  03
         1152
        -------
         (48)
          12
        -------
   192)  576 (3
         576
        -----
```

28. *Exam-*

28. *Example.*

If an Ounce of Gold be worth 3 *l.* 11 *s.* 6 *d.* what is a Peny Weight worth? Answer, 3 *s.* 6 *d.* $3\frac{12}{20}$ *qrs.* What is a grain worth? Ans. 1 *d.* $3\frac{72}{480}$ *qrs.*

```
oz.      l.  s.  d.         p.w.    s.  d.  qrs.
 1   :   3  11  06    : :   01   :  3  06  3 12/20
20      20
--      --
20      71
        12
       ---
       148
        71                        s.  d.  qrs.
       ---
20)   85|8 (42   12)42 (3   06  3 12/20  per p.w.
      ----           36
        18           --
                    (6)
         4
       ---
20)     72  (3 qrs.
        60
       ---
Rem.  (12)

oz.      d.      gr.     d.  qrs.
 1   :  858  : :  1   :   1  3 72/480  per grain
20
--
20
24
---                d.  qrs.
480)  858  (1  3 72/480  per grain
      480
      ---
      378
        4
     ----
480) 1512  (3 qrs.
     1440
     ----
       72
```

29. *Exam-*

29. *Example.*

What is due for a Pension of 3 *s.* 5 *d.* weekly, behind for 3 Years, 9 Months, and 10 Days? Answer, 33 *l.* 2 *s.* $4\frac{1}{7}$ *d.*

da.		*s.*	*d.*		*yea.*	*mo.*	*da.*		*l.*	*s.*	*d.*
7	:	3	5	: :	3	9	10	:	33	2	$4\frac{1}{7}$
		12			365	28					
		41			1095	252					
					252						
					10						
					1357						
					41						
					1357						
					5428						

7) 55637(7948 240)7948(33 2 $4\frac{1}{7}$ (*l.* *s.* *d.*)
Rem. (1)

30. *Exam-*

30. *Example.*

What is due to a Captain for 5 Months, 11 Days for his Pay, at 33 *s.* 6 *d. per* day? Answer, 252 *l.* 18 *s.* 6 *d.*

```
day      s. d.        mon. da.        l.  s. d.
1   :    33 6   : :   5  11   :     252  18  6
         12           28
       ----        -------
        402          151
                     402
                   -------
                     302
                    6040
                   -------      l.  s.  d.
          240)     60702     (252  18   6
                    480
                   -------
                    1270
                    1200
                   -------
                     702
                     480
                   -------
             12)     222     (18 Shillings
                      12
                   -------
                     102
                      96
                   -------
        Rem.  (6) Pence
```

Hitherto

Hitherto we have given Examples for the working the Golden Rule, only when the Queſtion is ſingle, and without any trouble in the Poſition, now I will give ſome few Examples, wherein ſome account muſt be made in ſetting down the Queſtion.

31. *Example.*

Two depart from one place, the one Eaſtward and the other Weſtward, the one travelleth 3 Miles a Day, the other 5 Miles a Day, how far are they diſtant the ninth Day after their Departure? Anſwer, 72 Miles. Here their firſt Days Diſtance is added together, which makes 8, and then it is 1 : 8 :: 9 : 72.

32. *Example.*

One ſelleth Cloth for 350 *l.* and gains after 10 *l.* in the 100 *l.* what was the Principal and what the clear Gain? Anſwer the clear Gain is 31 *l.* 16 *s.* 5 *d.* and the Principal 318 *l.* 5 *s.* 3 *d.*

The fourth Number in Proportion here is found to be 318 *l.* 3 *s.* 7 *d.* which is the Principal, and it being ſubſtracted from 350 *l.* the Remainder is 31 *l.* 16 *s.* 5 *d.* for the Gain.

l.		*l.*		*l.*		*l.* *s.* *d.*
110	:	100	: :	350	:	318 3 $7\frac{70}{110}$
				100		

$$
\begin{array}{r l}
 & \quad\quad\quad\; l.\;\; s.\;\; d. \\
110)\; 35000 & (318\;\; 3\;\; 7\tfrac{70}{100} \\
330 & \\
\hline
200 & \\
110 & \\
\hline
900 & \\
880 & \\
\hline
20 & \\
20 & \\
\hline
110)\; 400 & (3 \\
330 & \\
\hline
70 & \\
12 & \\
\hline
110)\; 840 & (7 \\
770 & \\
\hline
70 &
\end{array}
$$

l.	*s.*	*d.*
350	00	00
318	03	07
31	16	05

33. *Exam-*

33. *Example.*

There are 30 Cloths bought for 70 *l.* are sold for 80 *l.* Now if they had cost 80 *l.* how should they have been sold to have gained after the same rate? Answer, 91 *l.* 8 *s.* $6\frac{60}{70}$ *d.*

l.	l.	l.	l. s. d.
70 :	80 ::	80 .	91 08 $06\frac{60}{70}$

80

l. s. d.

70) 6400 (91 08 $06\frac{60}{70}$

630

100

70

Rem. (30)

20

s.

70) 600 (8

560

(40)

12

d.

70) 480 (6

420

60

Therefore had the Cloths cost 80 *l.* they should have been sold for 91 *l.* 8 *s.* $6\frac{60}{70}$ *d.*

34. *Exam-*

34. *Example.*

One ſold ſo much Velvet as yielded him 600 *l.* ready Money, wherein he gained clear above his Principal 60 *l.* what gained he upon every 100 of his principal? Anſwer 11 *l.* 2 *s.* 2 *d.* $2\frac{360}{540}$ *qrs.*

l.		*l.*		*l.*		*l.*	*s.*	*d.*	*qrs.*
540	:	60	: :	100	:	11	2	2	$2\frac{360}{540}$

```
          100
       ----------  l.
540) 6000 (11
      540
      ---
      600
      540
     -----
Rem. (60)
      20
     -------  s.
540) 1200 (2
     1080
     -----
Rem. (120)
      12
     -------  d.
540) 1440 (2
     1080
     -----
Rem. (360)
       4
     -------  qrs.
540) 1440 (2
     1080
     -----
Rem. (360)
```

35. *Example.*

One (suppose *A*) flying every day 40 Miles, is pursued the fourth day after by another (suppose B) 50 Miles a day, in how many days, and after how many Miles Travel will *A* be overtaken? Ans. at the end of 12 days: for *B* rideth 50 Miles, *A* but 40, and 50 less by 40 is 10 Miles, the clear gains of one day; but because *A* was fled 3 days Journy, that is 120 Miles before *B* pursued, I say, 10 *m.* 1 *d.* :: 120 *m.* 12 *days.*

Then I say again, if 1. 50 :: 120. 600 Miles that *B* must go before *A* be overtaken: Thus sometimes for the difficulty of a Question, the Golden Rule must be used more than once, before a full Answer be had.

12. Thus have I run over very many Questions, whereby the excellency of this Rule is perceived, the effects whereof are very many, as (1) by the Price or Value of one thing to find the Value of many, (2) by the Price or Value of many to find the Value of one, (3) by the Value of many to find the Value of many, *&c.*

13. If you will prove the Rule according to Sect. 3. the Product of the first Term given, and the last found, ought to be equal to the Product of the second and third.

CHAP.

CHAP. VIII.

Of the Rule of Three indirect, or of the Inverſe Rule of Proportion.

THIS Rule is called the *Inverſe or Backward Rule of Proportion*, becauſe it inverts the Practice of the former Rule: For in this Rule the firſt and the ſecond Numbers are to be multiplied, and the Product is to be divided by the third Number; and it is eaſily known, whether the Queſtion muſt be wrought by this Rule or no; for if more require leſs, or leſs more, the Queſtion is wrought by this Rule, as will more evidently appear by Examples. As if 4 Horſes in ſix days eat 10 Buſhels, 8 Horſes muſt needs eat 10 Buſhels in a leſſer time, and here 10 Buſhels (the common Term) is omitted in the figure, and the Queſtion ſtands thus; 4. 6 : : 8. 3. 8 Horſes will eat them in 3 Days.

2. *Example.*

If a Two-peny Loaf of Bread weighed *6 l. 3 oz.* when a Bole of Rye coſt *6 s. 6 d.* what is a Bole of Rye worth when a Two-peny Loaf of Bread weigheth but 2 *l.* 4 *oz.*?

The Numbers in the Queſtion being ſtated in order according to the 8*th.* Sect. of Chap. 7. Page 109. will be as followeth.

l.	*oz.*		*s.*	*d.*		*l.*	*oz.*
6	3	:	6	6	: :	2	4

Which is as much as to ſay, if 6 *l.* 3 *oz.* require 6 *s.* 6 *d.* what will 2 *l.* 4 *oz.* require?

Here I conſider that the leſſer the two-peny Loaf weigheth, the dearer is the Bole of Rye, wherefore if the Bole of Rye coſt 6 *s.* 6 *d.* when the two-peny Loaf weigheth 6 *l.* 3 *oz.* it muſt needs coſt more than 6 *s.* 6 *d.* when the two-peny Loaf weigheth but 2 *l.* 4 *oz.* and conſequently the third Number (which is leſſer than the firſt) requireth more than (6 *s.* 6 *d.*) the ſecond Number.

Therefore after due Reduction (of the given Terms) is made, I multiply the firſt and ſecond, and divide by the third, and find the Anſwer to be 17 *s.* $10\frac{1}{2}$ *d.* The whole Work followeth.

l.	oz.		s.	d.		l.	oz.		s.	d.
6	3	:	6	6	: :	2	4	:	17	$10\frac{1}{2}$
16			12			16				
99			78			36				
			99							

702
702

				s.	*d.*
36) 7722 (214		12)	214	(17	$10\frac{1}{2}$
Rem. (18)			12		
4			94		
36) 72 (2			84		
72		*Rem.*	(10) *pence*		

3. *Example.*

3. Example.

One borrowed Money, *viz.* 4000*l.* for 3 years of his Friend, which when he came to restore the Debt, his Friend would take no consideration, but only desired that he would make him satisfaction by lending him a Sum another time when he stood in need, and he afterwards lent him 7480*l.* how long was he to detain that Sum to requite the former Courtesie? Ans. 1 year, 7 months, 7 days.

The given Numbers being orderly stated will be as followeth,

l.		*yea.*		*l.*
4000	:	3	: :	7480

To answer which Question I consider that he who had the 7480 *l.* ought not to keep it so long as the other kept the 4000 *l.* because it is a greater Sum, so that here more requireth less, wherefore I multiply the first by the second, and divide by the third, and the Answer is 1 year, 7 months, and $7\frac{4040}{7480}$ days. See the Operation.

l.	*yea.*		*l.*	*ye.*	*m.*	*days.*
4000	: 3	: :	7480	: 1	7	$7\frac{4040}{7480}$

```
          4000
             3
          -----  yea. mo. days
7480)    12000  (1   7   7
          7480
          -----
Rem.     (4520)
            12
          -----  mon.
7480)    54240  (7
         52360
          -----
Rem.     (1880)
            30
          -----  days
7480)    56400  (7
         52360
          -----
Rem.     (4040)
```

4. *Example.*

A City besieged hath Victuals to maintain 5000 Souldiers six Months; it is desired to know what Souldiers it will maintain for 9 Months? Answer, 3333 Souldiers.

6 mon.

mon.		*Sould.*		*mon.*		*Sould.*
6	:	5000	: :	9	:	$3333\frac{3}{9}$
		6				

9) 30000 (3333

Rem. (3)

5. *Example.*

If 20 Pyoneers can finiſh a Work in 2 Months and 10 Days, how many Pyoneers will finiſh that Work in 20 Days? Anſwer, 69.

mon.	*da.*		*Pyon.*		*da.*		*da.*
2	10	:	20	: :	20	:	70
30			70				

70   2|0) 140|0 (70 *da.*

(0)

6. *Example.*

How much Pluſh is neceſſary to line a Cloak which hath in it 4 Yards of Cloth, of one Yard 3 quarters broad, when the Pluſh is but 3 quarters and ½ broad? Anſwer, 8 Yards.

Here I conſider that the narrower the Pluſh is, the more Yards in length will be required to line the ſaid Cloak, and that if it were a Yard and 3 Quarters wide, (as the Cloth is) then ought there

alſo to be 4 yards in Length, wherefore the Proportion is inverſe, as followeth.

yd.broad.	*yds.long.*		*qrs. broad.*	*yds.*
$1\frac{3}{4}$	: 4	: :	$3\frac{1}{2}$	: 2
4			2	
7 *qrs.*			7 *half qrs.*	
2			4	
14 *half qrs.*		14)	28 (2	
			28	
			(0)	

7. *Example.*

If for the matting of a Room there needs 100 yards of yard broad, how many yards will be needful of $1\frac{1}{2}$ broad? Anſwer, $66\frac{2}{3}$ yards.

This Example is of the ſame Nature with that foregoing, and therefore needs no further Explanation; the Operation is as followeth.

1 *yd. broad*

yd.broad.	*yds.long.*		*yds.broad.*	*yds.long.*
1 :	100	: :	$1\frac{1}{2}$:	$60\frac{2}{3}$
2	2		2	
2	3) 200 ($66\frac{2}{3}$	*yds.*	3 *half yds.*	

18

20
18

Rem. (2)

8. *Example.*

In an Acre of Land, if the breadth be 4 Perches, the length must be 40, now if the breadth be $8\frac{1}{2}$, what length must I have to make the Acre, Answ. $18\frac{14}{17}$ Perches.

4	:	40	: :	$8\frac{1}{2}$
2		8		2
8		320		17

17) 320 ($18\frac{14}{17}$
17

150
136

(14)

9. *Exam.*

9. *Example.*

If when a Tun of Wine is worth 40 *l.* a quantity worth 15 *l.* is sufficient for the Ordinary of 120 Men, how many Men will the same 15 *l.* worth suffice when the Tun is worth 60 *l.*? Ans. 80 men.

The dearer the Wine is, the less is to be had for the money, and consequently the fewer Men will the same 15 *l.* worth satisfie, wherefore by an inverted Proportion, I find the Answer to be 80 Men, See the Operation as followeth.

l.		*men*		*l.*		*men*
40	:	120	: :	60	:	80
		40				

6|0) 480|0 (80
48
(0)

10. *Example.*

A Traveller makes a Journey of 640 Miles in 30 days, when the Day is 16 hours long, in how many days will he perform the same, when the day is 10 hours long? Answer, in 48 days.

CHAP.

CHAP. IX.

Of the Double Golden Rule, or Compound Rule of five Numbers.

1. THIS Rule is called *double* or *compound*, because it is wrought by two Works of the *Golden Rule*; and is called the *compound Rule* of five Numbers, because the given Terms are always five, wherefore three are *conditional* and *Antecedents* or Suppositions, the other two are *interrogative*, and are Consequents, matching or answering some of the former Antecedents, insomuch that there are as many Consequents as Antecedents; one less of like kind or denomination with the Antecedents.

2. For the right placing the Terms in this Rule, first in three Terms set down the conditional part, let that which is the principal Cause of Loss or Gain, Interest or Decrease, Action or Passion be put in the first place, that whose Sirname betokeneth the space of Time, distance of Place, *&c.* be put in the second, and the other remaining be put in the third place; then the conditional part being rightly placed, the two Terms, wherein the demand lieth, must correspond to some of the conditional Terms, and be placed under them.

This Rule is likewise direct or converse, but lest the Reader should be troubled with too many Distinctions, take this one Rule for both: After you have

have placed the Queſtion by Section 2. then if the Term ſought fall under the third Term, the Rule is direct and thus to be wrought: (1) Multiply the three laſt Terms for a Dividend, and the two firſt for a Diviſor, finiſh Diviſion hereby, and the Quotient is the Term ſought. (2) But if the Term ſought fall under the firſt or ſecond Term of the conditional Part, then the Rule is, Multiply the firſt, ſecond, and laſt for Dividend, and the third and fourth for Diviſor, finiſh Diviſion hereby, and the Quotient will give the Term that is wanting. Examples will make the Rules more perſpicuous.

1. *Example.*

If 12 Rood of Ditching be wrought by 2 Men in 6 days, how many Rood ſhall be wrought by 8 Men in 24 days? According to Section 2. I place 2 for Men in the firſt place who are the principal Cauſe, 6 days in the ſecond, 12 in the third, and their match Terms under them; which done, I perceive the Blank, or Term unreſolved, falls under the third Term, therefore according to the firſt Rule, I multiply the three laſt Terms for Dividend, and the two firſt for Diviſor, which done the Quotient gives me 192 for the ſixth term or Reſolution of the Queſtion. See the following Work.

2 *men*

men	*days*	*Rood*
2.	6.	12
8.	24.	

$$\begin{array}{rrl} 12 & 6 & \\ 8 & 2 & \\ \hline 96 & 12 & \textit{Divisor} \\ 24 & & \\ \hline 384 & & \\ 192 & & \\ \hline \end{array}$$

12) 2304 (192

2. *Example.*

If 12 Rood require 2 Men to work it in 6 days how many men will 192 Rood require to work it in 24 days?

After the Position of the Terms given in the Queſtion according to the ſecond Sect. the Blank falls under the firſt place, and therefore the Rule is inverſe, and the Anſwer found to be 8 Men. See the Operation.

2. *men*

men	*days*	*Rood*
2.	6.	12.
	24.	192.

```
        2        24
        6        12
      ----     ----
       12       288 Divisor
      192
      ----
       24
     108
     12
     -----
288) 2304 (8 men
     2304
     ----
     (0)
```

3. *Example.*

If 2 men work 12 Rood in 6 days, in how many days will 8 men work 192 Roods?

The Numbers given in this Example being placed according to Sect. 2. the Blank falls under the ſecond place of the conditional part, and conſequently by the ſecond Rule in Sect. 3. the Rule is Inverſe, and the Anſwer to the Queſtion is 24 days. See the following Work.

2. *men*

men	*days*	*Rood*
2.	6.	12.
8.		192.

```
   2        12
   6         8
  ----     ----
  12        96 Divisor
 192
 ----
  24
 108
 12
 ----
96) 2304 (24 days
 192
 ----
  384
  384
 ----
  (0)
```

4. *Example.*

If 100 *l.* in 12 months gain 10 *l.* what shall 50 *l.* gain in 6 months? Answer, 2 *l.* 10 *s.*

Here 100 *l.* being the principal Cause of Interest must be put in the first place, and 12 months which signifie space of Time, must possess the second, and 10 *l.* the third place, and 50 *l.* and 6 months being placed

placed under their correſpondent Terms, in the conditional part, the Blank will fal' under the third place, therefore the Rule is direct, and the Anſwer by the firſt Rule in Sect. 3. is found to be 2 *l.* 10 *s.* See the Operation.

l.	*months*	*l.*
100.	12.	10.
50.	6.	

```
     10                        100
    ----                        12
    500                       -----
      6                       1200 Diviſor
   ------ l.  s.
1200) 3000 (2 10
```

5. *Example.*

If 10 *l.* gain come of 100 *l.* principal in 12 months, what Principal ſhall 50 Shillings gain come of in 6 months? Anſwer by the ſecond Rule 50 *l.*

The Numbers in the conditional Part of this Queſtion, are placed as in the laſt Example, only becauſe the gain mentioned in the Demand is given in Shillings (*viz.* 50 *s.*) therefore is the gain in the conditional part (*viz.* 10 *l.*) turned into Shillings (200 *s.*) the Blank falling under the firſt place, wherefore the Rule is inverſe, *&c.* and the Anſwer is found by the ſecond Rule to be 50 *l.* View the following Operation.

100 *l.*

l.	*months.*	*Shillings.*
100.	12.	200.
	6.	50.

```
           100
            12
         ------
          1200        200
            50          6
         ------ l.   -----
12|00)  600|00 (50   1200 Divisor
        60 |
        ---
        (00)
```

6. *Example.*

If in 12 months 100 *l.* give 10 *l.* Intereſt, in how many months ſhall 50 *l.* give 2 *l.* 10 *s.* Intereſt? Anſwer, 6 months.

For the reaſon given in the laſt Example, the Intereſt given in both parts of the Queſtion muſt be reduced to Shillings, and then by the ſecond Rule the Anſwer will be found to be 6 months. See the following Operation.

l.	*months*	*Shillings*
100.	12.	200.
50.		50.

100		200
12		50
1200		10000 *Diviſor*
50		

10000) 60000 (6 *months*

And after this manner may any Queſtion, reſolvable by the two firſt parts of this Rule, be performed.

4. But if 4 Terms of Explanation or Conditionality precede the Term of the Queſtion ; As,

How many { Of the firſt will make ſome Number of the laſt Sirname, the Queſtion is anſwerable by the compound Rule *deſcending*.
Of the laſt named will countervail ſome certain Number of the firſt named, the Queſtion is anſwerable by the compound Rule *aſcending*. }

The

The Work of the compound Rule descending.

Multiply the first, second, and fifth Terms for the Dividend, and multiply the second and fourth for the Divisor, finish Division thereby, the Quotient gives the Answer.

7. *Example.*

If 2 Angels countervail 20 *s.* and 12 *s.* countervail 2 Crowns, how many Angels will countervail 10 Crowns? Answer 6 Angels. See the Work.

2 *Ang.*	20 *Shillings*
12 *Shill.*	2 *Crowns*
24	40 *Divisor*
10 *Crowns*	

40) 240 (6 *Angels*

The Work of the compound Rule ascending.

Multiply the second, fourth and fifth Terms together for the Dividend, and multiply the first and third for the Divisor, finish Division, &c.

8. *Example.*

If 2 Angels countervail 20 *s.* and 12 *s.* countervail 2 Crowns, how many Crowns will countervail 6 Angels? Anſwer, 10 Crowns.

20	*Shill.*	2	*Ang.*
2	*Cr.*	12	
40		24	*Diviſor*
6	*Ang.*		

24) 240 (10 *Crowns*

5. If any Queſtion in any of theſe Rules have like Numbers in the Dividend and Diviſor, omit or cancel them to ſave Labour, and work only with the whole Numbers.

The Uſe and Effects of theſe four precedent Rules are of great moment; for upon the firſt two, not only all Queſtions of Merchandize, wherein time is bought and ſold, but alſo by them all Charges of War, touching Victuals, Soldiers Wages, Expences of Powder, caſting of Trenches, and other Military matters, are ſpeedily and aptly diſcuſſed: And upon the latter two of them depend the Equation of divers Barters, with the Exchange and Reduction of all manner of Moneys and Coyns, Weights and Meaſures of what nature ſoever, as may appear by the Queſtion following; wherein, leſt we may ſeem too tedious, the Operations are omitted. But how to ſolve Que-

ſtions

ſtions depending upon the two latter Rules,ſhall be more accurately ſhewed in the following Rule, called the Rule of *Exchange*, in Chap. 12.

Queſtions for practice of the precedent Rules.

1. If 30 Buſhels of Seed will yield in one year 360 Buſhels, how many will 80 Buſhels yield in 7 years? Anſwer, 6720.

2. If 6 Mowers will mow 45 Acres in 5 days, how many Mowers will mow 300 Acres in 6 days? Anſwer, $33\frac{2}{6}$.

3. If in ten days of 12 hours long a man may journey 300 miles, in how many days of 16 hours long may he travel 500 miles? Anſwer, $12\frac{1}{2}$.

4. If 12 peny worth of Wine ſatisfie 8 Perſons at a meal, when Wine is at 6 *d.* the Quart, how many ſuch Perſons will 20 peny worth of Wine ſatisfie when Wine is at 4 *d.* the Quart? Anſ. 20.

5. If 2 Plowes till 11 Acres in 6 days, how many Plows will till 45 Acres in 3 days? Anſ. $16\frac{4}{11}$.

6. If 500 Pyoneers caſt a Trench of 300 Rood long in 6 hours, how many Pyoneers will caſt a Trench of 160 Rood in 2 hours? Anſwer 800.

7. If 15 Ells of Stuff 3 quarters broad coſt 37 *s.* 6 *d.* what ſhall 40 Ells of the ſame Stuff coſt being Yard broad? Anſwer, 6*l.* 13 *s.* 4 *d.*

8. One buyeth Stuff for 22 *s.* 6 *d.* the piece ready money, and would ſell it again for 24 *s.* the piece, what time may he forbear his Money and yet gain after 9 *l.* in the 100 for 12 months? Anſwer, 8 months and 8 days.

9. One buyeth 20 yards of Satten for 12 *l.* 10 *s.* ready money, for how much more than it coſt may he ſell the Yard, giving 2 months day for Payment to gain after 10 *l.* in the hundred for 12 months?

For Anſwer, becauſe the Poſition is of 20 Yards and the Queſtion but what he may raiſe in the ſale of one yard only, and not in the whole 20 yards, I firſt learn the price of one Yard thus; 20 Yards, 12 *l.* 10 *s.* : : 1 *yd.* : 12 *s.* 6 *d.* Then having the price of one Yard, I place the Queſtion by the firſt Rule, and the Anſwer is 21 *s.* 8½ *d.*

10. A Town is beſieged wherein is 3000 Soldiers, who have ſufficient Victuals for 2 months; but they look for no aid to raiſe the Siege till 6 months; how many Soldiers may the Captain diſmiſs to make the Victuals ſerve ſo long? Anſ. he muſt diſmiſs 2000.

11. If 5 Cannons in 2 days ſpend 60 Barrels of Powder, what will 14 Cannons ſpend in 5 days? Anſ. 420.

12. If one Horſeman for one months Wages have 3 *l.* what Treaſure will pay the Wages of 4000 Horſemen 9 months? Anſ. 108000 *l.*

13. If 8 Tablers, when Wine is at 6 *d.* the Quart ſpend every Meal at their Ordinary 12 *d.* in Wine; when Wine is at 4 *d.* the Quart, what ſhall 16 Tablers ſpend a meal, not abating their uſual quantity?

For Anſwer, though the men ſeem to be the chief Cauſe of the ſpending of the Wine; yet becauſe their Number remaineth always one, and they drink no more at one time than at another;

the Cause therefore of the increase or decrease of this Expence, is rather to be attributed to the price of the Quart of Wine, whose alteration is the chief Cause of the increase or decrease of their Charges; therefore according to the Rule the Answer will be 1 *s.* 4 *d.*

14. If 4 peny worth of Bread suffice 12 Persons at a meal, when Wheat is at 20 *s.* the Quarter, how many peny worth will suffice 24 Persons when Wheat is at 15 *s.* the Quarter? Ans. 6 *d.*

15. If 15 Aulnes French make 100 Sticks Flemish, and 100 Sticks Flemish make 60 English Ells, how many Aulnes is 150 English Ells? Ans. by the third Rule 28 Aulnes.

16. At *Roan* *A* delivered *B* in exchange 100 Francks, every Franck being worth 50 Soulz Tourne, upon Condition to receive for the same at *London*, after 4 *s.* 11 *d.* for every Crown of 50 Soulz Tourne; how much sterling Money will pay the Bill of Exchange at *London*? For Answer, frame the Question thus, if 1 Franck countervail 20 Soulz, and 500 Soulz countervail 4 *s.* 11 *d.* how many Pence Sterling is 100 Francks? Ans. 2320 Pence.

17. If 4 *d.* *Paris* be worth 5 *d.* *Tournois*, and 10 *d.* *Tournois* be worth 12 *d* of *Savoy*, how many Pence of *Savoy* are 15 Pence *Paris*? Ans. 22$\frac{1}{2}$ of *Savoy*.

18. If 100 *l.* Weight of *Perose* be worth 90 *l.* Weight of *Sene*, and 100 Weight of *Sene* be worth 120 *l.* Weight of *Pyse*, how many Pounds Weight of *Perose* will weigh 324 Pound Weight of *Pyse*? Answer, 300.

19. If 4 Ells of *Antwerp* measure make 3 yards of *London* measure, and 15 yards of *London* measure contain 12 Ells of *Lyons* measure, how many Ells of *Antwerp* measure is contained in 60 Ells of *Lyons* measure? Answer 100.

20. If 35 Ells of *Vienna* make 24 at *Lyons*, and 3 Ells of *Lyons* 5 Ells of *Antwerp*, and 100 Ells of *Antwerp* 125 Ells of *Francfort*, how many Ells of *Francfort* make $42\frac{3}{4}$ Ells at *Vienna*? Ans. $60\frac{1}{4}$.

6. Questions in the double Rule of Three, may be solved by two single Rules of Three, if you dispose of the five given Numbers, according to the following Directions; *viz.*

Observe which of the given Numbers, in the conditional part of the Question, is of the same Denomination with the Number required, and let that be the second Number in the first Operation; and let either of the other two Numbers in the conditional part be the first Number, and let that Number in the Demand, which is of the same Name with the first, be placed for the third Number; As in the following Example.

Example.

If 120 Acres of Grass can be mowed by 12 men in 14 days, how many Acres may be mowed by 36 men in 56 days? Ans. 1440 Acres.

In order to resolve this Question at two single Rules of Three, I observe that the conditional Part of the Question is this, *viz.* If 120 Acres can be mowed by 12 men in 14 days, and the demand is

is how many Acres may be mowed by 36 men in 56 days.

Now the Number required is Acres, and the Number of the ſame Name in the conditional part is 120 Acres, therefore muſt 120 be the ſecond Number in the firſt Operation, and if I place 12 (which is in the conditional part and ſignifies 12 men) for the firſt Number, then muſt 36 in the Demand be the third Number:

Or if I place 14 days (in the conditional part) for the firſt Number, then muſt 56 days (in the Demand) be the third Number, and ſo, if the Numbers are placed according to the firſt Direction, then will they ſtand thus,

men		*Acres*		*men*
12	:	120	: :	36

Or if according to the ſecond Direction thus,

days		*Acres*		*days*
14	:	120	: :	56

There remains yet one Number in the conditional part, and another in the Demand, to be diſpoſed of, which muſt be thus ordered, *viz.* Let that Nnmber which is left in the conditional part, be put for the firſt Number in the ſecond Operation, and that left in the Demand for the third, and the fourth proportional Number found by the firſt Operation, muſt be the ſecond Number in the ſecond Operation.

This

This will eaſily be underſtood, by ſolving the foregoing Example.

And firſt I ſay (according to the firſt Direction) If 12 men can mow 120 Acres, how many Acres will 36 men mow? Which by the 1 ſt. Sect. of Chap. 8 in page 147. is found to be direct, and the Anſwer is 360 Acres; See the Operation.

men		*Acres*		*men*		*Acres*
12	:	120	: :	36	:	360
		36				
		720				
		360				
12)		4320	(360 *Acres*			

So that I have now diſcovered, that 36 Men can mow 360 Acres in 14 days, but the Queſtion is, how much they can mow in 56 days, which may be found out by a ſecond Rule of Three thus, *viz.* If 14 days require 360 Acres, what will 56 days require? This is likewiſe found to be direct, and the Anſwer is 1440 Acres. See the Operation.

Weeks		*Acres*		*Weeks*		*Acres*
14	:	360	: :	56	:	1440
		56				
		2160				
		1800				
14)		20160	(1440 *Acres*			

Or

Or if the Operation be according to the ſecond Direction, then the firſt Queſtion will be, *viz.*

If 14 Weeks require 120 Acres, what will 56 Weeks require? Anſwer, 480 Acres. See the Work as followeth.

Weeks		*Acres*		*Weeks*		*Acres*
14	:	120	: :	56	:	480

```
        120
         56
      -----
        720
       600
      -----
  14) 6720 (480 Acres
       56
      ---
       112
       112
      ----
       (00)
```

So that by this means I have found, that 12 men can mow 480 Acres in 56 Weeks, but the Queſtion is, how many Acres may be mowed in the ſame time by 36 men, which may be ſolved by a ſingle Rule Direct, and the Anſwer be the ſame as before, as in the following Operation, *viz.*

12 *men*

men		*acres*		*men*		*acres*
12	:	480	: :	36	:	1440

$$
\begin{array}{r}
480 \\
36 \\
\hline
2880 \\
1440 \\
\hline
\end{array}
$$

12) 17280 (1440 *Acres*

But if the Question fall under the double Rule of Three inverse, then will one of the Operations be direct, and the other inverse (for they never fall out to be both inverse) as in the following Example.

Example.

If Wine worth 15 *l.* is sufficient for the Ordinary of 100 men when it is worth 30 *l.* a Tun, how many men will be satisfied with Wine worth 3 *l.* when the Tun is worth 252 *l.* Answer 20 men.

The Number sought in the Question is *Men*, therefore must 100 Men be the second Number in the first Operation, wherefore I say, if 15 *l.* worth of Wine suffice 100 men, how many men will 03 *l.* worth suffice? This Question I find to be direct by the 1*st*. Sect. of Chap. 8. in Page 147. wherefore I multiply the second by the third, and divide the Product by the first, and the Quotient is 20 men. See the Operation.

15 *l.*

l.		*men*		*l.*		*men*
15	:	100	::	3	:	20
		3				
15)		300	(20 *Men*			
		30				
		(0)				

So that now I have found out, how many men will be ſatisfied with 3 pounds worth of Wine when it is worth 30 *l.* a Tun, but the Queſtion is, how many men will be ſatisfied with the ſame 3 pounds worth when the Tun is worth 25 *l.*? Wherefore I ſay in the ſecond place, if 30 *l. per* Tun require 20 men, how many men will 25 *l. per* Tun require? Anſwer 24 men.

This Queſtion is Inverſe, for the cheaper the Tun of Wine is, the more may be had for the money, and conſequently the more men will it ſuffice, therefore the ſecond being multiplied by the firſt, and the product divided by the third, the Quotient giveth 24 men for the Anſwer; See the Operation as followeth, *viz.*

l.		*men*		*l.*		*men*
30	:	20	: :	25	:	24
		30				

25) 600 (24

50

100
100

(0)

The Anſwer would have been the ſame, if the firſt and third Numbers in the ſecond Operation, had been made the firſt and third Numbers in the firſt Operation, for then the firſt Queſtion would have been, If 30 *l. per* Tun require 100 men, how many men will 25 *l. per* Tun require? Anſwer, 120 men.

This Queſtion is anſwered by the inverſe Rule of Three, becauſe leſs requireth more in the Queſtion; as before hath been ſhewed, wherefore the ſecond being multiplied by the firſt, and the Product being divided by the third, the Quotient is 120 men, as by the following Operation appears.

30 *l.*

l.		*men*		*l.*		*men*
30	:	100	::	25	:	120
		30				

25) 3000 (120 *Men*
  25

  50
  50
  ---
  (0)

So that now I have diſcovered, that Wine worth 15 *l.* will ſuffice 120 men, when the Tun is worth 25 *l.* But the Queſtion is, how many men 3 pounds worth will ſatisfie, when the Tun is at the ſame Rate; wherefore I ſtate a ſecond Queſtion and ſay, If 15 *l.* require 120 men, how many men will 3 *l.* require? Anſwer 24 men. This Queſtion is of the Direct Rule, wherefore the ſecond Number being multiplied by the third produceth 360, which being divided by (25) the firſt Number, the Quotient is 24 men, which is the Anſwer to the Queſtion, and is the ſame with that found before. See the Operation.

l.		*men*		*l.*		*men*
15	:	120	: :	3	:	24
		3				

```
15) 360 (24 Men
    30
    --
     60
     60
     --
     (0)
```

You may at your leisure try to resolve some of the foregoing Questions after this manner.

CHAP. X.

The Rule of Single Fellowship, commonly called Fellowship without time.

1. THE Rules of Fellowship are of great use in the Ballancing of Accompts depending between Merchants, when many put together a general Stock, so that every man must have his proportionable part of gain, or bear his proportionable share of Loss.

2. Fellowship

2. Fellowship is two fold, *viz.* Single, or Double; that is, without Time, or with Time.

3. Single Fellowship, or Fellowship without Time is, when divers Persons make a joynt Stock for the carrying on of Trade, and every Partner or Person concerned receiveth a Share in the Gains, or beareth a Share in the Loss proportionable to his Stock, without any consideration of Time, and consequently their Stocks must be of equal Continuance in Bank; As in the following Example, *viz.*

1. *Example.*

A and *B* buy a Tun of Wine for 20 *l.* for which *A* laid out 12 *l.* and *B* 8 *l.* and they gained in the Sale clearly 5 *l.* how much ought each man to have for his Share of the Gain?

In this Example the Stocks of *A* and *B* were of equal Continuance in Bank, and therefore are said to be of the Rule of Fellowship without Time.

4. In Single Fellowship the Proportion is as followeth, *viz.*

As the Sum of their Stocks is in proportion to the Total thereby gained or lost;

So is each mans particular Share in the publick Stock, to each mans proper Share in the Gain, or Loss.

Therefore that the Question may be rightly Stated and resolved, the several Moneys of all the several Partners must be gathered into one Sum, which must be placed for the first Term in the

Golden Rule, the common Gain or Loſs muſt be the ſecond Number in the Golden Rule, and in the third place muſt be placed every mans particular Stock, working the *Golden Rule direct* ſo many ſeveral times, as there are joynt Partners in the Work.

As in the foregoing Example.

	l.
The Stock of *A* is ————————	12
The Stock of *B* is ————————	8
The Sum of their Stocks is ———	20

Therefore muſt 20 be the firſt Number in the Rule of Three, and 5 *l.* which is the total Gain muſt be the ſecond Number, and the Stock of *A* which is 12 *l.* muſt be the third Number, and then will the Numbers ſtand as followeth, *viz.*

l. *l.* *l.*
20 : 5 : : 12

Then the ſecond Number being multiplied by the third, and the Product divided by the firſt, the Quotient is 3 *l.* which is the Share of *A.* See the Operation, *viz.*

l. *l.* *l.* *l.*
20 : 5 : : 12 : 3
12
—— *l.*
20) 60 (3

Then

Then to find out the Gain of *B*, repeat again the Rule of Three, making the Sum of their Stocks (20) the first Number, the total Gain (5) the second, and the Stock of *B* (which is 8 *l.*) the third Number, and the fourth proportional Number in a direct Proportion which here is 2 *l.* is the Gain of *B*. See the Operation.

$$
\begin{array}{l}
20 : 5 :: 8 : 2 \\
\quad\;\;\; 8 \\
\quad\;\; \overline{}\ l. \\
20)\ 40\ (2
\end{array}
$$

5. The Proof of these and the like Questions is found, by adding up into one Sum, all the proportional Numbers found by the Golden Rule, being the Partners Loss or Gain, which if it agree with the total Gain or Loss given, the Question is well proposed and wrought.

So in the foregoing Example

	l.
The Gain of *A* is —————————	3.
The Gain of *B* is —————————	2
The Sum is ————	5

So that I conclude the Operation to be truly wrought, the Sum of their particular Gains being equal to the general Gain given.

Therefore divers Questions impossible to be answered may be propounded, as when either

the Parts exceed the Total given, or when the Partners want of the due Shares appointed by the Question, and therefore it is best always to see, that the Partners Shares agree with the general Antecedent; for then consequently their other parts must agree with the general Consequent.

2. Example.

A, *B*, and *C* fraught a Ship with 70 Tuns of Wine, whereof *A* laded 30, *B* 24, and *C* 16 Tun: and by Extremity of Tempest 10 Tun is cast over Board, how much of this Loss ought each of these Merchants to bear?

First for the Loss of *A* the proportion is,

Tun		*Tun*		*Tun*		*Tun*
70	:	10	: :	30	:	$4\frac{20}{70}$
		30				

$$\begin{array}{r l}
70)\ 300 & (4\tfrac{20}{70}\ \textit{Tun} \\
280 & \\
\hline
(20) &
\end{array}$$

Secondly

Secondly for the Loſs of *B*, the proportion is,

Tun		*Tun*		*Tun*		*Tun*
70	:	10	: :	24	:	$3\frac{30}{70}$
		24				

$$\begin{array}{r} 70)\ 240\ (3\frac{30}{70}\ \textit{Tun.} \\ 210 \\ \hline (30) \end{array}$$

Thirdly for the Loſs of *C*, the proportion is,

Tun		*Tun*		*Tun*		*Tun*
70	:	10	: :	16	:	$2\frac{20}{70}$
		16				

$$\begin{array}{r} 70)\ 160\ (2\frac{20}{70}\ \textit{Tun} \\ 140 \\ \hline (20) \end{array}$$

So that the Anſwer to the Queſtion is as followeth, *viz.*

$$\text{The Loſs of} \left\{\begin{array}{c} A \\ B \\ C \end{array}\right\} \text{is} \left\{\begin{array}{l} \textit{Tun} \\ 4\frac{20}{70} \\ 3\frac{30}{70} \\ 2\frac{20}{70} \\ \hline 1 \\ \hline \textit{Proof}\ \ 10 \end{array}\right. \qquad \begin{array}{r} 20 \\ 30 \\ 20 \\ \hline 70)\ 70\ (1 \end{array}$$

Note that when any thing remaineth after Division in the ſeveral Operations is ended, in the proving of the Work ſuch Remainders are to be added together, and their Sum divided by the firſt Number, and the Quotient to be added to the ſeveral integral parts of the Anſwer.

So in the foregoing Example, the ſeveral Remainders after the ſeveral Diviſions are ended, are 20, 30, and 20, which being added together their Sum is 70, as you ſee in the foregoing Operation, which being divided by 70 (the firſt Number in each Operation) the Quotient is 1, which being added to the ſeveral Anſwers the Sum is 10, equal to the total given Loſs, therefore the Operation is Right.

3. *Example.*

One dyeth indebted to 8 ſeveral Perſons *viz.* to *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*, in the ſeveral Sums following, *viz.* to *A* 108 *l.* to *B* 96 *l.* to *C* 84 *l.* to *D* 72 *l.* to *E* 60 *l.* to *F* 48 *l.* to *G* 36 *l.* and to *H* 24 *l.* and all his Goods and Moneys will amount but to 240 *l.* whereof the Creditors agree to take their proportionable parts, what ought each Creditors Share to be?

Firſt, add their ſeveral Debts together, and you will find their Sum to be 528 *l.* and their ſeveral Shares in the ſaid Sum, will be diſcovered by the ſeveral proportions following, *viz.*

Firſt,

First, for the Share of *A*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	108	:	$49\frac{48}{528}$

240
108
———
1920
2400
——— *l.*
528) 25920 ($49\frac{48}{528}$
2112
———
4800
4752
———
Rem. (48)

Secondly, for the Share of *B*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	96	:	$43\frac{336}{528}$

240
96
———
1440
2160
——— *l.*
528) 23040 ($43\frac{336}{528}$
2112
———
1920
1584
———
Rem. (336)

Thirdly for the Share of *C*, the proportion is,

$$\begin{array}{rcrcrcl} l. & & l. & & l. & & l. \\ 528 & : & 240 & :: & 84 & : & 38\frac{96}{528} \end{array}$$

$$\begin{array}{r} 240 \\ 84 \\ \hline 960 \\ 1920 \\ \hline 528)\ 20160\ (38\frac{96}{528}\ l. \\ 1584 \\ \hline 4320 \\ 4224 \\ \hline Rem.\ (96) \end{array}$$

Fourthly for the Share of *D*, the proportion is,

$$\begin{array}{rcrcrcl} l. & & l. & & l. & & l. \\ 528 & : & 240 & :: & 72 & : & 32\frac{384}{528} \end{array}$$

$$\begin{array}{r} 240 \\ 72 \\ \hline 480 \\ 1680 \\ \hline 528)\ 17280\ (32\frac{384}{528}\ l. \\ 1584 \\ \hline 1440 \\ 1056 \\ \hline Rem.\ (384) \end{array}$$

Fifthly

Fifthly for the Share of *E*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	60	:	$27\frac{144}{528}$

$$\begin{array}{r}240\\60\\\hline\end{array}$$

528) 14400 ($27\frac{144}{528}$ *l.*

$$\begin{array}{r}1056\\\hline3840\\3696\\\hline\end{array}$$

Rem. (144)

Sixthly for the Share of *F*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	48	:	$21\frac{432}{528}$

$$\begin{array}{r}240\\48\\\hline1920\\960\\\hline\end{array}$$

528) 11520 ($21\frac{432}{528}$ *l.*

$$\begin{array}{r}1056\\\hline960\\528\\\hline\end{array}$$

Rem. (432)

Seventhly

Seventhly, for the Share of *G*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	36	:	$16\frac{192}{528}$

$$\begin{array}{r r l}
 & 240 & \\
 & 36 & \\
\hline
 & 1440 & \\
 & 720 & \\
\hline
 & & l. \\
528) & 8640 & (16\frac{192}{528} \\
 & 528 & \\
\hline
 & 3360 & \\
 & 3168 & \\
\hline
\textit{Rem.} & (192) &
\end{array}$$

Eighthly, for the Share of *H*, the proportion is,

l.		*l.*		*l.*		*l.*
528	:	240	: :	24	:	$10\frac{480}{528}$

$$\begin{array}{r r l}
 & 240 & \\
 & 24 & \\
\hline
 & 960 & \\
 & 480 & \\
\hline
 & & l. \\
528) & 5760 & (10\frac{480}{528} \\
 & 528 & \\
\hline
\textit{Rem.} & (480) &
\end{array}$$

Thus

Thus the Operation is finished, and by the several proportions foregoing, the Answer to the Question is found to be as followeth, *viz.*

					Remainders
The Share of	*A*	is	$49\frac{48}{528}$		48
	B		$43\frac{336}{528}$		336
	C		$38\frac{96}{528}$		96
	D		$32\frac{384}{528}$		384
	E		$27\frac{144}{528}$		144
	F		$21\frac{432}{528}$		432
	G		$16\frac{192}{528}$		192
	H		$10\frac{480}{528}$		480
			4		2112
			Proof 240		

528) 2112 (4

If you desire it, you may find the Value of every one of the foregoing Remainders, by the Rule laid down in the 11*th.* Sect. of Chap. 7. in the 119*th.* Page.

4. *Example.*

4. *Example.*

Two Merchants, *A* and *B*, make a joynt Stock, *A* put in 450 *l.* *B* put in I know not how much, And they gained 687 *l.* of which *B* had 229 *l.* for his Share; Now I demand the Stock of *B* and the Share of *A* in the Gain? Anſ. the Stock of *B* is 225 *l.* and the Gain of *A* is 458 *l.*

If from (687 *l.*) the total Gain, you ſubſtract (229 *l.*) the Gain of *B*, the Remainder is (458 *l.*) for the Gain of *A*.

Then may the Stock of *B* be found out by the following Proportion, *viz.*

As the Gain of *A* is in proportion to the Stock of *A*,

So is the Gain of *B* to the Stock of *B*.

So that the fourth Proportional Number is found to be 225 *l.* for the Stock of *B*. See the Operation as followeth,

	l.	
	687	*The total Gain.*
	229	*The Gain of* B.
Rem.	458	*The Gain of* A.

458 *l.*

l.		l.		l.		l.
458	:	450	: :	229	:	225

```
              229
            ------
             4050
             90
            90
            ------        l.
458)       103050    (225
             ...
            916
            ------
             1145
              916
            ------
              2290
              2290
            ------
              (0)
```

5. *Example.*

A and *B* make a joynt Stock, *A* put into the Stock 90 *l.* more than *B*, and they gained 280 *l.* of which *B* had 120 *l.* for his Share; I demand the Stocks of each? Anſ. the Stock of *A* is 360 *l.* and the Stock of *B* is 270 *l.*

Firſt, from the total Gain ſubſtract the Gain of *B*, and the Remainder is 160, for the Gain of *A*; Then, from the Gain of *A* ſubſtract the Gain of *B*, and the Remainder is 40 *l.* and ſo much doth the Gain of *A* exceed the Gain of *B*, and is called the

the Excess of the Gain of *A*. Then may the Stocks of each be discovered by the following Analogy, or Proportion, *viz.*

As the Excess of the Gain of *A* is to the Excess of the Stock of *A*,
So is the whole Gain of *A* to the whole Stock of *A*.

And,

So is the whole Gain of *B* to the whole Stock of *B*.

See the whole Operation practically performed as followeth, *viz.*

	l.	
	280	*the Total Gain.*
	120	*the Gain of B substracted.*
Rem.	160	*the Gain of A.*
	40	*the Excess of the Gain of A. above that of B.*

First for the Stock of *A*.

l.		*l.*		*l.*		*l.*
40	:	90	: :	160	:	360
				90		
						l.
			40)	14400		(360

So

So that the Stock of *A* by the foregoing Operation is found to be 360 *l.*

Secondly for the Stock of *B.*

$$\begin{array}{ccccccc} l. & & l. & & l. & & l. \\ 40 & : & 90 & :: & 120 & : & 270 \\ & & & & 90 & & \\ \hline & & & & & & l. \\ & & 4|0) & & 1080|0 & & (270 \end{array}$$

$$\text{The Stock of} \left\{ \begin{array}{c} A \\ B \end{array} \right\} \text{is} \left\{ \begin{array}{l} \overset{l.}{360} \\ 270 \end{array} \right.$$

Proof 90

6. Example.

Three Merchants make a joynt Stock, *A* put in 80 *l.* more than *C*, and they gained 720 *l.* of which *A* had 260 *l.* and *B* had 300 *l.* for their Shares; Now I demand the Stock of each, and the Gain of *C*? Anſw. the Gain of *C* is 160 *l.* the Stock of *A* 208 *l.* the Stock of *B* 246 *l.* the Stock of *C* is 128 *l.*

Firſt, add the Share of *A* in the Gain (260 *l.*) to the Share of *B* (300 *l.*) and the Sum is 560 *l.* which being ſubſtracted from (720 *l.*) the total Gain there remaineth 160 *l.* for the Gain of *C.*

Then

Then substract (160 *l.*) the Gain of *C* from (260 *l.*) the Gain of *A*, and there remaineth 100 *l.* for the Excess of the Gain of *A* above that of *C*.

So that now there is a Way opened for the Discovery of each mans Stock, by the following Analogy, *viz.*

As the Excess of the Gain of *A*, above that of *C* (100 *l.*) is in Proportion to the given Excess of the Stock of *A* above that of *C* (80 *l.*)

So is the whole Gain of *A* (260 *l.*) to the whole Stock of *A* (208 *l.*)

And

So is the whole Gain of *B* (300 *l.*) to the whole Stock of *B* (40 *l.*)

And

So is the whole Gain of *C* (160 *l.*) to the whole Stock of *C* (128 *l.*)

See the whole Operation as followeth;

	l.
The Share of A *in the Gain* ————	260
The Share of B *in the Gain* ————	300
The Sum of the Gains of A *and* B ——	560
to be subtracted from the Total gain —	720
The Remainder is the Gain of C ——	160
The Gain of C *being subtracted from the Gain of* A *remaineth*	—100

First

First, for the Stock of *A*, the Proportion is,

l.		l.		l.		l.
100	:	80	: :	260	:	208
				80		
						l.
		100)		20800		(208

Secondly, for the Stock of *B*, the Analesis is,

l.		l.		l.		l.
100	:	80	: :	300	:	240
				80		
						l.
		100)		24000		(240

Thirdly, for the Stock of *C*, the Proportion is,

l.		l.		l.		l.
100	:	80	: :	160	:	128
				80		
						l.
		100)		12800		(128

7. *Example.*

Three Merchants make a joynt Stock, *A* put in 120 *l.* more than *B*, and for every 4 *l.* that *B* put in, *C* put in 5 *l.* and when they came to share their Gains, *A* had 260 *l.* and *B* had 135 *l.* Now I demand the Stocks of *A* and *B*, and the Stock and Gain of *C*?

Answer the Stock of *A* is 249 *l.* 12 *s.* the Stock of *B* is 129 *l.* 12 *s.* and the Stock *C* is 162 *l.* and the Gain of *C* is 168 *l.* 15 *s.*

First, take the Difference between the Shares of *A* and *B* in the Gain, which here is found to be 125 *l.* and is the Excess of the Gain of *A* above that of *B*; and the Excess of the Stock of *A* above that of *B* is given to be 120 *l.* wherefore the whole Stocks of *A* and *B* may be found as in the fifth Example, the Work of which is as followeth;

First, for the Stock of *A*.

l.		*l.*		*l.*		*l.*	*s.*
125	:	120	:	260	:	249	12

```
          120
          260
         ----
         7200
          24
         ----        l.   s.
  125)  31200   (249  12
          250
         ----
          620
          500
         ----
         1200
         1125
         ----
         (75)
          20
         ----       s.
  125)  1500   (12
```

Facit 249 *l.* 12 *s.*

Secondly

Secondly, for the Stock of *B.*

l.		*l.*		*l.*		*l.*	
125	:	120	: :	135	:	129	12

```
          120
          135
          ---
          600
         36
        12
        -----            l.   s.
125)  16200  (129   12
        125
        ---
         370
         250
        ---              l.    s.
        1200      Facit  129   12
        1125
        ----
        (75)
         20
        ---        s.
125)  1500  (12
```

Having found out the Stocks of *A* and *B* by the foregoing Analogies, to be 249 *l.* 12 *s.* and 129 *l.* 12 *s.* the Stock of *C* may be likewise found out, for the Conditions of the Question require *C* to have 5 *l.* for every 4 *l.* that *B* hath, wherefore I say,

As 4 *l.* is in proportion to 5 *l.*

So is 129 *l.* 12 *s.* (the Stock of *B*) to the Stock of *C*, which is found to be 162 *l.* See the following Operation.

l.		*l.*		*l.*	*s.*		*l.*
4	:	5	: :	129	12	:	162
				20			
				2592			
				5			
			4)	12960	(324|0		
					162		

The Stock of *C* being found to be 162 *l.* it remaineth now only to find the Gain of *C* which may be thus performed, *viz.*

The Stock of *A* is 120 *l.* more than that of *B*, and his Share in the Gain is likewise 125 *l.* more than the Gain of *B*; Therefore is 125 *l.* gained by 120 *l.* Stock, wherefore may the Gain of *C* be found out by the following Analogy, *viz.*

As 120 *l.* Stock, is to 125 *l.* Gain

So is the Stock of *C* (162 *l.*) to the Gain of *C*, which is here found to be 168 *l.* 15 *s.* View the following Operation.

```
 l.      l.        l.       l.   s.
120  :  125   ::  162  :  168   15
        162
       -----
        250
       750
       125
       -----     l.   s.
12|0)  2025|0  (168  15
        ...
       12
       ---
        82
        72
       ---
        105
         96
        ---
         (9)
         20
        ---   s.
12)     180  (15
         ..
        12
        ---
         60
         60
        ---
         (0)
```

CHAP. XI.

Double Fellowship, or Fellowship with Time.

1. DOuble Fellowship, by some called Fellowship with Time, is when divers Partners make a joynt Stock without an Equality of Time, but each Partner draweth out of or putteth into the Stock, as his Occasion or Conveniency may serve or require; As in the following Example, *viz.*

1. *Example.*

Two Merchants make a joynt Stock, *A* put in 90 *l.* for 8 Months, and *B* put in 80 *l.* for 12 Months, and they gained 160 *l.* I demand each Mans Share in the Gain, proportionable to his Stock and his Time?

2. In double Fellowship multiply each particular Persons Stock, by the Time of it's Continuance in Bank, and then add all the Products together, so shall the Sum be the first or general Antecedent, or first Number in the Rule of Three; and the Total Gain or Loss, the general Consequent, or second Number in the Rule of Three, and any ones particular Product of Stock and Time for the third Number, and the fourth Number in a direct Proportion, is the Share of that Person in the

the Gain or Loſs, the Product of whoſe Money and Time you made the third Number.

Or the Operation may be performed by the following Analogy, *viz.*

As the Sum of all the Products of Stock and Time, is in Proportion to the general Gain or Loſs: So is the Product of any ones particular Stock and Time to his Share of Gain or Loſs.

So that the Rule of Three being repeated as often as there are Partners, the Share of each Partner is diſcovered.

So in the foregoing Example, the Stock and Time of each Perſon with their Product are as followeth, *viz.*

	l.	
The Stock of A *is*	90	
Its time	8	*Months*
The Product	720	
The Stock of B	80	
Its time	12	
The Product	960	
	720	
The Sum of the Products or general Antecedent	1680	
The general Conſequent	160	

Therefore, first for the Gain of *A.*

$$\begin{array}{rcccccl} & & l. & & & & l. \\ 1680 & : & 160 & :: & 720 & : & 68\frac{960}{1680} \\ & & 720 & & & & \end{array}$$

$$\begin{array}{r} 3200 \\ 112 \\ \hline 1680)\ 115200\ (68 \\ 10080 \\ \hline 14400 \\ 13440 \\ \hline \textit{Rem.}\ (960) \end{array}$$

So that the Gain of *A* is 68 *l.* $\frac{960}{1680}$, or 68 *l.* 11 *s.* 05$\frac{240}{1680}$ *d.*

Secondly

Secondly, for the Share of *B*.

$$\begin{array}{rcccccc} & & l. & & & & l. \\ 1680 & : & 160 & :: & 960 & : & 91\tfrac{720}{1680} \\ & & 960 & & & & \end{array}$$

$$\begin{array}{rll} & \underline{960} & \\ & 9600 & \\ & \underline{144} & l. \\ 1680) & 153600 & (91 \\ & \underline{15120} & \\ & 2400 & \\ & \underline{1680} & \\ \textit{Rem.} & (720) & \end{array}$$

So that the Gain of *B* is $91\frac{720}{1680}$ *l.* the Value of which Fraction by the 11*th*. Sect. of Chap. 7. page 119. will be found to be 91 *l.* 8 *s.* $6\frac{1440}{1680}$ *d.*

2. *Example.*

There is an Army of 6000 Foot and 800 Horse-men, the Stipend of each Foot-man is 4 Shillings a Week, and of every Horse-man is 9 *s.* a Week, and they are to divide a Booty of 2000 Shillings, so as their particular Parts may be proportionable to their Wages; how much belongs to the Foot-men, and how much to the Horse-men? Answer, the Horse-men must have 23 *l.* 01 *s.* $06\frac{14400}{31200}$ *d.* and the

the Footmen must have 76 *l.* 18 *s.* $05\frac{16800}{31200}$ *d.* See the Work.

The Number of Horse-men	800
Their Pay per *Week*	9
The Product	7200
The Number of Foot-men	6000
Their Pay per *Week*	4
	24000
	7200
The Sum of the Products or general Antecedent	31200
The general Consequent	2000

First, for the Share of the Horse-men.

	Shillings			*Shillings*
31200	: 2000	: :	7200	: $461\frac{16800}{31200}$
	7200			
		s.	*d.*	
31200)	14400000	(461	$06\frac{16800}{31200}$	

Secondly, the Share of the Foot.

	Shillings			*Shil.*	*d.*
31200	: 2000	: :	24000	1538	$5\frac{16800}{31200}$
	24000				
		s.	*d.*		
31200)	48000000	(1538	$5\frac{16800}{31200}$		

3. *Example.*

3. *Example.*

A, *B*, and *C*, take a Paſture for 30 *l* a year Rent, wherein *A* feedeth 20 Oxen 70 days, *B* feedeth 46 Oxen 56 days, and *C* feedeth 32 Oxen 60 days; what ſhall each pay toward the Rent in proportion to his Stock and his Time? Anſwer, *A* muſt pay $7\frac{728}{5896}$ *l.* *B* $13\frac{612}{5896}$ *l.* *C* $9\frac{5536}{5896}$ *l.*

A *his Stock*	20 *Ox.*	B 46 *Ox.*	*C.* 32 *Ox.*
A *his Time*	70 *days*	56 *da.*	60 *da.*
Their Prod.	1400	276	1920
		230	
		2576	

The Product of the Stock and Time of A 1400, B 2576, C 1920

Their Sum, or the general Antecedent — 5896

For

For the Share of *A*.

$$5896 : \overset{l.}{30} :: 1400 : \overset{l.}{7\tfrac{728}{5896}}$$

$$\begin{array}{r l}
 & 1400 \\
 & 30 \\
\hline
 & \qquad l. \\
5896) & 42000 \;(7 \\
 & 41272 \\
\hline
\textit{Rem.} & (728)
\end{array}$$

Secondly, for the Share of *B*.

$$5896 : \overset{l.}{30} :: 2576 : \overset{l.}{13\tfrac{632}{5896}}$$

$$\begin{array}{r l}
 & 2576 \\
 & 30 \\
\hline
 & \qquad l. \\
5896) & 77280 \;(13 \\
 & 5896 \\
\hline
 & 18320 \\
 & 17688 \\
\hline
\textit{Rem.} & (632)
\end{array}$$

Thirdly

Thirdly, for the Share of *C.*

$$\begin{array}{lllllll} & l. & & & & l. \\ 5896 & : 30 & :: & 1920 & : & 9\frac{4536}{5896} \\ & & & 30 & & \end{array}$$

$$\begin{array}{rrl} & & l. \\ 5896) & 57600 & (9 \\ & 53064 & \\ \hline \textit{Rem.} & (4536) & \end{array}$$

You may at your Leisure find out the Value of the several Remainders, as before hath been directed.

4. *Example.*

A Colledge hath 20 Fellows and 30 Scholars which may dispend yearly 2600 *l.* but every Fellow hath 5 *l.* for every Scholars 1 *l.* how much of this Revenue belongs to the Fellows, and how much to the Scholars? Answer, to the Fellows 2000 *l.* and to the Scholars 600 *l.* See the Work.

$$\begin{array}{r} 20 \\ 5 \\ \hline 100 \\ 30 \\ \hline 130 \end{array}$$

Therefore

Therefore first,

	l.		*l.*
130 :	2600 : :	100 :	2000

Secondly,

	l.		*l.*
130 :	2600 : :	30 :	600

5. *Example.*

A, *B*, *C*, and *D*, make a Stock to traffick with for a year, *A* disbursed 600 *l.* for 10 Months, *B* 400 *l.* for 8 Months, *C* 800 *l.* for 6 Months, and *D* 500 *l.* for 12 Months, and they gain 760 *l.* clearly, what Portion ought each to have? Answer, *A* gained 228 *l.* *B* 121 *l.* 12 *s.* *C* 182 *l.* 8 *s.* and *D* 228 *l.*

The Operation.

A *his* $\begin{cases}\textit{Stock}\text{---}600\\ \textit{Time}\text{---}\ 10\ \textit{months}\end{cases}$

The Product—6000

B *his* $\begin{cases}\textit{Stock}\text{---}400\\ \textit{Time}\text{---}\ 8\ \textit{months}\end{cases}$

The Product--3200

C *his* $\begin{cases}\textit{Stock}\text{----}800\\ \textit{Time}\text{----}\ 6\ \textit{mon.}\end{cases}$

The Product—4800

D *his* $\begin{cases}\textit{Stock}\text{---}500\\ \textit{Time}\text{---}\ 12\ \textit{mon.}\end{cases}$

The Product—6000

The

The Product of the Stock and Time of $\left\{\begin{array}{c} A \\ B \\ C \\ D \end{array}\right\}$ is $\left\{\begin{array}{r} 6000 \\ 3200 \\ 4800 \\ 6000 \end{array}\right.$

The Sum 20000

First, for the Gain of *A*.

		l.				*l.*
20000	:	760	: :	6000	:	228
				760		

2|0000) 456|000 (228

Secondly, for the Gain of *B*.

		l.				*l.*	*s.*
20000	:	760	: :	3200	:	121	12
				760			

192000
224

l. *s.*
20|000) 2432|000 (121 12

Thirdly

Thirdly, for the Gain of *C*.

```
              l.                   l.   s.
20000  :  760  : :  4800  :  182  08
                     760
                  --------
                  288000
                  336
                  --------         l.   s.
      20|000)  3648|000  (182  08
```

Fourthly, for the Share of *D*.

```
             l.                   l.
20000  :  760  : :  6000  :  228
                     760
                  --------     l.
      2|0000)  456|0000  (228
```

The Proof.

		l.	s.
The Share of	A	228	00
	B	121	12
	C	182	08
	D	228	00
	Sum.	760	00

3. The Proof of Fellowship with Time is the same with that in Fellowship without Time, for if the Sum of the particular Shares of the Partners in

in the Gain or Loss, be equal to the total Gain or Loss, you may conclude the Operation to be truly performed.

As in the last Example, the Sum of all the particular Gains of *A*, *B*, *C*, and *D*, is 760 *l.* equal to the total Gain given.

6. *Example.*

A, *B*, and *C*, enter Partnership upon the first of *January*, for a whole year, *A* the same day disbursed 100 *l.* whereof he received back again the first of *April* 20 *l.* *B* delivereth the first of *March* 60 *l.* and the first of *August* he putteth in 100 *l.* more, *C* layeth out 140 *l.* the first of *July*, but the first of *October* withdraweth 40 *l.* and at the years end their clear Gains is 142 *l.* how much ought every particular Partner to have?

Answer, *A* must have 51 *l.* *B* 55 *l.* and *C* 36 *l.*

In the foregoing and such like Questions, the particular Antecedents are gotten by Multiplying the several Disbursments by the proper time of its Continuance. And because within the whole time they took out and put in again, therefore after you have multiplied the Sums put in, by their proper Time, add the Products together, and so shall the Sum be the Product of Stock and Time: So in the last Example, because *A* had 100 *l.* betwixt the first of *January* and the first of *April* which is 3 months, and then 80 *l.* continued the rest of the Time which is 9 months longer, after you have multiplyed 100 by 3 (which is 300) and 80 *l.* by 9 (which is 720) you must add

 them

them together and the Sum is 1020; and so of the rest as by the following Operation.

	l.	*l.*		*l.*	
A	100	80		100	
	3	9		20	*subtr.*
	300	720	*Rem.*	80	
	720				
	1020				

	l.	*l.*		*l.*	
B	60	160		60	
	5	5		100	*add*
	300	800		160	
	800				
	1100				

	l.	*l.*		*l.*	
C	140	100		140	
	3	3		40	*subtr.*
	420	300	*Rem.*	100	
	300				
	720				

The them

The particular Antecedents are as followeth, viz. $\left\{\begin{array}{ll} A & 1020 \\ B & 1100 \\ C & 720 \end{array}\right.$

The general Antecedent —— 2840

Therefore the ſeveral Shares may be found out by the following Analogies,

And firſt, for the Share of *A.*

$$2840 : \overset{l.}{142} :: 1020 : \overset{l.}{51}$$

So that the Share of *A* is found to be 51 *l.*

Secondly, for the Share of *B.*

$$2840 : \overset{l.}{142} :: 1100 : \overset{l.}{55}$$

Thirdly, for the Share of *C.*

$$2840 : \overset{l.}{142} :: 720 : \overset{l.}{36}$$

The Proof.

$$\text{The Share of} \left\{ \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix} \right\} \text{is} \left\{ \begin{matrix} l. \\ 51 \\ 55 \\ 36 \end{matrix} \right.$$

Sum 142

7. *Example.*

A, *B*, *C*, and *D*, Partners by Contract for two years, make and manage their Stock thus, *viz.* *A* put in presently 30 *l.* but at 8 months end received back 10 *l.* for which at the end of 19 months, he restored 12 *l.* *B* put in presently 24 *l.* but at the end of 6 months he took back 8 *l.* for which at the end of 15 months he put in 14 *l.* *C* put in at first 20 *l.* but at 7 months end received it whole again, yet at the beginning of the 18*th.* month he returned 16 *l.* *D* put in nothing until the beginning of the 7*th.* month, and then he put in 18 *l.* but after 4 months received back 9 *l.* for which in the beginning of the 17*th.* month he restored 15 *l.* Now at the two years end they had gained 100 *l.* how much thereof ought each Partner to have? Answer, *A* $35\frac{820}{1748}$ *l.* *B* $31\frac{1612}{1748}$ *l.* *C* $14\frac{728}{1748}$ *l.* *D* $18\frac{336}{1748}$ *l.*

The

The Operation.

A

	l.	*l.*	*l.*
	30	30	20
	8 *mon.*	10	12
	240	20 *l,*	32 *l.*
	220	11 *mon.*	5 *mon.*
	160	20	160
Sum	620	20	
		220	

B

	l.	*l.*	*l.*
	24	24	16
	6 *mon.*	8	14
	144	16	30
	144	9 *mon.*	9 *mon.*
	270	144	270
Sum	558		

C

l.	*l.*
20	16
7 *mon.*	7 *mon.*
140	112
112	
Sum 252	

D

l.	*l.*	*l.*
18	18	9
4 *mon.*	9 *ſubtr.*	15 *add*
72	9	24
54	6 *mon.*	8 *mon.*
192	54	192
Sum 318		

Thus have we Proceeded to multiply each mans Stock by its appropriate Time of putting in and taking out,according to the Tenour of the Queſtion,which is but little trouble,in this or in any other, if you regard the time agreed upon,for Continuance of the Partnerſhip, and then how many months every Partner ſuffereth his proper ſhare to continue in Bank.

As

As for Example.

In the ſolving of the foregoing Queſtion, under the Letter *A* are contained the ſeveral Products of *A* his Money and Time, *viz.* it is ſaid that *A* put in at firſt 30 *l.* which continued 8 months, wherefore 30 *l.* is multiplied by 8 months, and the Product is 240; then becauſe *A* received 10 *l.* of 30 back, I ſubtract 10 from 30, and the Remainder is 20 *l.* which continued in Bank to the beginning of the 20*th.* month, which is 11 months after the taking out of the 10 *l.* wherefore I multiply 20 *l.* by 11 months and the Product is 220; then becauſe to the ſaid 20 *l.* he added 12 *l.* more, therefore is 12 *l.* added to 20 *l.* and the Sum (which is 32 *l.*) continued in Bank the Remainder of the two years, which is 5 mon. wherefore 32 *l.* being multiplied by 5 produceth 160, and ſo is the Work finiſhed for the Multiplication of the Stock and Time of *A*, and the particular Products being added together are 620 for the total Product of his Stock and Time; in the ſame manner are found out the Product of *B*, his Stock and Time, and likewiſe of *C* and *D*, which you will find to be as followeth.

The Product of the Stock and Time of	A	is	620
	B		558
	C		252
	D		318
		The Sum	1748

Therefore may each Partners Share in the Gains be discovered by the following Analogies, *viz.*

First, for the Share of *A.*

$$1748 : \overset{l.}{100} :: 620 : \overset{l.}{35\tfrac{820}{1748}}$$

Secondly, for the Share of *B.*

$$1748 : \overset{l.}{100} :: 558 : \overset{l.}{31\tfrac{1612}{1748}}$$

Thirdly, for the Share of *C.*

$$1748 : \overset{l.}{100} :: 252 : \overset{l.}{14\tfrac{728}{1748}}$$

Fourthly for the Share of *D.*

$$1748 : \overset{l.}{100} :: 318 : \overset{l.}{18\tfrac{336}{1748}}$$

So

So that by the foregoing Proportions the Share of each Partner is discovered to be as followeth, *viz.*

		l.		*l.*	*s.*	*d.*
The Share of	A	$35\frac{820}{1748}$,	*or*	35	09	$04\frac{1024}{1748}$
is	B	$31\frac{1612}{1748}$,	*or*	31	18	$05\frac{572}{1748}$
	C	$14\frac{728}{1748}$,	*or*	14	08	$03\frac{1668}{1748}$
	D	$18\frac{336}{1748}$,	*or*	18	03	$10\frac{232}{1748}$

The Accounting of the Partners Times of putting in, and taking out of their Stocks will be little Trouble, if you regard the Time agreed upon for the Continuance of the Partnerſhip, and then how many Months every Partner ſuffereth his proper Share to continue in Bank, as was ſaid before.

And the ſame may as eaſily be wrought if the Stock remain not for whole Months; as in the Example following for Days.

8. *Example.*

Two Graſiers, *A*, and *B*, took a Paſture for 40 *l. per Annum*, *viz.* 800 Shillings, and they both put in on the firſt of *January* 50 a peice; but *A* the 25*th.* Day of *March* put in 20 more, and the third Day of *May* took out 22, and the 13th. of *September* put in 7 more. *B* took out the ſecond Day of *February* 9, and the 10*th.* of *June* took out 32, How much muſt each man pay of the Rent? Anſwer,

If

If you multiply each particular Stock by the Number of Days it continued in Bank, and then add the Products together as before is taught, you will find them to be as followeth, *viz.*

$$\textit{The Product of the Stock and Time of} \begin{Bmatrix} A \\ B \end{Bmatrix} \text{is} \begin{Bmatrix} 19307 \\ 14892 \end{Bmatrix}$$

Sum 34199

Now may be discovered the Share of each in the Rent by the following Proportion, *viz.*

First, for the Share of *A.*

$$34199 : \overset{s.}{800} :: 19307 : \overset{s.}{451}\tfrac{21851}{34199}$$

Secondly, for the Share of *B.*

$$34199 : \overset{s.}{800} :: 14892 : \overset{s.}{348}\tfrac{12348}{34199}$$

So that the Shares of each are as followeth, *viz.*

$$\textit{The Share of} \begin{Bmatrix} A \\ B \end{Bmatrix} \text{is} \begin{Bmatrix} 451\tfrac{21851}{34199} \\ 348\tfrac{12348}{34199} \end{Bmatrix}$$

Observe the third Example of this Chapter.

CHAP.

CHAP. XII.

Rules of Practice.

1. THese Rules depending much upon the *Aliquot* or even *Divisions* of Shillings and Pence, and upon the *Aliquot Parts* of Weight, Measure, *&c.* therefore the Tables following must be gotten perfectly, and so fixed, that without them the Parts may easily and quickly be remembred.

Even

Even parts of a pound.		Uneven parts of a pound.	D.	Even parts of C. weig.	
l. s. d.	n.1.	s. s. s.			
10 · 0	½	19 10 · 5 · 4	2 · 4 · 5	56	½
6 · 8	3	18 10 · 4 · 4	2 · 5 · 5	28	4
5 · 0	4	17 10 · 5 · 2	2 · 4 · 10	16	7
4 · 0	5	16 10 · 4 · 2	2 · 5 · 10	14	8
3 · 4	6	15 10 · 5 ·	2 · 4	8	14
2 · 6	8	14 10 · 4 ·	2 · 5	7	16
2 · 0	10	13 4 · 4 · 5	5 · 5 · 4	4	28
1 · 8	12	12 10 · 2 ·	2 · 10	2	56
1 · 3	16	11 5 · 4 · 2	4 · 5 · 10	1	112
1 · 0	20	9 5 · 4	4 · 5		
10	24	8 4 · 4	5 · 5		
8	30	7 5 · 2	4 · 10		
6	40	6 4 · 2	5 · 10		
		3 2 · 1	10 · 20		

fractions of C. weight.	
98	7 · 8
96	6 · 7
84	3 · 4
80	5 · 7
70	5 · 8
64	4 · 7
48	3 · 2
32	2 · 7

Even parts of a Shill.		Uneven parts of a Shilling.	
d.		11 6 · 3 · 2	2 · 4 · 6
6	½	10 6 · 4	2 · 3
4	3	9 3 · 3 · 3	4 · 4 · 4
3	4	8 4 · 4	3 · 3
2	6	7 4 · 3	3 · 4
1	12	5 3 · 2	4 · 6
1 d. ob.	8		

The

The Explanation of the Tables.

1. The firſt are the even parts of a Pound, as 10 *s.* the ½ or one two parts of a Pound, 6 *s.* - 8 *d.* the third part, 5 *s.* the fourth part, 4 *s.* the fifth part, *&c.* and for all the *Aliquot* parts of a Pound in the firſt Column is the part of a Pound, and againſt it in the next Column is its part.

The ſame is for the even parts of a Shilling, as 4 *d.* the third part, 3 *d.* the fourth part, *&c.*

2. In the ſecond part of this Table are the *uneven parts* of a Pound or of a Shilling, and how thoſe may be divided into even parts.

Example.

19 *s.* may be parted into 10 *s.*——5 *s.*——4 *s.* whoſe even parts are 2, 4, and 5, for 2 anſwereth 10 *s.* 4 anſwereth 5 *s.* and 5 anſwereth 4 *s.*

The like for Pence, ſuppoſe 11 *d* which may be divided into 6 *d.*——3 *d.*——2 *d.* whoſe even parts are 2, 4, 6.

3. The third Part contains the even and uneven parts of a *C.* weight.

2. *The Uſes of theſe Tables follow.*

Firſt, theſe Rules only depending upon ſuch Queſtions as concern one Pound, one Yard, one Ounce, one *C.* weight, *&c.* then what ſhall many Pounds, Yards, Ounces, *&c.* coſt; take theſe general Rules.

1. If

1. If the Queſtion be upon Pounds, multiply all by Pounds, makes Pounds.

Example.

At 5 *l.* the Ounce, what comes 33 Ounces to? Anſwer, 165 *l.* Multiply 33 by 5 makes 165 Pounds.

2. If the Queſtion be upon Shillings or any other even parts of a Pound, divide by the Aliquot part under *N.* 1. the Anſwer will be *l.*

Example.

At 6 *s.*—8 *d.* the *l.* what comes 38 *l.* to? Anſ. 12 *l.*——13 *s* ——4 *d.*

For 6 *s.* 8 *d.* I divide by 3 the part for in 38 three is 12 times, and 2 remains, which is 2 Nobles.

```
                  l.  s.  d.
    3)  38   (12--13--4
        36
    ---------
    Rem. (2)
```

Example.

At 12 *s.* the Yard, what comes 134 Yards to? Anſwer, 90 *l* 8 *s.*

For 12 *s.* I divide by 2, and 10, as the Table ſhews for 10 *s.* and 2 *s.*

```
     2)  134   (67
    10)  13|4  (13 - 8 s.
               ----------
         Sum   80—8 s.
```

3. If

3. If the Queſtion be upon pence or any even part of a Shilling, divide by the Aliquot part and the Anſwer will be Shillings.

Example.

At 4 *d.* the Ell, what comes 57 Ells unto? Anſwer, 19 *s.*

For 4 *d.* I divide by 3. 3) 57 (19 *s.*

Example.

At 7 *d.* the Pint, what comes 57 Pints to? Anſ. 33 *s.* 3 *d.*

For 7 *d.* I divide by 3 and 4 the parts of 4 *d.* and 3 *d.*

3) 57 (19 *s.*
4) 57 (14 - 3 *d.*

Sum 33 - 3 *d.*

4. Laſtly, if the Queſtion lie upon Pounds, Shillings, and Pence, work by the three former Rules as the conveniency will ariſe.

Example.

At 3 *l.* 14 *s.* 8 *d.* the yard, what will 174 yards come to? Anſwer, 649 *l.* 12 *s.*

174	2) 174 (87 *l.*	3) 174 (58 *s.*
3	5) 174 (34 *l.* - 16 *s.*	3) 174 (58 *s.*
522 *l.*		

All

All which being summed up together do make 649 *l.* 12 *s.*

l.	*s.*
522	
87	
34	16
2	18
2	18
649	12

More Examples.

At 6 *d.* the yard, what will 375 yards give? Answer, 187 *s.* 6 *d.* or 9 *l.* 7 *s.* 6 *d.* by taking $\frac{1}{2}$ of 375.

At 9. *d.* the yard, what comes 5271 yards to? Answer, 197 *l.* 13 *s.* 3 *d.*

4) 5271 (1317—9 *d.*
4) 5271 (1317—9
4) 5271 (1317—9

395|3—3

197-13-3

At $5\frac{1}{2}$ *d.* the Ell what comes 120 Ells to? Ans. 2 *l.* 15 *s.* $5\frac{1}{2}$ is 4 *d.* and $1\frac{1}{2}$, which is $\frac{1}{3}$ and $\frac{1}{8}$ of a Shilling.

3)

	s.	
3)	120	(40
8)	120	(15
		5|5

	l.	s.
facit	20	15

At 16 s. the Pound, what doth 2312 l. come to at that Rate? Anſwer, 1849 l. 12 s.

16 s. is $\frac{1}{2}$ and $\frac{1}{5}$ and $\frac{1}{10}$ of a Pound, as by the foregoing Table; wherefore the Diviſors are 2, 5, and 10, as in the following Work.

	l.		
2)	2312	(1156	
5)	2312	(462	8 s.
10)	2312	(231	4 s.
	facit	1849	12

Note that if any thing remains after Diviſion, it is of the ſame Name that the Dividend is of, or ſuppoſed to be of.

As in the laſt Example in the ſecond Diviſion, where the Diviſor is 5, the Dividend is 2312 and is ſuppoſed to be ſo many 4 Shillings, wherefore every Unite that remains is 4 Shillings; and in that Diviſion, becauſe the Remainder is 2, it is accounted (2 times 4 Shillings) 8 Shillings; and in the laſt Diviſion, the Dividend is ſuppoſed to be ſo many 2 Shillings; and therefore, becauſe the

Remainder is 2, it is accounted (2 times 2 Shillings) 4 Shillings.

But when the given Price of the Integer is an even Number of Shillings,the Anſwer may be more conciſely found thus, *viz.*

When it is required to divide any Number by 10, the Quotient may be diſcovered at firſt Sight thus: Let the place of Unites in the Dividend be ſeparated from the reſt of the Figures by a Daſh of the Pen, which Figure ſo cut off, denoteth the Remainder after Diviſion is ended, and the Figures on the left Hand of the Daſh are the integral Part of the Quotient ſought.

Wherefore 2 *s.* being a tenth Part of a Pound, when the given Price of the Integer is ſuch, if you cut off the Figure in the Place of Unites with a Daſh of the Pen, and double it, it will give you the odd Shillings, and the other Figures remaining on the left Hand will be Pounds; an Example will make the Rule plain.

At 2 *s.* the Pound, what comes 2447 *l.* to? Anſw. 244 *l.* 14 *s*

In this Example I cut off 7 (in the place of Unites) with a Daſh of the Pen, and double it, it makes 14 Shillings, and the other Figures (*viz.* 244) on the left hand are Pounds, ſo that 2447 *l.* at 2 *s. per l.* comes to 244 *l.* 14 *s.* See the Example wrought.

	244|7	
	l.	*s.*
facit	244	14

Another

Another Example may be as followeth, *viz.* At 2 Shillings *per* Pound, what comes 749 *l.* to? Answer 74 *l.* 18 *s.* as appears by the following Operation.

	74|9	
	l.	*s.*
facit	74	18

In the like manner, when the given Price of the Integer is any other even Number of Shillings, the Price of any other Number of Integers may be found out thus, *viz.*

Take half the Number of Shillings, which is the given Price of the Integer, and thereby multiply the Number of Integers whose Price is required, doubling the Figure that should be first set down in the Product, for Shillings, and carrying the Tens to the Product of the next Figure, as in Multiplication has formerly been taught; so shall the rest of the Product (except the first Figure which is doubled for Shillings) be Pounds, and so the Answer to the Question is gained at one Operation;

As in the foregoing Example, where the Price of 2312 *l.* at 16 *s. per l.* is required.

Here the given Price of the Integer is 16 *s.* whose half is 8, wherefore by multiplying 2312 by 8, the Answer to the Question is gained thus: First I say, 8 times 2 is 16, and I should by the Rule of Multiplication put down 6, and carry 1 to the next, but instead thereof I double 6 (and it makes 12) for Shillings, and carry 1 to the Product of the next Figure for the Ten, and so proceed

to finish Multiplication, and you will find the Answ. to be as before 1849 *l.* 12 *s.* See the Operation as followeth.

	l.		*s.*	
	2312	*at*	16	*per l.*
	8			
	l.		*s.*	
facit	1849		12	

Another Example may be as followeth, *viz.*

At 12 *s.* the Pound, what comes 364 *l.* to? Answer 218 *l.* 08 *s.*

Here according to the Rule I take half the given Number of Shillings (6) and thereby multiply the given Number of Pounds (364) whose Price is required, doubling the first Figure of the Product for Shillings, saying, 6 times 4 is 24, and here I should set down 4 and carry 2, but instead thereof, I double the 4 and it makes 8, which I set down for Shillings; then I proceed saying, 6 times 6 is 36, and 2 that I carry is 38, wherefore I set down 8 and carry 3 to the next Product, *&c.* so the Answer is found to be 218 *l.* 08 *s.* See the following Operation.

	l.		*s.*	
	364	*at*	12	*per l.*
	6			
	l.		*s.*	
facit	218		08	

The

The like may be done in any other Question when the given Price of the Integer is an even Number of Shillings. More Examples of the like Nature follow.

l. *s.* 548 *at* 14 *per l.* 7 *l.* *s.* *facit* 383 12	*l.* *s.* 1462 *at* 8 *per l.* 4 *l.* *s.* *facit* 584 16
l. *s.* 1487 *at* 16 *per l.* 8 *l.* *s.* *facit* 1189 12	*yds.* *s.* 764 *at* 6 *per yd.* 3 *l.* *s.* *facit* 229 4

At 3 *l.* 15 *s.* 9 *d.* the Ounce of Gold Plate, what will 175 Ounces come to? Anſwer, 662 *l.* 16 *s.* 3 *d.* The whole Operation followeth.

oz.
175
3 *l.*
525 *l.*

s. 15 { 2) 175 (87--10 ; 4) 175 (43--15 — *l.* *s.* — 131--05

d. 9 { 2) 175 (87--6 ; 4) 175 (43--9 — *s.* *d.* — 131--3 — *l.* *s.* *d.* 6--11--3

Add all

l.	*s.*	*d.*
525	00	00
131	05	00
6	11	03
facit 662	16	03

 But

But if the Question be put upon 2 Denominations, then reduce them to one.

Example.

At 3 *d.* the Ounce, what will 5 *l.* 7 *oz.* of Plate come to at that Rate? Reduce the 5 *l.* into Ounces, it makes 60, therefore the whole is 65 Ounces at 3 *d.*

oz.	*s.*	*d.*
4) 65	(16	03

Answer is 16 *s.* 3 *d.* by dividing 65 by 4, because 3 *d.* is $\frac{1}{4}$ of a Shilling.

CHAP. XIII.

Of Alligation Medial.

1. THIS Rule is very Necessary to mix Quantities of several Rates, and to discover the mean Price; As also in the Composition of Medicines it is necessary both for the Quantity and the Price, as shall be shewed in the 15*th.* Chapter.

2. Alligation is either *Medial* or *Alternate*.

3. Alligation *Medial* is when, by having the Prices and Quantities of several Simples to be mingled, we discover the mean Rate or Price, of any part of such Mixture or Composition.

4. When

4. When the Quantities and Prices of ſeveral Simples are given to make a Compoſition, the Price of any Part of ſuch Compoſition may be found out by the following Proportion, *viz.*

As the Sum of the given Simples, is in Proportion to their Total Value,

So is any Part of the given Compoſition, to its Price or Value.

Wherefore, when a Queſtion in *Alligation medial* is propounded, firſt find out the Sum of the given Simples, and let that be the firſt Number in the Rule of 3, then find out the total Value of the given Simples, and let that be the ſecond Number in the Rule of 3: And let the Quantity whoſe Price is required be the third Number; ſo will the fourth Number in a direct Proportion be the Anſwer to the Queſtion; as in the following Examples.

1. *Example.*

A Mealman mingleth 40 Buſhels of Wheat Meal at 6 *s. per* Buſhel, with 20 Buſhels of Rye Meal at 4 *s.* 6 *d. per* Buſhel, now I demand how much 1 Buſhel of that Compoſition is worth?

In order to anſwer this Queſtion, I firſt find that

	s.
40 *Buſhels of Meal at* 6 s. per *Buſh. is*	240
20 *Buſh. of Meal at* 4 *s.* 6 *d.* per *Buſh. is*	90
In all 60 *Buſh. worth* ————————	330 *Shill.*

So that now I have diſcovered that the total Compoſition being 60 Buſhels is worth 330 Shillings,

lings, wherefore the Price of 1 Bushel of the same is discovered to be 5 *s.* 6 *d.* by the following Proportion, *viz.*

Bush.		*Shill.*		*Bush.*		*s.*	*d.*
60	:	330	: :	1	:	05	06

2. *Example.*

A Vintner mingleth 40 Gallons of Wine at 8 Shillings *per* Gallon, with 60 Gallons of another sort of Wine at 3 *s. per* Gallon, now I demand the Price of 6 Gallons of that Mixture? Answer 30 Shillings, or 1 *l.* 10 *s.* For

	s.		
40 *Gallons at*	8	*come to* 320	} *Shillings.*
60 *Gallons at*	3	*come to* 180	}

In all 100 *Gallons which come to* 500 *Shillings.*

Then say,

Gall.		*s.*		*Gall.*		*s.*
100	:	500	: :	6	:	30
		6				

100) 30|00 (30 *Shillings.*

3. *Example.*

A Corn-Chandler mingleth 10 Quarters of Oats at 16 Shillings the Quarter, with 20 Quarters of Barley at 24 Shillings the Quarter, and 14 Quarters of Beans at 20 Shillings the Quarter; now I demand

demand how much 1 Quarter of this Mixture is worth? Anſwer 20 *s.* 10 *d.* $3\frac{28}{44}$ *qrs.*

		s.		
10	*Quarters of Oats at*	16	*is*-160	*Shill.*
20	*Quarters of Barley at*	24	*is*-480	
14	*Quarters of Beans at*	20	*is*-280	
In all 44	*Quarters amount to*		920	

Wherefore by the Rule of Proportion ſay,

qrs.		*s.*		*qr.*		*s.*	*d.*	*qrs.*
44	:	926	: :	1	:	20	10	$3\frac{28}{44}$

The Proof of Alligation Medial.

When you have wrought a Queſtion in *Alligation medial*, the Operation may be proved by the following Proportion, *viz.*

As that part of the Compoſition whoſe Price you ſought, is in proportion to its Value found;

So is the total Compoſition to the total Value.

As in the ſecond Example of this Chapter, where the Value of 6 Gallons of the Compoſition in the Queſtion was found to be 1 *l.* 10 *s.* or 30 *s.* therefore to prove the Work, I ſay,

gall.		*s.*		*gall.*		*s.*
6	:	30	: :	100	:	500
		100				

6) 3000 (500 *Shill.*

So

So that the Value of 100 Gallons is 500 Shillings, which is the given Price in the Question, therefore I conclude the Work to be right.

4. *Example.*

A Gold-ſmith having 8 *l.* of Silver Bullion of 7 *oz.* fine, 15 *l.* of 8 *oz.* 10 *p.w.* fine, and 13 *oz.* of 10 *oz.* fine, melting all together would know what fineneſs 1 *l.* Weight of the whole Maſs would be? Anſw. 8 *oz.* 14 *p.w.* fine.

Here becauſe in one of the given Numbers of fineneſs, there is mentioned 10 Penny weights, therefore muſt all the given Numbers of fineneſs be reduced to Penny weights, and each Quantity multiplied by its fineneſs in Penny weights, and the Sum of the Products divided by the Sum of the Quantities, and the Quotient is the Anſwer, which likewiſe is in Penny weights for the former Reaſon.

The

The Operation.

	oz. *p.w.*	*oz.*	*l.*
oz.	8 10	10	8
7	20	20	15
20		200	13
	170	13	
140	15	600	36 *divisor*
8		200	
	850	2600	
1120	170		
2550			
2600	2550 *oz.* *p.w.*		
	20)174 (8 $14\frac{6}{16}$ *fine*		
36)6270(174	160		
36	(14) *p.w.*		
267			
252			
150			
144			
6			

5. *Example.*

A Gold-Smith hath melted 12 *l.* of Gold Bullion of 18 Carracts fine, with 4 *l.* of 21 Carracts fine, I demand of how many Carracts fine is 1 *l.* of this Mass? Answ. $18\frac{11}{16}$ Carracts fine.

The

The Operation.

12 *l.*	4 *l.*	12 *l.*
18 *Car.*	21 *Car.*	4
96	84	*Sum* 16
12		

Prod $\begin{cases} 216 \\ 84 \end{cases}$

Sum 300

Car.

16)300($18\frac{12}{16}$, *or* $18\frac{3}{4}$ *Car. fine.*
16
140
128
12

CHAP. XIV.

Of Alligation Alternate.

1. ALligation Alternate is when by having the Values of ſeveral Simples given, we diſcover what Quantities of each to mingle together, ſo that the whole Compoſition may bear a certain Rate, or Price propounded.

2. Theſe

2. Theſe general Obſervations are to be conſidered, firſt, that all the ſeveral Quantities and Meaſures may be *Homogenial*, that is, alike and of the ſame Kind. Secondly, that the Price of the mixed Quantity be a *mean Price* betwixt ſome of the Prices of the given Simple Quantities.

3. *Alligation Alternate* doth alter or change the *Location* of the *Exceſſes* or Differences, falling out between the mean Price of both the Extreams, aſcribing that to the greater Extream which proceeds from the leſſer, *& contra*; and this being done, the ſaid Exceſſes ſo Alternately and Interchangeably ſituated, do forthwith declare the due Proportion of every Simple, entring the Mixture. In the Alligating the Extreams of the Prices, you are to obſerve theſe 3 following Rules.

1. That each greater Extream muſt be linked with the leſs, the thing whereof the Rule taketh his Name, as in the Example in the Margent, where the given Prices of the Simples are 18, 15, 9, and 4. and the *mean Price* required is 10. And

here you ſee that 18 which is bigger than the *mean Price* (10) is joyned, linked, or alligated to 4, which is leſſer than the ſaid mean Price, and alſo 15, which is greater, is alligated to 9, leſſer than the *mean Price*.

2. That when both the greater and the leſſer Extreams are not ſingle, that then they may be linked ſo diverſly and ſo often, that ſundry Exceſ-

fes

ſes or Differences may be applied for one, and there will be Diverſity of Anſwers, and yet all true. As in the foregoing Example, the biggeſt Extream (18) may be linked with 9, and 15 with 4, the leſſer Extream or 18 and 15 the biggeſt Extreams, may each of them be linked to 9 and 4 the leſſer Extreams, as you ſee in the Margent.

$$10\left\{\begin{array}{l}18\\15\\9\\4\end{array}\right.$$

$$10\left\{\begin{array}{l}18\\15\\9\\4\end{array}\right.$$

3. If either of the Extreams be ſingle, and the other Extreams be plural, the ſingle Extream muſt be linked with all the reſt, and then there can be but one Anſwer, as in the Example in the Margent 18 only is the biggeſt Extream, therefore is joyned with 9 and 4 the leſſer Extreams.

$$10\left\{\begin{array}{l}18\\9\\4\end{array}\right.$$

4. Of Alligation Alternate there are theſe 3 ſeveral Varieties, *viz.*

1. If the Price of every of the Simples be expreſſed, but no Quantity given, and it be demanded how I may mix the Simples to ſell one Meaſure or Quantity at a *mean Rate*, then meer *Alligation Alternate* anſwereth ſuch Queſtions.

2. If the Meaſure or Quantity of one of the Simples, and the Prices of every of the Simples be expreſſed

expreſſed, and it be demanded how much or what Quantity of every of the other Simples may I commix with the Simple expreſſed, to ſell at a *mean Rate*, then *Alligation Alternate*, and the golden Rule joyntly perform ſuch Queſtions.

3. If the Price of every Simple, but none of their Quantities be expreſſed, and it be demanded how much muſt be taken of each Simple to make up a certain Quantity to be ſold at a *mean Rate* propounded, then *Alligation Alternate*, *Addition of the Exceſſes*, and the *golden Rule* perform ſuch Queſtions.

Examples of the firſt Variety.

1. There is Wheat at 28 *d.* the Buſhel, Rye at 20 *d.* Barley at 14 *d.* and Oats at 10 *d.* the Buſhel, to be mingled together, how much of each muſt there be that the whole Compoſition may be worth 16 *d.* the Buſhel? Anſwer, there muſt be 6 of Wheat, 2 of Rye, 4 of Barley, and 12 Buſhels of Oats.

To reſolve this Queſtion, firſt I place the given Prices of the ſeveral Simples, in order one over the other, and on the left Hand thereof draw a line of Connection, and behind that I place 16 the *mean Price* of the Compoſition as you ſee in the Margin.

$$16 \left\{ \begin{array}{l} 28 \\ 20 \\ 14 \\ 10 \end{array} \right.$$

Note here that 28, 20, 14, and 10, are called Extreams in reſpect of (16) the mean, and 28 and 20 are called bigger Extreams, and 14 and 10 are called leſſer Extreams, becauſe they are leſſer than 16 the mean.

Then

Then I proceed in the Work, and joyn 28 a bigger Extream with 10, a leſſer Extream, and alſo 20 a bigger Extream with 14 a leſſer, by Arches or crooked Lines, and behind thoſe Arches I draw a ſtreight upright Line as you ſee in the Margin.

16	28	
	20	
	14	
	10	

Then I proceed to finiſh the Operation thus, firſt, I take the Difference between 16 and 28 (which is 12) and ſet it againſt that Extream with which it is linked, *viz.* 10, and likewiſe the Difference between 20 and 16, (which is 4) and place it againſt 14, and alſo the Difference between 16 and 14, (which is 2) and place it againſt 20, and the Difference between 16 and 10 (which is 6) and place it againſt 28 with which it is linked, and ſo the Work is finiſhed, and I find that if there be mingled together 6 Buſhels of Wheat, 2 of Rye, 4 of Barley, and 12 of Oats; ſuch Compoſition will bear the mean Rate propounded, *viz.* 16 Pence *per* Buſhel.

16	28	6 *Wheat*
	20	2 *Rye*
	14	4 *Barley*
	10	12 *Oats*
		24

The Proof of this Queſtion and ſuch as are of this ſort is, That if the Sum of all the Quantities found being multiplied by the mean Price propounded, its Product be equal to the Sum of the ſeveral

Products

Products of each Quantity multiplied by its Rate or Price, then you may conclude the Operation to be right, otherwise not.

As in the last Example, the sum of the Quantities found is 24, which being multiplied by 16 (the mean Price) the Product is 384. And

28 *times* 6 *is* ———— 168
20 *times* 2 *is* ———— 40
14 *times* 4 *is* ———— 56
10 *times* 12 *is* ———— 120

The Sum of the Product is 384

which is equal to the Product found as before; wherefore the Work is right.

According to *Sect*. 3. *Ob*. 2. the Extreams being plural, *viz*. 14, and 10 less than 16, and 28 and 20 greater, the Alligation may be altered, and yet the Answer true, as followeth.

So that by this Alligation there must be taken 2 Bushels of Wheat, 6 of Rye, 12 of Barley, and 4 of Oats, to sell a Bushel at 16 *d*. The Proof of this is the same with the foregoing Alligation.

16	28	2
	20	6
	14	12
	10	4
		24

Or thus,

16	28	2, 6	8 *Wheat*
	20	2, 6	8 *Rye*
	14	12, 4	16 *Barley*
	10	12, 4	16 *Oats*
			48

In the laſt Alligation I ſet (12) the Difference between 16 and 28 againſt 14 and 10, becauſe it is linked with them both, and alſo the Difference between 16 and 20 againſt the ſame Numbers for the ſame Reaſon, and the Difference between 16 and 14, and between 16 and 10 againſt 28 and 20, they being thereto linked; then draw an upright ſtreight Line on the right Hand of the ſaid Differences, and add the Differences which ſtand againſt each Number together, placing the Sums before the ſaid upright Line, and the Work is finiſhed; ſo that by this Alligation I find there muſt be mingled 8 Buſhels of Wheat, 8 of Rye, 16 of Barley, and 16 of Oats.

Proof.

The Sum of the Particulars is 48, which being multiplied by 16, the Product is 768. Then

8 times

8 *times* 28 *is*——————224
8 *times* 20 *is*——————160
16 *times* 14 *is*——————224
16 *times* 10 *is*——————160

The Sum of the Products is--768

which is equal to the Product first found, and therefore the Work is right.

But if according to *Sect.* 3. *Ob.* 3. the mean Price given had been 12 *d.* then the Alligation would be as full, and the Answer thus,

d. 12	28	2	2 *Wheat*
	20	2	2 *Rye*
	14	2	2 *Barley*
	10	16,8,2	26 *Oats*
			32

So that there must be 2 Bushels of Wheat, 2 of Rye, 2 of Barley, and 26 of Oats, to sell a Bushel of that Mixture for 12 pence.

Proof

32 *times* 12 *makes* 384

2 *times* 28 *is*————56
2 *times* 20 *is*————40
2 *times* 14 *is*————28
26 *times* 10 *is*————260

Sum is 384 *as before.*

2. I would know what Quantity of *Sugar* at 14 *d.* the Pound, *Carroway Seeds* at 3 *s.* or 36 pence the Pound, *Eggs* at 1 *d.* the Pound, and *Flower* at 2 *d.* to make a Compoſition worth 15 *d.* a Pound?

15	36	1, 13, 14	28	*Carroway Seeds*
	14	21	21	*Sugar*
	2	21	21	*Flower*
	1	21	21	*Eggs*

Examples of the ſecond Variety.

1. I have 10 Buſhels of Wheat at 28 *d.* the Buſhel, to mix with Rye at 20 *d.* Barley at 14 *d.* and Oats at 10 *d.* the Buſhel; I demand how many Buſhels of each I muſt mingle with the 10 Buſhels of Wheat, that the Price of the whole Compoſition may de 16 *d.* a Buſhel? Let the Work be firſt managed as in the firſt Example of the firſt Variety, which will be as followeth,

16	28	6	*Wheat*
	20	2	*Rye*
	14	4	*Barley*
	10	12	*Oats*

So that by the foregoing Alligation I find, that to 6 Buſh. of Wheat I muſt put 2 of Rye, 4 of Barley, and 12 of Oats. But I muſt know what Quantity of each I muſt mingle with 10 Buſhels of Wheat, which I may diſcover at 3 ſingle Rules of 3, as followeth.

6

6	:	2	: :	10	:	$3\frac{2}{6}$	*Rye.*
6	:	4	: :	10	:	$6\frac{4}{6}$	*Barley.*
6	:	12	: :	10	:	20	*Oats.*

Therefore I conclude that for every 10 Bushels of Wheat, I must put in $3\frac{2}{6}$ of Rye, $6\frac{4}{6}$ of Barley, and 20 of Oats at the same Prices, and afford the Mixture at 16 *d.* the Bushel.

The same Work will hold in all the several Observations of linking the Values, as in the first Variety; and yet all true.

2. *Example.*

I have 100 Quarts of Canary at 12 *d.* the Quart, to be mixed with Sherry, at 9 *d.* and with Malaga at 6 *d.* the Quart; I demand how many Quarts of Sherry, and of Malaga must be mingled with the 100 Quarts of Canary, that the whole Composition may be sold at 10 *d.* the Quart?

10	12	1, 4,	5	*Canary*
	9	2	2	*Sherry*
	6	2	2	*Malaga*

So that by the foregoing Alligation I find that to every 5 Quarts of Canary, I must put 2 of Sherry and 2 of Malaga; therefore by the Rule of Proportion, I may find how much of each I must mingle with 100 Quarts of Canary, *viz.*

5	:	2	:	::	100	:	40	*of Sherry*
5	:	2	:	::	100	:	40	*of Malaga*

And for Proof I say,

	s.
100 *Quarts of Canary at* 12 d. *comes to*	100
40 *Quarts of Sherry at* 9 d. *comes to*	30
40 *Quarts of Malaga at* 6 d. *comes to*	20
Sum	150

And 180 Quarts of the Composition, at 10 *d.* the Quart come to the same Sum.

Questions of the third Variety.

1. I have Wheat at 20 *d.* the Bushel, Rye at 16 *d.* Barley at 8 *d.* and Oats at 6 *d.* the Bushel, and would mingle 100 Bushels, so as I might afford it at 10 *d.* the Bushel; how much of every sort must I take?

The Operation.

10	20	4 *Wheat.*
	16	2 *Rye.*
	8	6 *Barley.*
	6	10 *Oats.*
		22

So

So that I find, that for every 4 Bushels of Wheat, I must take 2 of Rye, 6 of Barley, and 10 of Oats, which Composition is but 22 Bushels; but I must have a Composition that may be 100 Bush. wherefore I say by the Rule of Proportion,

							Bush.
22	:	4	::	100	:	$18\frac{4}{22}$	*of Wheat.*
22	:	2	::	100	:	$9\frac{2}{22}$	*of Rye.*
22	:	6	::	100	:	$27\frac{6}{22}$	*of Barley.*
22	:	10	::	100	:	$45\frac{10}{22}$	*of Oats.*

2. Mix Gold of 20, 22, and 24 Carracts fine, with Alloy; so that 100 Ounces of that mixture may be but 18 Carracts fine; how much of each sort of Gold, and how much Alloy will be sufficient to perform the same?

The Operation.

18	20	18	18
	22	18	18
	24	18	18
	0	2, 4, 6.	12
			66

So that I have found that 18 *oz.* of each sort of the Gold mingled with 12 *oz.* of Alloy, the Composition will be 18 Carracts fine; but the whole Mass is but 66 Ounces, and should be 100 Ounces; and therefore I say by the Rule of 3, *viz.*

$$66 : 18 :: 100 : 27\tfrac{18}{66}$$
$$66 : 12 :: 100 : 18\tfrac{12}{66}$$

Therefore, if you mix $27\frac{18}{66}$ of each ſort of Gold, with $18\frac{12}{66}$ *oz.* of Alloy, then will the whole Compoſition be 18 Carracts fine.

3. Abate Silver Bullion of $9\frac{1}{2}$ Ounces fine, unto 6 Ounces fine; how much Copper Alloy muſt be mixed, and how much of the Bullion to make 100 weight of the fineneſs required?

The Operation.

$$6\left\{\begin{array}{c} 9\frac{1}{2} \\ 0 \end{array}\right. \left|\begin{array}{l} \text{oz.} \\ 6 \textit{ of Silver.} \\ 3\frac{1}{2} \textit{ of Copper Alloy.} \\ \hline 9\frac{1}{2}. \end{array}\right.$$

So I conclude that for every 6 Ounces of Silver I muſt take $3\frac{1}{2}$ Ounces of Alloy; what Quantity of each muſt be taken for the compounding the Quantity propoſed, may be found out as before.

4. Compound Silver of 3, 5, 8, and 10 Ounces fine, to be juſt 6 Ounces fine.

The Operation.

$$6\left\{\begin{array}{c} 3 \\ 5 \\ 8 \\ 10 \end{array}\right. \left|\begin{array}{c} 4 \\ 2 \\ 1 \\ 3 \\ \hline 10 \end{array}\right.$$

Therefore I conclude I muſt take 4 *oz.* of 3 fine, 2 of 5, 1 of 8, and 3 of 10, to make a Compoſition of 6 Ounces fine.

5. Com-

5. Compound 500 *l.* weight of Silver out of Bullion of 3, 5, 8, and 10 Ounces fine, that the Maſs may be 6 Ounces fine.

The Operation.

6	3	4
	5	2
	8	1
	10	3
		10

Then ſay,

10	:	4	::	500	:	200
10	:	2	::	500	:	100
10	:	1	::	500	:	50
10	:	3	::	500	:	150
						500 *l.* Weight.

6. A certain Vintner having divers ſorts of Wines, *viz.* the beſt at 5 *s.* 3 *d. per* Gallon, other at 3 *s.* 6 *d.* the Gallon, other ſome at 2 *s.* the Gallon, and ſome at 1 *s.* 6 *d.* and he is deſirous to ſophiſticate the Wine, and fill an Hogſhead containing an 100 Gallons with the Mixture of theſe Wines, that he may afford it at 2 *s.* 3 *d.* the Gallon; now I demand how much of each ſort he muſt take to compound the ſaid Quantity?

The

The Operation.

27	63	9
	42	3
	24	15
	18	36
		63

Then say,

63	: 9	: :	100	: $14\frac{18}{63}$
63	: 3	: :	100	: $4\frac{36}{63}$
63	: 15	: :	100	: $23\frac{51}{63}$
63	: 36	: :	100	: $57\frac{9}{63}$

Therefore I conclude that the Vintner must take $14\frac{18}{63}$ Gallons of that Wine which is 5 *s.* 3 *d. per* Gallon, $4\frac{36}{63}$ of that which is 3 *s.* 6 *d. per* Gallon, $23\frac{51}{63}$ of that which is 2 *s. per* Gall. and $57\frac{9}{63}$ Gall. of that which is 1 *s.* 6 *d. per* Gallon.

If the Vintner desires to take yet more of some sort than other, he may link the Terms as is directed in the 3 Observations of Sect. 3. and yet satisfie the Quantity.

7. *Example.*

A Goldsmith hath divers sorts of Silver, *viz.* some of 14 Ounces 12 Penny weight fine, other some of 10 Ounces, another sort of 9 Ounces 11 Penny weight, and some of 8 Ounces 10 Penny weight

weight fine; and he desires to produce a Mass of Silver, weighing 30 *oz.* 6 *p.w.* and bearing 6 Ounces 10 Penny weight and 6 Grains fine; how much of each ought he to take?

These Numbers in the Question are to be linked together according to the foregoing Examples; but I perceive that the given *mean Rate* is less than any Fineness propounded, therefore must there be some Alloy mixed to satisfie the Demand; which being always accounted of no Value, therefore for it put 0.

The Operation.

	oz. p.w. gr.	oz. p.w. gr.
oz. p.w. gr.	14 : 12 : 0	6 : 10 : c6
6 : 10 : 6	10 : 00 : 0	6 : 10 : 06
	9 : 11 : 0	6 : 10 : 06
	8 : 10 : 0	6 : 10 : 06
	0 : 00 : 0	16 : 12 : 00
	Sum	42 : 13 : 00

oz.	p.w.	gr.	oz.	p.w.	gr.
14	12	00	10	00	00
6	10	06	6	10	06
8	01	18	3	09	18

9 oz.

oz.	*p.w.*	*gr.*		*oz.*	*p.w.*	*gr.*
9	11	00		8	10	00
6	10	06		6	10	06
3	00	18		1	19	18
8	01	18				
3	09	18				
1	19	18				
16	12	00				

So that by the foregoing Alligation, I have found that there muſt be 6 *oz.* 10 *p.w.* 06 *gr.* of each ſort of Silver, and 16 *oz.* 12 *p.w.* of Alloy to make a Maſs of the fineneſs required; the Sum of which is 42 *oz.* 13 *p.w.* 00 *gr.* But the whole Maſs ſhould be but 30 *oz.* 6 *p.w.* wherefore I ſay by the golden Rule, if 42 *oz.* 13 *p.w.* require 6 *oz.* 10 *p.w.* 6 *gr.* what will 30 *oz.* 6 *p.w.* require? Anſwer, 4 *oz.* 12 *p.w.* $12\frac{696}{851}$ *gr.* is the Quantity of each ſort of Silver to be mingled: Then for the Alloy ſay, if 42 *oz.* 13 *p.w.* require 16 *oz.* 12 *p.w.* what will 30 *oz.* 6 *p.w.* require? Anſwer 11 *oz.* 15 *p.w.* $20\frac{628}{851}$ *gr.* and ſo much Alloy muſt there be, as appears by the following Work.

First

First for the Quantity of each sort of Silver.

```
oz. p.w.  oz.p.w. gr.    oz.p.w.  oz. p.w. gr.
42--13 : 6--10--06  ::  30-06  : 4--12--12 696/851
 20       20              20
----     ----            ----
853      130             606
          24
        ----
         526
        260
        -----
        3126
         606
        -----
       18756
      187560
      -------      24)        20)  oz. p.w. gr.
853)  1894356    (2220       (92  (4--12--12 696/851
        ....        ..        80
       1706        216       ----
      ------     -------Re.(12)p.w.
        1883        60
        1706        48
       ------     -----
        1775  Rem.(12)gr.
        1706
       ------
Rem.(696)
```

Secondly,

Secondly, for the Quantity of Alloy.

```
oz. p.w.   oz. p.w. gr.    oz. p.w.  oz. p.w. gr.
42--13  :  16--12--0   ::  30--6  :  11-15-20 628/853
20         20              20
---        ---             ---
853        332             606
            24
          ----
          1328
          664
          ----
          7968
           606
          ----
         47808
        478080
        -------    24)       20)  oz. p.w. gr.
853)   4828608    (5660     (235 (11-15-20 628/853
       4265        48        20
       ----        --        --
       5636        86        35
       5118        72        20
       ----        --        --
       5180        140  Re.(15)p.w.
       5118        120
       ----        ---
Rem. (628) Rem.(20)gr.
```

If you take 4 *oz.* 12 *p.w.* 12$\frac{696}{853}$ *gr.* of each sort of Silver, and 11 *oz.* 15 *p.w.* 20$\frac{628}{853}$ *gr.* of Alloy and add them all together, the Sum will be 30 *oz.* 6 *p.w.*

6 *p.w.* which proves the Work to be right.

8. *Example.*

A Mint-Maſter hath 100 *l.* weight of Gold of 21 Carracts fine, and 20 *l.* weight of 15 Carracts fine, and he would ſo mix them together, that every Pound of the whole Maſs might be 18 Carracts fine; whether needeth he to mix any Alloy with it, and how much?

For Anſwer, I muſt firſt find out by *Alligation Medial*, what fineneſs 1 *l.* of this Mixture will be of, when both ſorts of this Gold are mixed together.

The Operation.

100 *l.*	20 *l.*		*l.* 100
21 *Car.*	15 *Car.*		20
100	100	*Sum*	120 *Diviſor*
200	20		
2100	*Prod.* { 300 / 2100		

12|0) 240|0 (20 *Carracts fine.*

So that by the foregoing Work I have found 1 *l.* of the ſaid Mixture or Maſs to be 20 Carracts fine, but I would have it but 18 Carracts fine; therefore there muſt be ſome Alloy mingled with it; and by the following Work I find the Anſwer to be thus, *viz.* That for every 18 Pound of Gold there muſt be 2 Pounds of Alloy taken, as by

by the following Alligation appeareth.

$$18 \begin{cases} 20 \\ 0 \end{cases} \Big| \begin{matrix} 18 \\ 2 \end{matrix}$$

Now to find what Quantity of Alloy must be put to the whole Quantity of Gold, I say by the *Golden Rule*,

l.		*l.*		*l.*		*l.*
18	:	2	: :	120	:	$13\frac{1}{3}$

```
        120
     ———————   l.       l.
18)  240  (13 6/18, or 13 1/3
     18
     ———
      60
      54
     ———
     (6)
```

So that to the 120 *l.* of Gold there be put $13\frac{6}{18}$ *l.* or 13 *l.* 04 *oz.* of Alloy to make it 18 Carracts fine.

9. Example.

A Goldsmith having several sorts of Silver Bullion, *viz.* 24 *l.* Weight of 7 *oz.* fine, 30 *l.* weight of 9 *oz.* fine, and 16 *l.* weight of 12 *oz.* fine, would melt all these together, so as 1 *l.* weight of the Mass might bear 10 Ounces fine; whether ought he to melt any Alloy therewith, and how much? First, find by Alligation Medial the fineness of 1 Pound. *The*

The Operation.

24 *l.*	30 *l.*	16 *l.*	*l.* 24
7 *oz.*	9 *oz.*	12	30
			16
168	270	32	
		16	*Sum* 70 *Divisor*
		192	

Prod. $\left\{\begin{array}{l}168\\270\\192\end{array}\right.$

Sum 630 *Dividend*

70) 630 (9
630

(0)

So that I find, when the given Quantities are melted together, they will make a Maſs of but 9 Ounces fine; but by the Tenor of the Queſtion the Goldſmith ſhould have 10 Ounces fine, wherefore this Mixture is not fine enough by one Ounce in every Pound Weight; ſo that no Alloy is to be mixed, but contrary-wiſe more fine Silver, which is 12 *oz.* fine, muſt be put unto it, which to find, the Work by Alligation Alternate is as followeth,

$10\left\{\begin{array}{l}9\\12\end{array}\right.\Big|\begin{array}{l}2\\1\end{array}$ Which Alligation ſignifieth that for every 2 *l.* of the 3 Bullions mixed together, there muſt be taken 1 *l.* of fine Silver.

Laſtly to find what Quantity of fine Silver muſt be mingled with the whole Quantity of the 3 given Bullions, I ſay by the Golden Rule,

l.		*l.*		*l.*		*l.*
2	:	1	::	70	:	35

Therefore the whole Mass of 70 *l.* will require 35 *l.* of fine Silver to be Incorporated with it, to make 1 *l.* Weight 10 *oz.* fine.

Proof.

70 *l. Weight, multiplied by* } 630
9 *oz. its fineness, produceth* }

And

35 l. *by* 12 oz. *its fineness, produceth.*---420

Sum 1050

Now,

105 l. *multiplied by* 10 oz. *its* } 1050
fineness, produceth ———--- }

Which is equal to the Sum of the former Products, therefore I conclude the work to be right.

CHAP.

CHAP. XV.

Of divers useful Propositions concerning the Composition of Medicines.

1. WE Consider all Medicines, either in their *Quantities*, *Qualities*, or *Prices*.

2. The Weights which Apothecaries use, are Pounds, Ounces, Drams, Scruples, and Grains: In a Pound there are 12 Ounces, in an Ounce 8 Drams, in a Dram 3 Scruples, in a Scruple 20 Grains. These alter from Troy Weight only in the Subdivisions. The Character for Pounds is ℔, for Ounces ℥, for Drams ʒ, for Scruples ℈, for Grains, *gr*.

3. You are to observe in the Works following, that all the Quantities are to be brought to one Denomination by Reduction, *viz.* into the least Term named, as, Pounds into ℥ by multiplying by 12; ℥ into ʒ by multiplying by 8; Drams into ℈, by multiplying by 3; ℈ into *gr*. by multiplying by 20: Or otherwise, ℔ into ʒ, by multiplying by 96; and ℔ into ℈, by multiplying by 288.

4. The *Qualities*, *Faculties* and *Vertues* of *Medicines* are considered in respect of us and not of themselves; for those Simples are called temperate, that bring no Change in our Bodies, in respect of Heat, Cold, Moistures and Dryness; those hot which have Power of Heat, those cold

which work cold, and so likewise some moist and some dry; so that all Medicines and Simples are considered in Quality in some of these Ways, either as they are *hot* or *cold*, *moist* or *dry*, or as they are *temperate*, which may likewise be taken two Ways; for either a Medicine may be said to be temperate, in that it is neither hot nor cold, and yet may be moist or dry, or in respect it is neither moist nor dry, and yet may be hot or cold; or lastly and generally it may be said to be temperate, in respect it is neither hot, cold, moist or dry.

5. The Differences of these Qualities are distinguished into Degrees; for a Simple or Medicine, according to its express in Quality, is accounted to proceed from the Temperature towards any other Quality in 4 Degrees, and so a Medicine is said to be hot, cold, moist or dry, in the first, second, third or fourth Degree. As concerning the Prices and the due proportioning thereof, it shall be spoken of in the Propositions following.

Prop. 1. *To augment a Medicine in Quantity, keeping the Proportion given.*

Sum up the Quantities of the Medicine given; then say, as the Sum of the Medicine given is to the Augmentation, so is the Quantity of each Parcel of the Medicine given to the Quantity of the augmented Medicine desired.

Example.

Example.

Unguentum album Camphoratum.

	℥		ʒ
℞. *Oyl of Roses* ———	12	*or*	96
White Wax ———	3	*or*	24
Ceruse ———	6	*or*	48
Camphyre beat with } *Oyl of Roses* ——— }	0	*or*	02
			170

I desire to augment this, or make up an Oyntment consisting of 210 ℥.

Therefore say,

$170.\ 210 :: 96.\ 118\tfrac{10}{17} :: 24.\ 29\tfrac{11}{17} :: 48.\ 59\tfrac{5}{17} :: 2.\ 2\tfrac{8}{17}$

```
     96               210        210
    ----             ----       ----
     126               24         48          420(2
    189               48         96           340
   ---------         ------     ------       ------
170)20160(118        5040(29    10080       (5980
    316               164        85
    170               153        158
    ----             -----      -----
    1460              110        153
   -----                        -----
     100                         50
```

Answer, you must take of Oyl of Roses $118\tfrac{10}{17}$ ℥, of white Wax $29\tfrac{11}{17}$ ℥, of Ceruse $59\tfrac{5}{17}$ ℥, of Camphyre $2\tfrac{8}{17}$ ℥, to make up the Medicine 210 ℥.

Prop. 2. *To diminiſh a Medicine in Quantity, keeping ſtill the Proportion of the given Quantities.*

This is performed as the laſt Propoſition; for as the Sum of the Quantities of the given Doſe, is to the Sum of the Quantities of Diminution, ſo is the ſeveral Quantities of the Doſe given, to the ſeveral Quantities of Diminution. The Work as in the former.

Prop. 3. *To find out what Quantity of any Ingredient or Simple is contained in any Quantity of a Compoſition.*

Conſider the Compoſition, and ſum up the Ingredients; then ſay, as the Sum of the great Compoſition is to the Quantity of your Doſe; ſo is the Quantity of the ingredient propoſed in the Compoſition, to the Quantity of that Ingredient in the Doſe propoſed.

Example.

I have 1 ℥ of *Pilulæ ſine quibus*, I would know what Quantity of *Scamony* is contained therein? For Anſwer, I conſider the Compoſition, and find that in the whole there is contained 328 ʒ: Therefore I ſay, 328 ʒ. 96 :: 1. 1¼; and I conclude that there being 6 Drams contained in the whole Doſe, there muſt be 1¼ ʒ in 1 ℥ of that Compoſition.

Prop. 4. To know the Exact Temperament and Quality of any Medicine whatsoever.

Hitherto we have ſpoken of the Quantities, now come we to the Qualities, and for finding the Quality emergent,

1. Diſpoſe the Quantities of all your Simples into one Row, ſetting orderly by them their ſeveral Quantities, and if any of the Ingredients be compounded, you muſt firſt learn its Quality.

2. Multiply each Quality by its own Quantity, and ſubſtract the hot from cold, moiſt from dry, or contrarily, and ſet down the Difference of theſe Products: For as the Sum of all the Quantities is to the Difference of the Products, ſo is Unity or one to the Quality emergent, which is always of the ſame Kind that the greater Product was of.

Example.

I have 4 ℥ of a Simple cold in 2°, moiſt in 1°, 5 ℥ hot in 3° and temperate, 3 ℥ hot in 2°, and dry in 2°, 6 ℥ hot in 1°, moiſt in 4°, 4 ℥ cold in 3°, and moiſt in 2°. I would know if theſe were compounded, what ſhould be the Temper reſulting? For Anſwer, I order them as in the Example.

	℥	o		
Hot.	5	3	15	
	3	2	6	
	6	1	6	27
Cold.	4	2	8	
	4	3	12	20
	22			

$\frac{27}{20}$ $\frac{7}{22} = \frac{1}{3}$ *Heat.*

	℥	o		
Moist.	4	1	4	
	5	0	0	
	6	4	24	
	4	2	8	36
Dry.	3	2	6	
	5	0	0	6
	27			

Moist.
$\frac{30}{17}$ ($1\frac{3}{27}$

I first set down my Quantities of heat and cold, and multiply them by their Qualities: I add up both the Products, and subtract the lesser from the greater, and divide the Remainer by the Quantity of Ounces, the Quotient gives me $\frac{1}{3}$ o of heat, and $1\frac{3}{27}$ of moist, because the Products 27 of dry, and 36 of moist, were the biggest; and now therefore I conclude that the Resultment of this Medicine in Quality was $\frac{1}{3}$ Deg. of heat, and $1\frac{3}{27}$ of moist.

2. *Example*

2. *Example.*

Where the Quantities are the same, what is the Quantity emergent of 1 ℥ hot in 3°, moist in 2°, mixed with 1 ℥ cold in 1°, and dry in 3°? Answ. in 1° of heat, and $\frac{1}{2}$ of drought.

$$\begin{array}{lccl} \textit{Hot.} & 1 - 3 & 3 & \\ \textit{Cold.} & 1 - 1 & 1 & \\ \hline & 2 &)\,2\,(& 1\ \textit{Heat.} \end{array} \qquad \begin{array}{lccl} \textit{Moist.} & 1 - 2 & 2 & \\ \textit{Dry.} & 1 - 3 & 3 & \\ \hline & 2 & \frac{1}{2} & \textit{Dry.} \end{array}$$

3. *Example.*

$$\begin{array}{l} \begin{array}{lr|r} & ℥ & \\ \textit{Hot.} & 31 - 3 & 93 \\ & 11 - 3 & 33 \\ & 5 - 2 & 10 \\ & 2 - 0 & 0 \\ \hline \textit{Cold.} & 17 - 2 & 34 \\ & 15 - 3 & 45 \\ & 2 - 0 & 00 \\ \hline & 83 & \end{array} \quad \begin{array}{l} 136 \\ \\ 79 \end{array} \quad \frac{57}{83}\ \textit{Heat.} \end{array}$$

$$\begin{array}{lr|r} \textit{Dry.} & 11 - 4 & 44 \\ & 2 - 3 & 6 \\ & 7 - 1 & 7 \\ & 9 - 4 & 36 \\ \hline \textit{Moist.} & 5 - 2 & 10 \\ & 9 - 1 & 9 \\ \hline & 43 & \end{array} \quad \begin{array}{r} 93 \\ \underline{19} \\ \underline{74} \\ \underline{43} \\ 31 \end{array} \quad (1\tfrac{31}{43}\ \textit{Drought.}$$

4. *Exam-*

4. *Example.* *Pilulæ Synoglossi.*

	ℨ	call	fri.	hum.	sic.	tem.
Radicum Synoglossi sic.	48	0	2	0	2	0
Sem. Hyoscijami albi.	48	0	3	0	1	0
Opii præparat.	48	0	4	0	0	0
Myrrhæ.	6	2	0	0	2	0
Thuris Musc.	5	2	0	0	1	0
Croci.	2	5	0	0	1	0
Castorei.	2	5	0	0	2	0
Styracis Calamitæ.	2	1	0	0	0	0

161 The Sum of the Quantities in Drams.

		q.	c.	f.	h.	sic.
The Work.	1	48	0	96	0	96
	2	48	0	144	0	48
	3	48	0	192	0	0
	4	6	12	0	0	12
	5	5	10	0	0	5
	6	2	10	0	0	2
	7	2	10	0	0	4
	8	2	2	0	0	0
		161	44	432	0	167

432 —— 44 = 388
167 —— 0 = 167

161 (388 (2 $\frac{66}{161}$ *frig.*
322
66

161) 167 (1 $\frac{6}{161}$ *sic.*

Wherefore

Wherefore I conclude the ſaid Pills to be cold in 2 Degrees and almoſt an half, and dry in one Deg. and ſomething more; the ſame may be pronounced of any other Doſe.

Prop. 5. *To augment in Quality a Medicine to any Degree propoſed.*

It is requiſite for an Apothecary to know the due Reſultment of any Medicine, as *Sennertus* affirms, *Lib.* 5. *Inſtit. Pars.* 3. *Chap.* 1. which is taught by the laſt; and in the ſame Chapter, amongſt many other Obſervations, he ſaith, *Interdum vis Medicamenti eſt debilis, quam validioris admiſtione intendere; contra nonnunquam vehementior, quam delilioris additione remittere, oportet.*

The Rule.

1. Diſpoſe all the Quantities given either of the Simples or Compounds in their exact Degrees, which are to be taken and tempered out of Medicines contrary betwixt themſelves, *viz.* hot with cold, and moiſt with dry, otherwiſe no exact temperament can be choſen.

2. As in the laſt Prop. multiply the Quantities and Qualities, and ſubſtract the Products, as was there taught.

3. Then always chuſe one of the Simples a Degree higher than the propounded Degree, as if I would raiſe it from 1° of Heat to 2°: Then I chuſe among the Simple Medicines one in 3° Degrees of heat, or if I find none, then I muſt chuſe a Simple of

of that Degree; this done, by the Degree you have given, multiply the Sum of the Quantities, and from the Product subtract the Difference of the Product of either Temper; the Remainder divide by the Difference of the Degree desired, and the Degree from which it is desired to be augmented.

1. *Example.*

I have of Simples, 1 ℥ dry in 4°, 2 ℥ dry in 3°, 1 ℥ dry in 1°, and 2 ℥ dry in 2°, this being mixed with other Simples, *viz.* 1 ℥ cold in 2°, and 1 ℥ cold in 1°; I finding the Temper of this Mixture to be only $1\frac{1}{2}$ Degrees of Heat, desire that it may be augmented to two Degrees.

```
     ℥.       o.
     1—4 ⎫ 4 |
H.   2—3 ⎬ 6 |
     1—1 ⎪ 1 | 15 ⎫
     2—2 ⎭ 4 | —  ⎬ 12
C.   1—2 | 2 | 3  ⎭
     1—1 | 1 | —
     8
     2        1) 4 (4.
     —
     16
     12
     —
     4
```

I find that by adding 4 ℥ to the 2 ℥, that is, hot in 3° (because it is one Degree higher than the Degree desired) to augment the Heat to two Degrees.

The Proof.

```
             ℥.     o.
     2      1—4 |  4 |
     4    —6—3  | 18 | 27
     —          |    |  3
     6 H.   1—1 |  1 |  —
            2—2 |  4 |        24 (2
            —        |  3)    12
            1—2 |  2 |
            1—1 |  1 |
     12
```

2. *Exam-*

2. Examp. Where the Heat is raised 2 Degrees.

3—3	9			3 3	9				
15—3	45			15 3	45				
7—4	28	90	7	66 · 4	264				
8—1	8	41	8	8 - 1	8	326			
11—2	22	49		11 · 2	22	41			
5—1	5	56		5 · 1	5	285			
7—2	14			7 · 2	14				
56				115					
3									
168									
49									
2) 119 (59½									

Prop. 6. To diminish a Medicine in Quality, from any Degree whatsoever.

This is but the Converse of the former, for taking one degree of Cold, or that Way from the Degree of Heat given more than the Degree proposed, and working as before in the last Proposition, you have the Quantities to be added to the Quantities of the Degree taken.

Prop. 7. To reduce any Medicine proposed to any Degree of Quality whatsoever.

1. This is wrought by the Rule of *Alligation*, and therefore you must have your Qualities to ascend from 1 to 10, that a Difference amongst them may

may be found out, accounting the Temperate equal to 5, and then adding Degrees of Hot and Dry to 5, and substracting Cold and Moist from 5, as in these Figures,

Cold and Hot,	9 8 7 6 5 4 3 2 1.	Cold
Moist and Dry,	4 3 2 1 0 1 2 3 4.	and Moist.

where the upper Figures may be termed the Differences, the lower being the Quantities, the Cypher standing for Temperate, and the Degrees of Hot and Dry being accounted towards the right Hand, Cold and Moist towards the left.

2. In this Rule the Operation is still wrought by the Differences or higher Numbers, and not with the Degrees themselves; as if you would set down 3° of Heat, and 1° of Cold, you must use the Numbers above, *viz.* 8 and 4.

3. Therefore when a Question in this Way is proposed, set down the Differences in their Order, from the highest Degree of Heat downwards, or contrarily, and then by the Rule of Alligation alternate, take the Differences from the Degree proposed alternate, and subscribe the Doses or Quantities, as in the Example following.

1. *Example.*

I have Simples some hot in 3°, in 2°, some temperate, some cold in 2°, and 4°; It is desired that a Medicine may hereof be compounded hot in the 1°, what Doses or Quantities must be taken of each?

Answer,

Answer, for to make the Medicine 14 ℥, you must take 8 ℥ of the first, or of hot Quantity in 3°, of the second and third, of either 1 ℥, and of the two last of either two, or changing the Alligation, you may alter the Quantities as in the second Alligation.

Hot 1°	3° Hot.	*In their differen-ces to be set thus.*		*diff.*	*quan.*	*q.*
	2° Hot.		6	8	5 + 3	8
	0° Temp.			7	1	1
	2° Cold.			5	1	1
	4° Cold.			3	2	2
				1	2	2
						14

The Proof.

	diff.	*Or thus.*	*q. d.*			*q. d.*	
6	8	5 + 1	6 - 8	48		8 - 8	64
	7	3	3 - 7	21		1 - 7	7
	5	2	2 - 5	10		1 - 5	5
	3	1	1 - 3	3		2 - 3	6
	1	2	2 - 1	2		2 - 1	2
			14)	84 (6	14)		84 (6
				84			14
							84

The Proof of this Rule ariseth out of Prop. 4. For if you multiply the Quantities by the Differences, and divide the Product by the Sum of the Quantities, and if the Quotient be equal to the Quality proposed, then you have wrought well.

2. *Example.*

2. *Example.*

And here are all the ſeveral Caſes of Alligation to be applied, as in this Example, where the Qualities of all, but the Quantity of one Simple is expreſſed, and it is deſired what Quantities of the reſt muſt be taken to make up a Medicine of a Quality given.

As for Example.

I have 4 ℥ of a Simple hot in 3° to be mixed with Qualities hot of 2° and 1°, and with Qualities cold of 2° and 4°; what Quantities of each of the latter muſt I take to make the Medicine only in 1° of heat? Anſwer,

$$1^{\circ}\,h.\left\{\begin{array}{l}3\,h.\\2\,h.\\1\,h.\\2\,h.\\4\,h.\end{array}\right\}6\left\{\begin{array}{l}8\\7\\6\\3\\1\end{array}\right. \quad \begin{array}{|l|l}℥. & ℥.\\5 & 5\\3 & 3\\1,3 & 4\\1 & 1\\2 & 2\end{array}$$

Then if
$5:3::4:2\tfrac{2}{5}$
$5:4::4:3\tfrac{1}{5}$
$5:1::4:\tfrac{4}{5}$
$5:2::4:1\tfrac{3}{5}$

The ſeveral Ounces and Parts to be taken of the reſpective Qualities to be mixed with the 4 ℥ of 3° in heat, and yet the Quality to be 1° in heat.

Another Variety may be in ſuch a Queſtion as this; I have Simples of ſeveral Qualities; what Quantity muſt I take of each to make a Doſe of 14 ℥, and yet the Quality to be ſome Mean a-

mongſt

mongſt the Qualities of the Simples given? But becauſe theſe and the like have been largely handled in Alligation alternate, I remit the Reſolution.

And here I might again obſerve to the Reader, how by theſe Differences of Qualities the former Propoſitions may as eaſily be wrought; as in the Proof of Examp. 1. Prop. 5. I would know the exact Temperament of that Medicine.

1—4° H.		1—9	9
6—3° H.		6—8	48
1—1° H.	*In their Differences thus.*	1—6	6
2—2° H.		2—7	14
1—2 Co.		1—3	3
1—1 Co.		1—4	4
		12)	84 (7
			84

I find the Difference of the Quantity to be 7, which ſignifieth 2° of Heat, anſwerable to the Proof of that Example in Prop. 5. I will now paſs to the Prices of Medicines.

Prop. 8. *To find out the Value or Price of any Quantity of a Medicine, having the Values of the Simples firſt given.*

This is found out by Alligation medial, for as the Sum of the Quantities is to the Sum of the ſeveral Products of the Quantities in their Prices, ſo is the Quantity propoſed to its Price.

As for Example.

If an Apothecary compound the Oyntment *Unguentum pectorale, viz.* New Butter 6 ℥ at *ob.* the ℥, 4 ℥ of the Oyl of Sweet Almonds at 1 *d. ob.* the ℥, 3 ℥ of the Oyl of Camomil at 3 *d.* the ℥, of Violets at 4 *d.* the ℥, 2 ℥ of the Fat of a Duck at 3 *d.* the ℥, of a Hen at 2 *d.* the ℥, 1 ℥ of *Flower de Lys* Roots at 2 *d.* the ℥, and 3 ℥ of white Bees Wax at 1 *d.* the ℥; how may he sell 4 Ounces of this Medicine? Answer, 7 *d. ob.* View the following Work.

oz.		*d.*		*d.*
06	at	$0\frac{1}{2}$	comes to	03
04		$1\frac{1}{2}$		06
03		3		09
03		4		12
02		3		06
02		2		04
01		2		02
03		1		03

In all 24 Ounces which cost 45 Pence.

Wherefore I say by the Rule of 3,

If 24 *oz.* cost 45 Pence, what will 4 *oz.* cost? multiply and divide, and you will find the Answer to be $07\frac{1}{2}$ *d.*

An Apothecary useth in Powder 6 *l.* of Sugar of 03 *s. per l.* 2 *l.* of Liquorice at 6 *d. per* Pound, 3 *l.* of Annis Seeds, at 10 *d. per l.* and 1 *l.* of Fennel Seeds

Seeds at 6 *d.* Now I desire to know what 3 *l.* of this Mixture is Worth? Answer 05 *s.* 06 *d.*

The Operation.

l.		*d.*		*d.*
6	*at*	36	*is* ———	216
2	*at*	6	*is* ———	12
3	*at*	10	*is* ———	30
1	*at*	6	*is* ———	6

In all 12 *Pounds come to* 264 *Pence.*

Then by the Rule of 3 *I say*

l.		*d.*		*l.*		*d.*	*s.*	*d.*
12	:	264	: :	3	:	66 or	5	06.

Thus have I delivered 8 useful Propositions concerning Medicines, never to my Knowledge written on before.

CHAP. XVI.

Of Fractions.

Section 1. Of finding the greatest common Measure of two Numbers.

1. A Number is said to be *prime* which cannot be measured by any Number of Multitude: such are 2.3.5.7.11.13.17.19, *&c.* which no Number of Multitude (that is) above an Unite, can divide evenly, but there will be a Remainer, and therefore are called *prime Numbers.*

2. A Number is said to be *compound*, when it may be divided by a Number of Multitude; as 4. 6. 8. 10. 12. *&c.* are compound Numbers, because they may every of them be divided by a Number of Multitude, as 8 may be evenly divided by either 2 or 4.

3. Numbers may be said to be *prime amongst themselves*, when no one Number of Multitude can measure them both: as 8. and 9. 14. and 15. 21. and 34. *&c.*

4. Numbers are said to be *compound amongst themselves*, when one Number of multitude will divide or measure them, and the Number so measuring them, then is called their common Measure; and if it be the greatest that can be, it is called their greatest common Measure, as 8 and 12 may be

be meaſured by 2, but their *greateſt common Meaſure* is 4.

5. The *greateſt common Meaſure* of two Numbers is found by a continual Diviſion of the greater by the leſſer, and of the Diviſor by the Remainer; for the firſt Diviſor that divideth the Dividend without a Remainer, is the greateſt common Meaſure of both Numbers.

Thus the greateſt common Meaſure of 24 *and* 38 *is* 2,

```
24) 38 (1
    24
    --
    14) 24 (1
        14
        --
        10) 14 (1
            10
            --
             4) 10 (2
                 8
                 --
                 2) 4 (2
                    4
                    --
                    0
```

of 36 and 54 is 18.

```
36) 54 (1
    36
    --
    18) 36 (2
        36
        --
```

6. And therefore all Fractions whatſoever (if poſſible) ought to be reduced into their leaſt Terms, before you inſtitute any Work in them, as $\frac{36}{54}$ by dividing them by 18 become $\frac{2}{3}$, and ſo likewiſe $\frac{432}{728}$ by dividing them by 8 become $\frac{54}{91}$, and have the ſame Value that the firſt Fraction had, for as 432. 728 : : 54. 91.

7. Therefore if the Terms of a Fraction be in-

 commenſurable

commensurable together, then such a Fraction is in his least Terms already.

8. If both the terms have outwardly Cyphers, (I mean towards the right Hand) such a Fraction may be much abridged speedily by cutting off a like Number of Cyphers from them both.

Thus $\frac{2|00}{3|00}$ *will be forthwith* $\frac{2}{3}$ *so* $\frac{150}{300}$ *will be* $\frac{15}{30}$ *or* $\frac{1}{2}$.

9. Even Numbers may be much abridged by halfing of both as long as they will be halfed; and so may any Number which appeareth presently to the Eye, to be equally divisible or commensurable by any other Number.

Sect. 2. *Of the Reduction of Fractions.*

1. In Fractions, as the *Denominator* is to the *Numerator*, so *one whole* is to *the Parts signified by the Fraction.*

As for Example.

In the Fraction $\frac{2}{3}$ of a Shilling, as 3 is to 2, so 12 *d.* is to 8 *d.* which is two third Parts of a Shilling.

2. *Hence* it followeth necessarily, that the Tables of the same Fraction may be infinite in a Number, so that they still bear the same Proportion one to another, as in the Example before $\frac{2}{3}$ it may be expressed by $\frac{4}{6}$. $\frac{6}{9}$. $\frac{8}{12}$. $\frac{10}{15}$. $\frac{12}{18}$. $\frac{14}{21}$. $\frac{20}{30}$. $\frac{200}{300}$. *&c.* and so infinitely, every one of which Fractions is but still two Thirds.

3. *There-*

3. *Therefore* alſo it followeth, that if the *Numerator* be leſſer than the *Denominator*, that then the Parts ſignified are leſs than one Whole, be the Terms never ſo big, as in $\frac{1579}{5797}$, and this is a Fraction properly ſo called, or a proper Fraction.

4. *Therefore* alſo if the Numerator be equal to the Denominator, then the Fraction is equal to one Whole, as $\frac{2}{2}$. $\frac{3}{3}$. $\frac{20}{20}$. *&c.* every one of which is equal to one Whole, and ought be expreſſed by 1.

5. *Therefore* alſo if the Numerator be greater than the Denominator, then the Fraction is greater than the Whole, as $\frac{379}{239}$. And theſe two laſt kinds are improperly called Fractions, becauſe they do include in them one or more Integers.

6. To *reduce* a whole Number into an *improper Fraction* of any Denomination given, is done by multiplying the Number by the Denominator, and ſetting the Product over the Denominator in form of a Fraction: as to reduce 5 whole into halfs, it is thus done, $5 \times 2 = 10$ that is $\frac{10}{2}$. ſo 7 into 3 Parts, it is $\frac{21}{3}$, and 6 into Quarters is $\frac{24}{4}$.

7. *Therefore* if a mixt Number (*viz.* Integers and a Fraction) be propounded to be reduced into an improper Fraction of the ſame Denomination with the Fraction annexed, the whole Number muſt be multiplied by the Denominator of the Fraction, and unto the Product the Numerator muſt be added, and ſo the Sum will be the Numerator of the improper Fraction required.

As for Example.

Let $35\frac{3}{5}$ a mixt Number, be reduced into an improper Fraction of the ſame Denomination with $\frac{3}{5}$ it

will be thus, $35\times5=175+3=178$ and the improper Fraction will be $\frac{178}{5}$, which is the ſame in Value with $75\frac{3}{5}$.

8. *Therefore* alſo on the contrary, if an improper Fraction propounded, be required to be reduc'd into the *Integers* or *whole Numbers* contained in it, the Numerator muſt be divided by the Denominator, and the Quotient will ſhew the Integers, and the Remainder (if any be) the odd Parts.

Example.

Let $\frac{10}{2}$ be reduced into Integers, it will be 5 Integers thus, $2)\,10\,(5$. ſo $\frac{178}{5}$ is reduced into $35\frac{3}{5}$ thus $5)\,178\,(35\frac{3}{5}$.

9. Two Fractions of divers Denominations are to be reduced to the ſame Denomination, their Values ſtill remaining thus, *viz.* divide both Denominators by their greateſt common Meaſure, ſetting the Quotients alternately, as you ſee in the Examples, then multiply the common Meaſure and thoſe Quotients together for a new Denominator, and either Numerator by the Quotient under him for a new Numerator.

Example.

$\frac{5}{12}$ and $\frac{7}{18}$ are reduced into two Fractions of the ſame Denomination, *viz.* $\frac{15}{36}$ and $\frac{14}{36}$ dividing by their greateſt common Meaſure 6. See the following Work.

But if the Denominators cannot be meaſured, then multiply them together for a new Denominator,

tor, and the Numerators cross by the Denominators for new Numerators. See the Example,

$$\begin{array}{l} \quad 15\ \ 14 \\ \quad \cdots\cdot \\ 6)\ \tfrac{5}{12}\ \ \tfrac{7}{18}\ \ \tfrac{15}{36}\ \ \tfrac{14}{36} \\ \quad \cdots\cdot \\ \quad \underbrace{3\ \ \ 2}_{36} \end{array}$$

The two Fractions reduced to the same Denomination 36. where $\frac{15}{36} = \frac{5}{12}$ and $\frac{14}{36} = \frac{7}{18}$.

$$\begin{array}{l} \qquad\quad 21\ \ 10 \\ \qquad\quad \cdots\cdots \\ \text{So likewise } \tfrac{3}{5}\ \ \tfrac{2}{7}\ \ \tfrac{21}{35}\ \ \tfrac{10}{35} \\ \qquad\quad \cdots\cdots \\ \qquad\qquad 35 \end{array}$$

The two Fractions reduced to the same Denomination 35. where $\frac{21}{35} = \frac{5}{12}$ and $\frac{10}{35} = \frac{2}{7}$.

10. If there be many *Fractions* of *unlike Denominations*, whether they be Compound or Prime, they are to be reduced to one Denomination thus, *viz.* Multiply all the Denominators together for the common Denominator, and multiply every Numerator in every Denominator (except his own) so the several Products shall be for the several new Numerators for every Numerator.

Thus $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$ and $\frac{4}{5}$ reduc'd thus $\dfrac{\overline{60.\ 80.\ 90.\ 96,}}{\underline{\tfrac{1}{2}\ \ \tfrac{2}{3}\ \ \tfrac{3}{4}\ \ \tfrac{4}{5}}}$ The
$\qquad\qquad 120$

common Denominator being 120. for $\frac{60}{120} = \frac{1}{2}$. $\frac{80}{120} = \frac{2}{3}$ $\frac{90}{120} = \frac{3}{4}$ and $\frac{96}{120} = \frac{4}{5}$.

11. Yet

11. Yet if any of the *Denominators* be *compounds together*, sometimes the Terms may be lessen'd thus, multiply the Denominator with the greater Compound, only rejecting the lesser for the common Denominator, then dividing the common Denominator by every particular Denominator, multiply that Quotient by his own Numerator, and place that Product for the new Numerator.

	30	40	45	48
Thus	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
		60		

Because 2 and 4 in the Denominators are compounded, I reject the less 2, and multiply $3\times4\times5=60$, for a new Denominator; then I divide 60 by $2=30$, which I multiply by 1, and set 30 over 1, so dividing 60 by 3, the Quotient is 20, which multiply by $2=40$, for the Numerator over 2, &c. And now having these Fractions to a like Sirname or Denomination with the former, they may be easily applied to Addition or Substraction, as shall after appear.

12. If *Fractions of Fractions* (or *Particulars*) be proposed, they are likewise to be reduced to a new Denominator thus, *viz.* Multiply the Numerators each into other for new Numerators, and likewise Denominators into Denominators for new Denominators.

$$\text{Thus } \frac{3}{4} \text{ of } \frac{2}{3} \text{ of } \frac{6}{7} \text{ of } \frac{7}{9} \text{ that is } \frac{252}{756}$$

(252 over 756)

And

And Fractions of Fractions, when they are reduced to one Denominator, they are appliable to Addition or Substraction with any other Fraction, whether of like Base or unlike, for then they may be reduced according to Rule 9. of this Sect.

13. If you will know what the Fraction of any whole thing signifieth in common Parts, you must divide the whole thing in common Parts by the Denominators, and the Quotient must be multiplied by the Numerator.

As for Example.

How much is the $\frac{5}{12}$ of a Crown? Here the Crown is in common Parts 60 Pence; but the Fraction requireth of them $\frac{5}{12}$, that is, five twelfth Parts: Divide 60 by 12 the Quotient is 5, and 5 multiplied by the Numerator 5 is 25, which sheweth that $\frac{5}{12}$ of a Crown or 60 Pence is 25 Pence, or two Shillings one Penny.

Sect. 3. *Of Addition and Substraction of Fractions.*

1. First the Numbers proposed to be added or substracted, are either Fractions with Fractions, Fractions with whole Numbers or mixt, or mixt with whole Numbers or mixt; and in all these observing the Rules in the last Section, you are to reduce the proposed Quantities to one Denomination, and after that according to the Sign of Addition + or Substraction — to add together or substract the Numerators.

Examples

Examples of Fractions, with Fractions of the same Denomination.

$$\frac{2}{5}+\frac{1}{5}=\frac{3}{5} \text{ or } \frac{1}{5}.\quad \frac{2}{7}+\frac{4}{7}+\frac{6}{7}=\frac{12}{7} \text{ or } 1\frac{5}{7}.\quad \frac{54}{60}+\frac{13}{60}=\frac{67}{60}.$$

Examples of Fractions with Fractions of unlike Denomination, reduced by Rule 9. of the last Sect.

$$\begin{array}{c} 6 \overset{+}{-} 5 \\ \cdots\cdots \\ 3)\ \frac{2}{3}+\frac{5}{9}=\frac{11}{9} \text{ or } 1\frac{2}{9}. \\ \cdots\cdots \\ \underbrace{3\quad 1.}_{9} \end{array} \qquad \begin{array}{c} 10\ \ 12 \\ \cdots\cdots \\ 1)\ \frac{2}{3}+\frac{4}{5}=\frac{22}{15} \text{ or } 1\frac{7}{15}. \\ \cdots\cdots \\ \underbrace{5\quad 3}_{15} \end{array}$$

$$\begin{array}{c} 15 \cdot 14 \\ \cdots\cdots \\ \text{So } 6)\ \frac{5}{12} \overset{+}{-} \frac{7}{18}=\frac{29}{36} \text{ or } \frac{1}{36}. \\ \cdots\cdots \\ \underbrace{3\quad 2}_{36} \end{array}$$

Examples

Examples of Fractions with whole Numbers.

$5+\frac{2}{3}$ that is $4\frac{3}{3}+\frac{2}{3}=4\frac{5}{3}$ or $5\frac{1}{3}$. where you ſee I take 1 from the 5 and put in an improper Fraction, that is $\frac{3}{3}=1$.

$8\pm\frac{17}{19}$ that is $7\frac{19}{19}\pm\frac{17}{19}=7\frac{36}{19}=8\frac{17}{19}$ or $7\frac{2}{19}$.

Examples of mixt Numbers with Fractions.

Of like Baſes, $2\frac{3}{5}\pm\frac{2}{5}=2\frac{5}{5}$ or $2\frac{1}{5}$. but if the latter Fraction be greater than the Fraction of the mixt Number, you muſt ſubtract 1 from the Integer, and turn it into an improper Fraction with the firſt thus,

Of like Baſe, $2\frac{3}{5}\pm\frac{4}{5}$ added is $2\frac{7}{5}$ or $3\frac{2}{5}$ but ſub. you muſt ſay $1\frac{8}{5}-\frac{4}{5}=1\frac{4}{5}$,

$$\begin{array}{c} 8 \quad 15 \\ \cdots\cdots \\ 3\frac{2}{5}\pm\frac{3}{4} \\ \cdots\cdots \\ \underbrace{4 \quad 5}_{20} \end{array}$$

Of unlike Baſes $3\frac{2}{5}\pm\frac{3}{4}$ Reduced is $3\frac{8}{20}\pm\frac{15}{20}$ added is $3\frac{23}{20}$ or $4\frac{3}{20}$ but ſubtracted thus $2\frac{28}{20}-\frac{15}{20}=2\frac{13}{20}$.

Examp.

Examp. Of whole Numbers and mixt.

$7 \pm 4\frac{2}{3}$ added would be $11\frac{2}{3}$ but subftracted thus $6\frac{3}{3} - 4\frac{2}{3} = 2\frac{1}{3}$.

$3 \pm 2\frac{1}{3}$ added would be $5\frac{1}{3}$ but fubftracted thus, $2\frac{3}{3} - 2\frac{1}{3} = \frac{2}{3}$.

Examp. Of mixt Numbers with mixt.

Of like Bafes $5\frac{4}{9} \pm 4\frac{3}{9}$ = by Add. $9\frac{7}{9}$ Sub. $1\frac{1}{9}$,
$7\frac{3}{8} \pm 3\frac{5}{8}$ add. is $10\frac{8}{8}$ or 11. but fub.
$6\frac{11}{8} - 3\frac{5}{8} = 3\frac{6}{8}$.

8 . . . 9

. . . .

Of unlike Drs. $4\frac{2}{3} \pm 1\frac{3}{4}$ added is $6\frac{17}{12}$ or $7\frac{5}{12}$ or fub.

. . . . $3\frac{20}{12} - 2\frac{9}{12} = 1\frac{11}{12}$.

4 3

12

6 5

. . . .

Again $4\frac{3}{5} \pm 3\frac{1}{2}$ added is $7\frac{11}{10}$ or $8\frac{1}{10}$ or fub. $1\frac{1}{10}$.

. . . . . .

2 5

10

So

So to add $6\frac{3}{4}$ to $10\frac{1}{2}$ and to $14\frac{5}{6}$ by the 11 Rule of the last I reduce the Fractions, then it will be $6\frac{18}{24} + 10\frac{12}{24} + 14\frac{20}{24} = 30\frac{50}{24} = 32\frac{2}{24}$.

$$\begin{array}{ccc} 18. & 12. & 20. \\ \ldots & \ldots & \ldots \\ \frac{3}{4} & \frac{1}{2} & \frac{5}{6} \\ \hline & 24 & \end{array}$$

So to sub. $4\frac{3}{4}$ from $6\frac{2}{3}$ thus,

$$\begin{array}{ccc} 8 & \ldots & 9 \\ 6\frac{2}{3} & \rightarrow & 4\frac{3}{4} \\ \ldots & \ldots & \ldots \\ 4 & & 3 \\ \ldots & \ldots & \ldots \\ & 12 & \end{array}$$

Then $5\frac{20}{12} - 4\frac{9}{12} = 1\frac{11}{12}$.

Sect. 4. *Of Multiplication of Fractions.*

1. In *Multiplication* and *Division* of Fractions; make mixt Numbers improper Fractions, and make whole Numbers like Fractions by subscribing an Unite.

2. If the Byas Terms be compounded, bring them to the least Terms, and the Work will be in the least Terms. And

Therefore if the Byas Terms be equal, the other Terms stand for the Multiplication.

And therefore if the Terms of any two Fractions be cross-wise equal, the Product is always an Unite.

The Rule for Multiplication of Fractions is, multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator.

1. *Example.*

1. Examp. *How to multiply a mixt Number.*

1. *By a mixt Number.*
2. *By a whole Number.*
3. *By a Fraction.*

(1) $2\frac{3}{4} \times 3\frac{4}{5}$ that is $\frac{11}{4} \times \frac{19}{5} = \frac{209}{20}$.

(2) $2\frac{3}{5} \times 7$. that is $\frac{13}{5} \times \frac{7}{1} = \frac{91}{5}$.

(3) $2\frac{3}{4} \times \frac{2}{5}$ that is $\frac{11}{\cancelto{2}{4}} \times \frac{\cancelto{1}{2}}{5} = \frac{11}{10}$

2. Examp. *To multiply a whole Number.*

1. *By a mixt Number.*
2. *By a Fraction.*

(1) $6 \times 3\frac{2}{3}$ that is $\frac{\cancelto{2}{6}}{1} \times \frac{11}{\cancelto{1}{3}} = \frac{22}{1} = 22$.

(2) $7 \times \frac{3}{4}$ that is $\frac{7}{1} \times \frac{3}{4} = \frac{21}{4}$.

3. *Examp.*

3. *Examp. How to multiply a Fraction.*

1. *By a mixt Number.*
2. *By a whole Number.*
3. *By a Fraction.*

$$(1)\ \frac{2}{3} \times 2\frac{4}{5} \text{ that is } \frac{2}{3} \times \frac{14}{5} = \frac{28}{15}.$$

$$(2)\ \frac{3}{4} \times 6 \text{ thus } \frac{3}{\cancel{4}_{\,2}} \times \frac{\cancel{6}^{\,3}}{1} = \frac{9}{2} \text{ or } 4\frac{1}{2}.$$

$$(3)\ \frac{2}{7} \times \frac{3}{5} = \frac{6}{35}. \text{ and } \frac{\cancel{2}^{\,1}}{5} \times \frac{3}{\cancel{4}_{\,2}} = \frac{3}{10}.$$

More Examples.

$$(1)\ \frac{3}{5} \times \frac{6}{9} = \frac{18}{45} \text{ by the second Rule.}$$

$$(2)\ \frac{9}{12} \times \frac{12}{9} = 1 \text{ by the second Rule.}$$

Sect. 5. *Of Division of Fractions.*

1. In Division, if the Numerators or Denominators be compounded, reduce them into the least Terms, and the Work will be in the least Terms, And

Therefore if the Numerators be equal, cancel them, and set the Denominator of the Divisor over the other, in form of a Fraction.

And therefore if the Denominators be equal, reject them, and the Work is in the Numerators.

2. The Rule for *Division*, multiply *alternately* the *Numerator* of the second by the *Denominator* of the first for a new Numerator, and the other Numerator and Denominator for a new Denominator.

Examp. How to divide a mixt Number

1. *By a mixt Number.*
2. *By a whole Number.*
3. *By a Fraction.*

(1) $3\frac{4}{5})\,2\frac{3}{4}\,(\frac{55}{76}$ or thus $\frac{19}{5})\,\frac{11}{4}\,(\frac{55}{76}$.

(2) $7)\,\frac{13}{2}\,(\frac{13}{14}$ or thus $\frac{7}{1})\,\frac{13}{2}\,(\frac{13}{14}$.

(3) $\frac{2}{5})\,2\frac{3}{4}\,(\frac{55}{8}$ or thus $\frac{2}{5})\,\frac{11}{4}\,(\frac{55}{8}$

How

How to divide a whole Number.

1. *By a mixt Number.*
2. *By a Fraction.*

(1) $3\frac{2}{3})\ \frac{6}{1}\ (\frac{18}{11}$ or thus $\frac{11}{3})\ \frac{6}{1}\ (\frac{18}{11}$.

(2) $\frac{3}{4})\ 7\ (\frac{28}{3}$ or thus $\frac{3}{4})\ \frac{7}{1}\ (\frac{28}{3}$.

How to divide a Fraction.

1. *By a mixt Number.*
2. *By a whole Number.*
3. *By a Fraction.*

(1) $2\frac{4}{5})\ \frac{2}{3}\ (\frac{10}{42}$ thus $\frac{14}{5})\ \frac{2}{3}\ (\frac{10}{42}$.

(2) $6)\ \frac{3}{4}\ (\frac{3}{24}$ thus $\frac{6}{1})\ \frac{3}{4}\ (\frac{3}{24}$.

(3) $\frac{3}{4})\ \frac{2}{5}\ (\frac{8}{15}$.

More Examples.

$$\overset{5}{\underset{4}{\frac{15)}{16)}}}\ \overset{1}{\underset{1}{\frac{4}{3}}}\ \frac{(64}{(45}\quad \frac{3)}{16)}\ \frac{15}{16}\ \frac{(15}{(3}$$

$$7\,\frac{2)}{3)}\ \frac{8}{15}\ \frac{(8}{(115}\ \text{thus}\ \frac{23)}{\underset{1}{3)}}\ \frac{8}{\underset{5}{15}}\ \frac{(8}{(115}$$

3. How to know the greater Fraction of any two propounded, when a Fraction is divided by another, whether it be a true or improper Fraction, the Quotient doth always expreſs the Proportion betwixt the Dividend and Diviſor.

1. If therefore both the Fractions be equal, both Terms of the Quotient will be equal.

2. If the Dividend be the greater Fraction, the Numerator of the Quotient muſt be greater.

1. Thus $\frac{2}{3} = \frac{6}{9}$ Becauſe in Diviſion the Terms of the Quotient are equal. $\frac{2}{3}) \frac{6}{9} (\frac{18}{18}$

2. But $\frac{3}{4}$ is greater than $\frac{3}{12}$ Becauſe in Diviſion the Numerator is greater. $\frac{3}{12}\ \frac{3}{4} (\frac{36}{12}$

3. But $\frac{2}{3}$ is leſſer than $\frac{3}{4}$ Becauſe in Diviſion the Denominator is greater. $\frac{3}{4}) \frac{2}{3} (\frac{8}{9}$

Thus have you a plain Way delivered for Fractions: Now come we to the Practice of them in the Golden Rule.

Sect. 5. *The Golden Rule in Fractions.*

1. If any of the Terms be mixt Numbers, reduce them into improper Fractions.

2. If any of the Terms be whole Numbers, you may make them improper Fractions by ſubſcribing an Unite.

3. If the ſame Figures be in the Dividend and Diviſor, reject or cancel them, and work Diviſion with the reſt.

4. The Rule direct is wrought thus, *viz.* Multiply the Denominator of the firſt Term into the Numerators

merators of the ſecond and third Terms for your Dividend, and multiply the Numerator of the firſt in the Denominators of the ſecond and third Terms for the Diviſor; work out your Diviſion, the Quotient gives the Anſwer.

Example.

If $\frac{3}{4}$ Yard coſt 8 *s.* what ſhall $2\frac{1}{2}$ Yards coſt? Anſ. $1\frac{1}{4}$ of a Pound.

$$\frac{3}{4}\ \frac{2}{5} :: \frac{5}{2}\ 1\frac{1}{4}. \quad \begin{array}{l} 4\times2\times5=40\ (1\frac{1}{4}. \\ 3\times5\times2=30) \end{array}$$

5 If the ſecond Term only be a Fraction, make the firſt Term of like Denomination, reject the Denominators and work as in whole Numbers.

If 2 Yards give $\frac{3}{4}$ *l.* what ſhall 7 Yards coſt? Anſwer, $2\frac{5}{8}$.

$$\frac{2}{1}\ \frac{3}{4} :: \frac{7}{1}.\ 1\frac{5}{8}. \quad \text{For } \begin{array}{l} 1\times3\times7=21\ (2\frac{5}{8}. \\ 2\times4\times1=8 \end{array}$$

Or by the Rule thus $\frac{8}{4}\ \frac{3}{4} :: 7.\ 2\frac{5}{8}$ by making 2 of the ſame Denomination.

6. If either of the Homogeneal Terms be a Fraction, and the other not, reduce it to like denomination, with his like Term, cancel the Denominators and work as in whole Numbers.

If $\frac{3}{4}$ Yard. 2 s. : : (7) or reduced to like denomination with $\frac{3}{4}$) $\frac{8}{4}$ 18 $\frac{2}{3}$.

If 6 Yards or $\frac{24}{4}$ 12 : : $\frac{3}{4}$. $\frac{1}{2}$.

For 24) $\frac{36}{24}$ ($1\frac{12}{24} = 1\frac{1}{2}$.
12

Sect. 5. *The backward Rule, or Rule Inverse in Fractions.*

1. Multiply the Numerators of the first and second Terms in the Denominator of the third for the Dividend, and the Numerator of the third in the Bases of the first and second Terms for your Divisor; finish Division hereby, the Quotient gives the Answer.

Example.

If 3 Men do a Work in $6\frac{1}{2}$ of Hours, in how many Hours shall 12 Men do it? Answer, $1\frac{5}{8}$ Hours.

$\frac{3}{1}$. ($6\frac{1}{2}$) $\frac{13}{2}$: : $\frac{12}{1}$. $1\frac{5}{8}$. $3 \times 13 \times 1 = 39$ ($1\frac{15}{24}$ or $1\frac{5}{8}$.
$12 \times 1 \times 2 = 24$)

If $6\frac{1}{4}$. (*i.e.*) $\frac{25}{4}$ *oz.* $6\frac{1}{2}$. (*i.e.*) $\frac{13}{2}$: : $2\frac{1}{3}$. (*i.e.*) $\frac{7}{3}$. $17\frac{23}{56}$

$25 \times 13 \times 3 =$ 975 ($17\frac{23}{56}$.
$7 \times 4 \times 2 = 56$)

If

If a Two-penny Loaf of Bread weighed 6 *l.* 3 *oz.* when a Bole of Wheat coſt 6 *s.* 6 *d.* what is a Bole worth when a Two-penny Loaf of Bread weighed but 2 *l.* 4 *oz.*? Anſwer, $17\frac{23}{53}$ Shillings

$$\frac{25}{4}.\ \frac{13}{2} :: \frac{7}{3}.\ 17\frac{23}{56}. \qquad 25\times13\times3 = 975\ (17\frac{23}{56}.$$
$$7\times4\times2=56)$$

Sect. 6. *To work the double Golden Rule in Fractions.*

1. The Work of the Compound Rule direct, if any of the Terms be Fractions, or Fraction-like expreſſed, is thus; Multiply the 2 firſt Denominators in the 3 laſt Numerators for Dividend, and multiply the 2 firſt Numerators in the 3 laſt Denominators for Diviſor, and after that finiſh your Diviſion.

Example.

If 33 *l.* 6 *s.* 8 *d.* in $\frac{3}{4}$ of a Year bring $2\frac{1}{2}$ *l.* Gain, what ſhall 400 *l.* Stock bring in $\frac{1}{4}$ of a Year? Anſwer, 10 *l.*

$$33\frac{1}{3} \text{ or } \frac{100}{3}.\frac{3}{4}.\frac{2}{5} \qquad 3\times4\times5\times400\times1=24000\ (10$$
$$\frac{400}{1}.\frac{1}{4}. \qquad 100\times3\times2\times1\times4=2400)$$

2. The Work of the compound Rule converſe in Fractions is thus; Multiply the Numerators of the firſt, ſecond and fifth in the Denominators of the third and fourth Terms for your Dividend, and multiply the Denominators of the firſt, ſecond

and fifth Terms in the Numerators of the third and fourth for your Divisor, finish Division, *&c.*

Example.

If 400 *l.* Stock in $\frac{1}{4}$ of a Year yeild 10 *l.* Interest, in what time will 33 *l.* 6 *s.* 8 *d.* gain or yield 2 *l.* 10 *s.?* Answer, $\frac{3}{4}$.

$$\frac{400}{1}\cdot\frac{1}{4}\cdot\frac{10}{1} \qquad 400\times5\times3 = \frac{6|000}{8|000} = \frac{3}{4}.$$
$$\frac{100}{3}\cdot\ \cdot\frac{5}{2} \qquad 4\times2\times10\times100 = 8|000$$

3. The Work of the compound Rule descending, when any of the Terms are Fractions, is thus; Multiply the Numerators of the first, third and fifth in the Bases of the second and fourth for your Dividend, and multiply the Bases of the first, third and fifth in the Numerators of the second and fourth Terms for your Divisor, finish Division, *&c.*

Example.

If $\frac{3}{4}$ Duccats countervail $\frac{2}{3}$ Rose Nobles, and $\frac{4}{5}$ Rose Nobles countervail 2 Crowns, how many Duccats are equal to 20 Crowns? Answer, 9.

Duc. *R.N.*

R. N. $\frac{3}{4}$. $\cdot\frac{2}{3}$ $\qquad 3\times4\times20\times3=720\ (9.$

$\frac{4}{5}$. $\cdot$ $\cdot\frac{2}{1}$ *Cr.* $\qquad 4\times5\times1\times2\times2=80)$

Cr. 20. · *Duc.*

4. The Work of the compound Rule ascending when any of the Terms are Fractions is thus; Multiply the Numerators of the second, fourth and fifth,

fifth, with the Denominators of the first and third for the Dividend, and multiply the Denominators of the second, fourth and fifth with the Numerators of the first and third for the Divisor, finish Division, *&c.*

Example.

If $\frac{3}{4}$ Duccats countervail $\frac{2}{3}$ Rose Nobles, and $\frac{4}{5}$ Rose Nobles countervail 2 Crowns, how many Crowns will countervail 9 Duccats? Ans. 20.

$\frac{3}{4}$ *Duc.* . $\frac{2}{3}$ *R. N.* $2\times2\times9\times4\times5=720\ (20$

$3\times3\times4=36)\ 72.$

$\frac{4}{5}$ *R. N.* . $\frac{2}{1}$ *Cr.*

$\frac{9}{1}$ *Duc.* . 20 *Cr.*

Sect. 7. *The Rule of Fellowship in Fractions.*

Example.

Four Men, *viz.* *A*, *B*, *C*, *D*, take a Prize worth 8190 *l.* whereof *A* should have $\frac{1}{3}+\frac{1}{10}$ of the Value, *B* $\frac{1}{4}+\frac{1}{10}$, *C* $\frac{1}{6}$, and *D* $\frac{1}{20}$, what must each have for his Share? First, *A* and *B* share severally, *A*'s Share will be $\frac{13}{30}$, *B*'s Share $\frac{7}{20}$. Then reduce $\frac{13}{30}+\frac{7}{20}+\frac{1}{6}+\frac{1}{20}$ to like Base, so they will be $\frac{26}{60}+\frac{21}{60}+\frac{10}{60}+\frac{3}{60}=\frac{60}{60}$. Then work for *A*, 60. 8190 :: 26. 3549. *B*, 60. 8190 :: 21. 2866$\frac{1}{2}$. *C*, 60. 8190 :: 10. 1365. *D*, 60. 8190 :: 3. 409$\frac{1}{2}$. Therefore *A* must have 3549 *l.* *B* 2866 *l.* 10 *s.* *C* 1365 *l.* *D* 409 *l.* 10 *s.* = 8190.

Some-

Sometimes in this Rule, as was before noted, divers Questions impossible to be answered may be proposed;

As in this Example.

Four Men, *viz.* *A*, *B*, *C*, *D*, agree to divide 600 Crowns amongst them, so that *A* shall have $\frac{2}{3}$ thereof more by 9 Crowns, *B* shall have $\frac{3}{5}$ more by 8 Crowns, *C* shall have $\frac{5}{6}$ + 7 Crowns, and *D* shall have $\frac{7}{8}$ more by 6 Crowns, what shall each have?

For Answer, because $\frac{2}{3}+\frac{3}{5}+\frac{5}{6}+\frac{7}{8}$ make by Reduction $\frac{357}{120}$, and $9+8+7+6 = 30$ maketh $\frac{6}{120}$, for 600 represented by 120, being divided by 30 quoteth 20, and 120 being divided by 20 quoteth 6, that is $\frac{6}{120}$.

Now I say because $\frac{357}{120}+\frac{6}{120}$ maketh $\frac{363}{120}$, which amounteth to 3 Integers and $\frac{1}{40}$ more, all which ought only to be but $\frac{120}{120}$, or one Value, therefore there cannot be a Division made according to the Position of the Question: But when the *Overplus*, that is, 30 Crowns, are deducted out of the Total 600 Crowns, the Division must be made according to the Proportion, which the Numerators of the Fractions reduced beareth each to other.

$$\frac{\dfrac{80+72+100+105}{\frac{2}{3}+\frac{3}{5}+\frac{5}{6}+\frac{7}{8}}}{120} = 357.$$

Thus as oft as *A* shall have 16 Crowns, *B* 135, and *C* 200, and *D* 210, by making the Total of their Shares 705 the general Antecedent, 570 that

is

is 600 — 30 the general Consequent, and every Partners Part the particular Antecedent in the Golden Rule thus.

A, 357. 570 :: 80. $127\frac{261}{357}$.
B, 357. 570 :: 72. $114\frac{342}{357}$.
C, 357. 570 :: 100. $159\frac{237}{357}$.
D, 357. 570 :: 105. $167\frac{231}{357}$.

And now $\left\{\begin{array}{l} A\ 127\frac{261}{357}+9=138\frac{261}{357}. \\ B\ 114\frac{342}{357}+8=122\frac{342}{357}. \\ C\ 159\frac{237}{357}+7=166\frac{237}{357}. \\ D\ 167\frac{231}{357}+6=173\frac{231}{357}. \end{array}\right.$

All which Sums are equal to 600 which is the Proof of the Question.

Sect. 8. *The Consideration of the* Ratio *of two Numbers in Quantity, and of the Addition and Substraction thereof.*

1. The simple Reason betwixt two Numbers is found by dividing the Antecedent by the Consequent, and therefore they are expressed in Form of a Fraction, as 2 to 2 set thus $\frac{2}{2}$, is the Reason of Equality, because the first Antecedent, or Numerator, being divided by the latter Consquent or Denominator, is contained equally once in the same; so 2 and 1 or $\frac{2}{1}$ is of inequality and double, because 2 divided by 1 is contained twice therein.

2. The Addition of these *Ratios* is as though they were Fractions and to be multiplied, as if you were

were to add $\frac{1}{2}$ to $\frac{3}{4}$, that is, if the Proportion of 1 to 2 be to be added to the Proportion of 3 to 4, multiply them as though they were Fractions, and the Products ſhews the Proportion augmented to be $\frac{3}{6}$, or as 3 is to 6, $\frac{1\,3}{2\,6} = \frac{3}{6}$, ſo $\frac{3}{4}$ to $\frac{8}{5}$ added is $\frac{24}{20}$, ſo $\frac{4}{3}\,\frac{9}{8} = \frac{36}{24}$, ſo $\frac{8}{3}$ added to $\frac{1}{2}$ makes $\frac{24}{6} = \frac{4}{1}$, *Ex. Euclid. Lib.* 8. *Prop.* 5.

3. The Subſtraction of one *Ratio* from another is done by dividing the *Ratios* as though they were Fractions: for if you divide the *Ratio* from which the Subſtraction is to be made by the *Ratio* to be ſubſtracted, you have done that which is deſired: As if the *Ratio ſeſquialtera* $\frac{3}{2}$ be to be ſubſtracted from the double *Ratio* $\frac{2}{1}$, the Remainder will be $\frac{4}{3}$ *ſeſquitertia.* $\frac{3}{2}$) $\frac{2}{1}$ ($\frac{4}{3}$.

The Proof of Subſtraction in this kind is thus, If the *Ratio* found be added to the *Ratio* ſubducted, do make up the *Ratio* from which the Subduction was made, as in the Example where $\frac{3}{2}$ was ſubſtracted from $\frac{2}{1}$, the Remainder was $\frac{4}{3}$, which $\frac{4}{3}$ added to $\frac{2}{1}$ makes $\frac{3}{2}$, and ſo likewiſe you may examine Addition by Subſtraction.

1. *Examp. Of Addition of* Ratios.

The Proportion of a Penny to a Farthing is as 4 to 1, the Proportion of a Shilling to a Penny is as 12 to 1, what are theſe Proportions added? Anſwer, $\frac{4}{1}\frac{12}{1} = \frac{48}{1}$, which is the Proportion of 1 Shilling to 1 Farthing.

2. *Example*

2. *Example.*

An Horſe bears a Weight, but he may eaſily bear 3 times as much, and another Horſe bears a Weight, but he may well bear 3 quarters as much as he doth; the Queſtion how much theſe two might draw or bear betwixt them? Add the *Ratios* $\frac{3}{2}$ and $\frac{3}{4}$ makes $\frac{9}{4}$ or $2\frac{1}{4}$, and therefore they might well draw betwixt them twice as much and half as much as they did.

3. *Example.*

A Fountain runs at 3 Pipes, if the firſt Pipe run, it would fill the Conduit in 2 Hours, if the ſecond run, it would do the ſame in 5 Hours, if the third, in 12 Hours: It is deſired to know, that if the ſame run all at once, in how many Hours will it fill the Conduit? For Anſwer, the Proportion of the firſt is $\frac{1}{2}$, the ſecond $\frac{1}{5}$, the third is $\frac{1}{12}$, the Fractions $\frac{1}{2}+\frac{1}{5}+\frac{1}{12}$ reduced make $\frac{30+12+5}{60}$.

Therefore I conclude that 47 Conduits will be filled in 60 Hours, and therefore one Conduit would be filled if they all run, in one Hour and 13 Minutes.

Examples

Examples of Substraction of Ratios.

1. *Example.*

There are two Workmen, one doth the Work in 30 Days, but being joyned in Work with the other he performs the Work in 12 Days: It is desired to know the Ability of the other Workman, and in how many Days he could perform the same Work? For Answer, I substract the *Ratio* of $\frac{1}{30}$ out of $\frac{1}{12}$ reduced is $\frac{4\ 10}{120} = \frac{6}{120}$ or $\frac{1}{20}$. Therefore I conclude the latter Man could have done it in 20 Days by himself.

2. *Example.*

A Fountain hath two Pipes, that which fills the Conduit is the greater, that which empties is less; the greater will fill the Conduit in 8 Hours, the less will empty it in 22 Hours: It is desired, that seeing the Proportion of the filling the Conduit is greater than the Evacuation, both the Pipes running, in what time the Conduit will be filled? For answer I subtract the Proportion $\frac{1}{8}$ out of $\frac{1}{22}$, resteth $\frac{7}{88}$, and therefore the Conduit will be filled in $12\frac{4}{7}$ Hours.

$$(2 \quad \underset{11}{\underline{\tfrac{1}{8}}} \quad \underset{4}{\underline{\tfrac{1}{22}}}| \qquad \frac{4}{88} - \frac{13}{88} = \frac{7}{88}$$

These Precepts do extraordinarily conduce both to the Mixture of Medicines before handled, and

and to the ſpeculative Part of Muſick, for the *Diapente* being *ſeſquialter*, *Diateſſeron ſeſquitertia*, *Diapaſon dupla*, *Diapaſon* with the *Diapente* triple, and the Tone *ſeſquioctava:* From hence the Muſicians have found out, that the *Diapaſon* is made of *Diateſſeron* added to the *Diapente*, and that the *Diapente* is made of the *Diateſſeron*, added to the Tone, *&c.* as alſo to the Powers of all ſorts of Machinations or forcible Inſtruments.

Sect. 9. *Of the Multiplication and Diviſion of* Ratios.

1. Place the *Ratio* propoſed to be multiplied ſo many times over as there are Unites in the Multiplier, and multiply the Denominators for a new Denominator, and the Numerators for a new Numerator, if you were to multiply the *Ratio* of $\frac{2}{3}$ by 3, it is $\frac{8}{27}$.

Thus $2\times2\times2=\frac{8}{27}$. ſo $\frac{4}{2}$ by 2 is $\frac{16}{4}=4$
$3\times3\times3=$
Thus $\frac{4\times4}{2\times2}=\frac{16}{4}$.

So if you would double $2\frac{1}{4}$, it is $\frac{81}{16}$. $\frac{9}{4}\times\frac{9}{4}$—$\frac{81}{16}$.

2. Hence it appeareth that to double any *Ratio* is but to ſquare it, and to triple it, to cube it, *&c.*

3. And on the contrary to divide any *Ratio* by 2, is but to take the ſquare Root thereof, and to divide it by 3 is to take the Cube Root, *&c.*

As

As for Example.

To take the half of $2\frac{1}{4}$ or the Ratio $\frac{9}{4}$ or divide it by 2 is to take the square Root thereof, $\frac{3}{2}$ that is $1\frac{1}{2}$.

These have great Use in many hard Examples, for these Proportions do not only find out the Demonstration of the Rules of Proportion, but do explain many peculiar and hard Problems.

As in this out of Ptolomy Almag.

The Diameter of the Sun to the Diameter of the Earth hath the Proportion as 11 to 2 or $\frac{11}{2}$.

The Diameter of the Sun to the Diameter of the Moon hath the Proportion as 94 to 5 or $\frac{94}{5}$.

The Diameter of the Earth to the Diameter of the Moon hath the Proportion as 17 to 5, or $\frac{17}{5}$, from these Proportions the greatest and remotest Bodies fall under Measure.

For because Spheres or Globes have triple Proportion to their Diameters, *per ult. prop. Eu.* 12. Therefore triple $\frac{11}{2}$ the Proportion which the Sun hath to the Earth, and you have the Proportion that the Body of the Earth beareth to the Body of the Sun, the which by the Work will be $\frac{1331}{8}$, or $166\frac{3}{8}$, that is, the Sun's Body is bigger than the Body of the Earth 166 times and $\frac{3}{8}$.

$$\frac{11}{2}\times\frac{11}{2}\times\frac{11}{2}=\frac{1331}{8}.\ \text{and } 8)\,1331\,(166\tfrac{3}{8}.$$

After the same manner you will find the Sun to be greater than the Moon $6644\frac{84}{125}$, and lastly the Earth

Earth to be bigger than the Moon 39 times and almoſt $\frac{1}{3}$.

Sect. 10. *Of the ſeveral* Species *of* Ratios.

1. The Proportion of two Numbers conſiſts either in Equality or Inequality; Equality when both Numbers are equal, as $\frac{5}{5}$.

2. Inequality is either greater than Equality, as $\frac{7}{5}$, or leſs as $\frac{3}{5}$.

3. The Proportion of Inequality is either Prime or Conjunct.

4. Prime, when the Quotient is always Unity or a Fraction, and this is either manifold, ſuperparticular or ſuperpertient.

5. Proportion conjunct is when the Quotient exceedeth an Unite as $\frac{5}{2}$, and this is either manifold ſuperparticular, or manifold ſuperpertient; but theſe are ſufficiently explained in moſt Arithmetick Books, and being of no ſpecial Uſe I therefore I leave them and paſs now to *the Rule of Poſition.*

CHAP. XVII.

The Rule of Position.

1. THIS is called the *Rule of Position*, because any Number is taken for to work the Question by, and placed instead of the Number sought, after the Position; wherefore you must examine the Question according to the Tenour thereof, and if your Position fall true, then you have satisfied the Question; if false, (from whence it is called the Rule of *false Position*) then the true Number is found out by this Rule, whereof there are two kinds: 1. *The single Rule.* 2. *The double Rule.*

2. *The single Rule* is to be used when there is some partition of Numbers in parts proportional; and for finding out the truth, place the Number found by the argumentation first in the golden Rule; in the second place put down the *Hypothetical* or Number supposed, and in the third place the given Number; the Numbers being thus disposed, the fourth Proportional shall be the Number sought.

1. *Example.*

Three men, *viz.* *A*, *B*, *C*, buy a Mannor costing 2700 *l.* and *B* is to pay double what *A* must pay, and *C* triple to that which *B* payeth: how much ought every of them to pay? For Answer, I suppose *A* paid 6 *l.* therefore *B* paid 12 *l.* and *C* paid

paid 36 *l.* but 6+12+36 maketh 54 *l.* which by the intent of the Queſtion ought to have been 2700 *l.* wherefore by this Poſition I want of the truth; yet according to my Rule, I ſay, if 54 *l.* come of 6 *l.* :: 2700 *l.* 300. Therefore I conclude, *A* paid 300 *l.* *B* 600 and *C.* 1800: For

$$300+600+1800=2700\ l.\ \textit{The Total.}$$

2. *Example.*

One bought 30 Yards of Taffaty, and 40 Yards of Satten, which coſt him 330 *s.* but every Yard of Satten coſt double ſo much as a Yard of Taffaty, what did the Yard of Taffaty coſt? For Anſwer, I ſuppoſe a Yard of Taffaty coſt 4 *s.* therefore a Yard of Satten coſt 8 *s.* therefore 30 Yards of Taffaty coſt 120 *s.* and 40 Yards of Satten coſt 320 *s.* but $120+320=440$ *s.* which ſhould be but 330 *s.* I therefore ſay, if 440. 4 :: 330. 3. The Anſwer is 3 *s.* and ſo much a Yard of Taffaty coſt, and a Yard of Satten 6. For Proof, 30 Yards of Taffaty at 3 *s.* coſt 90 *s.* and 40 Yards of Satten at 6 *s.* coſt 240. and $90+240=330$.

If the Queſtion have a Fraction in it, it is beſt for moſt Facility in proceeding to chooſe ſuch a Number for the Poſition, as hath the parts expreſt in the Queſtion, as in the Example following.

3. *Example.*

One ſaid, he knew not what money he had in his Purſe, but he knew that the third part, and the

fourth part, and the fifth part thereof would make just 94 *l.* what money had he? For Answer, I chose 60 for my Position, which hath the parts exprest, (and such choice is easily made by multiplying the Denominators of the given Fractions together, as in this Example, the given Fractions are $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, whose Denominators are 3, 4, and 5, which being multiplyed together continually, the product is 60, for $3 \times 4 \times 5 = 60$) therefore I suppose he had 60 *l.* But the third part thereof (20) the fourth part thereof (15) and the fifth part thereof (12) that is to say $20+15+12=47$, and by the Question it should be 94 *l* but now I say, if 47 . 60 : : 94 . 120. wherefore I conclude he had 120 *l.* in his Purse: And for Proof I say the third part thereof is 40, the fourth part 30, and the fifth part 24, now $40+30+24=94$ according to the intent of the Question.

In Questions wherein a Number constant or permanent is given; substract it for a time from the given Sum, and after Operation restore it again, because it doth not rise and fall proportionably as the Fractions do.

4. *Example.*

One said if I had in my Stock as many more as I have, together with the half, the third and fourth parts of these I have, and one *overplus*, I should should justly have 630. How many had he? For Answer, if I find a Number which being twice taken or doubled, and the half, and third and fourth parts thereof will be 629, I satisfie the Question, for

for then adding to it the permanent Number 1, it makes up just 630. Suppose my Number was 12, (because it hath the parts required) but then 12+12+6+4+3 make up but 37, and should be 629. Therefore I say if 37 come of my Position 12, of what cometh 629? The Rule answereth 204, and for proof 204+204+102+68+51 will be just 629, and now restoring the 1 before substracted, 629 +1 will be 360 according to the Intent of the Question.

37. 12 :: 629. 204.
12
7548 (204
148

5. *Example.*

One had spent the $\frac{2}{3}$ and the $\frac{1}{5}$ of his Stock, and had only 36 shillings remaining; what was his Stock? For Answer, seek such a Number as having the $\frac{2}{3}$ and the $\frac{1}{5}$ thereof abated, leaveth remaining 36. Suppose therefore the Stock was 15, whose $\frac{2}{3}$ (10) and $\frac{1}{5}$ (3) that is 10+3 being taken from 15 leaveth 2 remaining, which by the intent of the Question should be 36, therefore by the Rule, if 2. 15 :: 36. 270. which was the Stock, for the two thirds thereof 180, and the fifth part 54 added, maketh 234, and 270—234=36, which was required.

Note, that when the Fraction or parts expressed in any Question to be substracted exceed an Unite, so that in reason it cannot be substracted from the

Position or Number put, that then it is a Question absurd, and utterly impossible, and therefore had the Question before required been put, that $\frac{2}{3}$ and $\frac{3}{5}$ thereof should be abated, as it was but $\frac{2}{3}$ and $\frac{1}{5}$ it had been impossible; for $\frac{2}{3}$ of 15 in the position is 10, and $\frac{3}{5}$ is 9, now $10+9=19$, and how is it possible to substract 19 from 15?

6. *Example.*

What Numbers are they whose $\frac{2}{3}$ of the one is $\frac{3}{4}$ of the other? For Answer, I take a Number that hath such parts, as 12, whose $\frac{2}{3}$ is 8, then I seek to know of what Number 8 is $\frac{3}{4}$, and put such a Number for my Position as hath $\frac{3}{4}$ for the Denominator, as 20, whose $\frac{3}{4}$ is 15, but should be but 8; and now if 15 come of my Position 20, of what cometh 8? Answer, $10\frac{2}{3}$ of 12. Therefore 12 and $10\frac{2}{3}$ are the Numbers required; for as 8 is $\frac{2}{3}$ of 12, so is 8 likewise $\frac{3}{4}$ of $10\frac{2}{3}$.

7. *Example.*

What two Numbers are they, whereof $\frac{1}{2}+\frac{1}{3}$ of the one is equal to $\frac{1}{4}+\frac{1}{5}$ of the other? For Answer, take such a Number as hath parts like the Fractions, as 24, whose $\frac{1}{2}$ 12 + $\frac{1}{3}$ 8, is 20. Then I put 40 for the second Number, whose $\frac{1}{4}$ 10 + $\frac{1}{5}$ 8, is 18, which, according to the Question, ought to have been 20. But now I say, if 18. 40 :: 20. $44\frac{3}{9}$. wherefore I conclude 24 and $44\frac{4}{9}$ are the two Numbers desired, for the $\frac{1}{2}$ more the $\frac{1}{3}$ of 24 is 20, and so much is the $\frac{1}{4}+\frac{1}{5}$ of $44\frac{4}{9}$.

In

In the two last Examples there is not two Positions made, for both 12 and 24, the Numbers I take in each Question, continue and always remain one of the true Numbers that serve to answer the Question without any Alteration, but neither 12 nor 24 (which I take for my Position) are any of the true.

4. But if there be no Partition in Numbers to make a proportion, then must you use the *Rule of double Position*; that is, you must twice make a Supposition, and if at either time you hit upon such a Number as will satisfie the Question, you have performed it; if not, observe the Errors, and whether they were greater or lesser than the Resolution required, and according mark them with the Signs of + or —.

5. After you have set down the Errors, multiply them by the contrary Positions, and if the Errors were both too great or both too little, substract both the one Product from the other, and the one Error from the other, and divide the Difference of the Products by the Difference of the Errors; but if the Errors be of divers Kinds, add the Products, as likewise the Difference together, and divide the Sum of the Products by the Sum of the Errors, the Quotient gives the Number sought; for the Proportion of the Errors is the same with the Proportion of the Excesses or Defects of the suppositious Numbers, from the Numbers sought.

Example.

A, *B*, and *C*, do agree to divide 100 *s.* amongſt them thus, *B* ſhall have more Shillings than *A* by 3, and *C* ſhall have 4 *s.* more than *B*, how many Shillings muſt each have?

The 1ſt. Poſition 33. *the 2d. po.* 31.
Error + 9. *Error* + 3.
Croſs Multip 99. *Prod.* 279
99
Diff. Prod. 180 (30.
9 6)
3
Differ. 6

For Anſwer I ſuppoſe *A* ſhall have 33 then *B* 36, and *C* 40. But the Total of 33 + 36 + 40 = 109 & ſhould have been but 100, therefore I erre 9 more than I ſhould; I make my Suppoſition to be 31 for *A*, therefore *B* 34, and *C* 38. But 31 + 34 + 38 are 103, and ſhould but be 100, therefore I erre again 3 too much; therefore multiplying croſs-wiſe, and becauſe the Errors are both too great, I abate both the Errors and the Products, and according to the Rule divide 180 the *Difference* of the Products by 6 the Difference of the Errors, and find that *A* had 30 *s.* therefore *B* 33, and *C* 37; and 30 + 33 + 37 = 100.

And this is the Reſolution where both the Errors are too great.

Now

Now suppose in the same Question *A* had 25, therefore *B* 28, and *C* 32; but 25+28+32 = 85. The Error too little by 15. Again if I suppose *A* 27, then *B* 30, and *C* 34: but 27+30+34=91, too little by 9: And because the Errors are alike, I work as before, and find the Answer for *A* to have 30 *s.* as before.

The 1 *Po.* 25		*The* 2 *Po.* 27	
Errors — 15		*Errors* — 9	
*Cross Pro.*225		— 405	
Differ. of Prod. 180			
Differ. of Er. 6)180(30			

Again, suppose in the same Question, I had supposed *A* had 34, therefore *B* 37, and *C* 41. Now 34+37+41 = 112, which ought to be but 100, therefore too much by 12: In the second Position let *A* be 20, therefore *B* 23, and *C* 27. But 20+23+27=70, too little by 30; and now seeing the Errors are unlike, after alterne Multiplication the Products and Errors are summed up, and according to the Rule, the Quotient is found 30 for *A* as before, and so have you this Example in every Variety.

The 1 *Po.* 34.	*The* 2 *Po.* 20	
The Er. +12	*Er.* — 30	
Cross Prod. 1020	— 240	
Sum of the Products	1260 (30	
Sum of Errors	42) 126	

2. *Example.*

Two men, *viz.* *A* and *B* snatched at 100 *s.* and after Agreement *A* said to *B*, give me the ⅓ of the Shillings

Shillings you ſnatched, and I will give you the $\frac{1}{5}$ of mine; this done they had either of them 50 *s.* How many therefore had each ſnatched?

For Anſwer, I ſuppoſe *A* ſnatched 30. Now 30 - 10 + $\frac{1}{5}$ of 70 *B* had that is 14, makes *B* to have 56 + 10, and *A* to have but 20+14=34, which ſhould be 50, and ſo 16 too little; I therefore now ſuppoſe *A* ſnatched 45; now 45 - 15 + $\frac{1}{5}$ of 55 which *B* had, that is, 11 make *B* to have 44+15, and *A* to have 30+11, that is 41, and ſhould have 50, ſo the Error is 9 too little.

The 1 *Po.* 30. *The* 2 *Po.* 45
Err. — 16. *Err.* — 9
The Croſs Prod. 270 — 720
The Diff. of Pro. — 450
The Diff. of Err. — 42 (64$\frac{2}{7}$
30

And now according to the Rule I find *A* ſnatched $64\frac{2}{7}$, and *B* ſnatched $35\frac{5}{7}$.

And for Proof,

A having $64\frac{2}{7}$ - $21\frac{3}{7}$ and $\frac{1}{5}$ of $35\frac{5}{7}$ which *B* had, that is $7\frac{1}{7}$. *A* had $42\frac{6}{7}+7\frac{1}{7}=50$
B had $28\frac{4}{7}+21\frac{3}{7}=50$ According to the Conditon of the Queſtion.

3. *Example.*

A having ſtoln ſome Number of Shillings was ſtayed by *B*, to whom he gave the $\frac{1}{2}$ of his Shillings, but he returned 12 *s.* again; after he was ſtayed by *C*, to whom he gave $\frac{1}{2}$ of his Shillings, and he returned back 7 *s.* after he was ſtayed by *D*,

D, to whom he gave $\frac{1}{2}$ of his Shillings, and he returned 4 *s*. And after all this he had 20 *s*. how many Shillings did he steal at first? For Answer, I suppose *A* had stoln 40.

Now $40-20+12 = 32$ and $32-16+7 = 23$ and $23-11\frac{1}{2}+4 = 15\frac{1}{2}$ but should have been 20, therefore the Error's too little by $4\frac{1}{2}$.

The 1 *Po.* 40. *The* 2 *Po.* 60
The Err. $-4\frac{1}{2}$ *Err.*——-2
The Cross Prod. 80 -- 270
The Diff. of Prod.——190 (76
The Diff. of Err. $2\frac{1}{2}$

I suppose therefore now *A* had stoln 60, now $60-30+12=42$ and $42-21+7=28$ and $28-14+4=18$, but should have been 20, therefore 2 too little, so now I find by the Work, that *A* stole at the first 76 Shillings.

And for Proof I say,
$76-38+12=50$
$50-25+7=32$
$32-16+4=20$ The true intent of the Question.

And for Conclusion of this Rule, any Question whatsoever, if not impossible, may thereby be resolved; if by Comparing, Adding, Substracting or Proportion, you could prove that Question if the true Resolution was given, for otherwise the Question cannot be resolved, because you cannot come to know what the Errors were at the Positions: but all such Questions being more easily and speedily resolved by the Doctrine of *Æquations*, taught in the fourth Book, I shall hereof say no more.

CHAP.

CHAP. XVIII.

Of Arithmetical and Geometrical Proportion continued.

1. ARithmetical and Geometrical Proportion continued, are termed by Authors the Comparative Parts of Arithmetick, and of them we ſhall here ſpeak in Order. And firſt,

2. *Arithmetical continued Proportion* is a continued *Progreſſion* or *Series* of Numbers, increaſing or diminiſhing by equal Differences, as the Numbers 3. 5. 7. 9. 11. *&c.* are continued in an Arithmetical Proportion, the common Difference being 2.

3. Here we are to conſider five things. (1) The *firſt* and leaſt Term 3. (2) The *laſt* and greateſt Term 11. (3) The *Number* of Termes which are here 5. (4) The *common Difference* 2. (5) The *Sum* of all the Terms 35. And *note*, that the Number of the Differences is the Number of the Terms made leſs by one, and that the Sum of the Differences is equal to the Product of the Number of the Terms drawn into (or multiplyed by) the common Difference, made leſs by that common Difference which is equal to the laſt Term made leſs by the firſt.

The Considerations.

(1) *The first Term.*

(2) *The last Term.*

(3) *The Number of Terms.*

(4) *The common Difference.*

(5) *The Sum of all the Terms.*

4. Having three of any of these five given, the other two are found out by these twenty Rules following, invented by Mr. *Oughtred* in his *Clavis* Chap. XVIII. Prob. VI. the which I have put into Words, and will be of good Use for explaining the Characters in the fourth part of this Arithmetick.

1. Having the 1. 2 and 3. given to find the 5.

To the Product of the 3 into the 2, add the Product of the third into the 1. the Sum of these two Products divided by 2, is equal to the 5. Which is thus to be understood:

Example.

Suppose 3 the first Term, 11 the last Term, and 5 the Number of the Terms, be given to find the Sum of all the Terms, the Rule directs you to multiply 5 the Number of the Terms by 11 the last Term, which makes 55, to which add the Product (of 5 the Number of the Terms multiplied by 3 the first Term) which is 15, whose Sum is 70, which divided by 2, makes 35 the Sum of all the Terms:

Terms: this being practised in ſome few, the reſt will be found very plain.

2. *Having the* 1. 2. *and* 3 *given to find the* 4.

Divide the 2*d.* made leſs by the 1*ſt.* by the 3*d.* made leſs by Unity or one, and the Quotient is equal to the 4.

3. *Having the* 1. 2. *and* 4 *given to find the* 3.

Subſtract the 1ſt. from the 2d. divide the Remainer by the fourth, and add unto the Quotient Unity, the Sum is equal to the 3d.

4. *Having the* 1. 2. *and* 5. *given to find the* 3.

Double the 5, and divide the Sum by the 1 added to the 2, the Quotient is equal to the 3.

5. *Having the firſt, third, and fourth given to find out the ſecond.*

I multiply the third and fourth, and take from the Product the fourth, to the Remains add the firſt, the Sum is equal to the ſecond.

6. *Having the firſt, third, and fourth given to find out the fifth.*

Multiply the third and fourth, from the Product take the fourth, to the Remain add the double of the firſt, the Sum multiplied by the third, and divided by two is equal to the fifth.

7. *Having the firſt, third, and fifth, given to find out the ſecond.*

Double the fifth, and ſubſtract from it the Product of the third multipled by the firſt, divide the Remains by the third, the Quotient is equal to the ſecond.

8. *Having*

8. *Having the ſecond, third, and fourth given to find out the firſt.*

From the Sum of the ſecond, added to the fourth, ſubſtract the Product of the third multiplied by the fourth, the Remainder is equal to the firſt.

9. *Having the ſecond, th rd, and fourth given to find out the fifth.*

From the doubled Sum of the ſecond, added to the fourth, ſubſtract the Product of the third multiplied by the fourth, and multiply the Remainder by the third, the Product is equal to twice the fifth.

10. *Having the ſecond, third and fifth given to find out the firſt.*

Double the fifth, divide it by the third, ſubſtract from the Quotient the ſecond, the Remainder is equal to the firſt.

11. *Having the ſecond, third and fifth given to find out the fourth.*

Double the Product of the third multiplied by the ſecond, and ſubſtract from it the double of the fifth, divide the Remainder by the third multiplied in it ſelf, and made leſs by it ſelf, the Quotient is equal to the fourth.

12. *Having the firſt, ſecond, and fourth given to find out the fifth.*

Subſtract the Square of the firſt from the Square of the ſecond, divide the Remainer by the fourth, to the Quotient add the firſt and ſecond, the half of the Sum is equal to the fifth.

13. *Having the first, third and fifth given to find out the fourth.*

Substract the Square of the first from the Square of the second, divide the Remainer by the fifth doubled, made less by the first and second, the Quotient is equal to the fourth.

14. *Having the first, third and fifth given to find out the fourth.*

Double the first multiplied by the third, and substract it from the fifth doubled, divide the Remainder by the Square of the third made less by it self, or by the third the Quotient is equal to the fourth.

15. *Having the third, fourth and fifth given to find out the first.*

Divide the fifth doubled, by the double of the third add to the Quotient half of the fourth, and substract from that Sum the half of the Product of the third, multiplied into the fourth, the Remainer is equal to the first.

16. *Having the third, fourth and fifth given to find out the second.*

Divide the fifth doubled, by the double of the third add to the Quotient the half of the Product of the third multiplied by the fourth, and from that Sum substract the half of the fourth, the Remainer is equal to the second.

The former 16 Prop. may be all wrought by the Rules before-going; the latter four are wrought by extracting the square Root taught in the fourth Book, and therefore are here omitted.

3. When any Question, resolvable by Arithmetical Progression, is propounded, consider on which

which of the five precedent Confiderations the Refolution depends, *viz.* Whether the Demand be to give the firft or laft Terms, the Number, general Difference or Sum of the Terms, that is, whether of the firft, fecond, third, fourth or fifth.

If it be of the firft, then the reft of the Terms given, and the Refolution of the Queftion will depend either on the eighth, tenth or fifteenth precedent Rules.

If of the fecond, it depends on the fifth or feventh.

If of the third, it depends on the third, fourth or eighth.

If of the fourth, it depends on the fecond, eleventh, thirteenth or fourteenth.

If of the fifth, it depends on the firft, fixth, ninth or twelfth.

The Refolution of the Work, when you have duly ftated the Confiderations, is found out by the Rule.

1. *Example*

If the firft Term of a Progreffion be 5, the laft Term 15, and the Number of the Terms 6, what is the Sum of the Terms, and what the equal Difference of each Term? Here is given the firft, fecond and third Confiderations to find the fifth and fourth, which by the firft and fecond Rules give the Sum to be 60, and the Difference 2.

The Work.

6 6	90	120 (60 *the Sum.*		15 6		10 (2 *the Difference.*	
15 5	30	3)		5 1		(5	
90 30	120			10 5			

2. *Example.*

Of eight Brethren the youngeſt was 27 years old, the eldeſt 50, each differed alike in Age from other; what Difference was there in their Ages, and what the Age of each Brother? (3) The Number of Terms 8. (1) the firſt Term 27. (2) the laſt Term 50 are given to find out (4) the common difference, by the ſecond Rule the Work finds the equal Difference to be $3\frac{2}{7}$.

$$8-1=7)\ 50-27=23\ (3\tfrac{2}{7}$$
$$\textit{for}\ 3\tfrac{2}{7}\times 7=23\ \textit{and}\ 23+27=50$$

3. *Example.*

One had divers Sons, the youngeſt 6 years old, the eldeſt 40, and every one in order ſtill exceeds his Brother by 2 years; How many Sons had he? Anſw. 18. And how much was the Number of all their Ages? Anſ. 414.

Here

Here is the first, second and fourth given to find the third and fifth.

By Rule 3d. 2) $40-6=36$ $(17+1=18$

By Rule 12th.

$$\begin{array}{r} 1600 \\ 36 \\ \hline 2)\ 1564\ (782 \\ 6 \\ 40 \\ \hline 2)\ 828\ (414\ (5) \end{array}$$

4. *Example.*

One travelled 50 miles, every day encreasing his Journey 2 Miles, till at 5 Days end he finished his Journey; how many Miles was his first Days Journey? Answer 6. And how many the last? Answer 14.

In this Question, the third, fourth and fifth are given to find the first by Rule 15. and the second by Rule 16.

$$\begin{array}{l} 10)\ 50+50=100\ (10+1=11 \\ \quad 5\times2=10\ (5=\underline{5} \\ \quad\quad 2) \quad\quad\quad 6\ (1) \end{array}$$

$$\begin{array}{l} 10)\ 50+50=100\ (10 \\ \quad\quad\quad \tfrac{10}{2}=\underline{5} \\ 5\times2=10 \quad\quad 15-1=14.\ (2) \end{array}$$

 5. *Example.*

5. *Example.*

One had 20 Cloths worth 5 *l.* a piece ready Money, and ſold the firſt for 1 Crown, and augmented his price 2 Crowns more for every Cloth than was paid for the former; what received he for his Cloths, and whether did he gain or loſe of the juſt price of 5 *l.* a Cloth? Anſw. he received 800 Crowns or 200 *l.* his Cloths at 5 *l.* the Cloth, coming to 100 *l.*

Here the third (20) *is given, the firſt* (1) *and the fourth* (2) *by the ſixth Rule.*

$$40\times2-2+2=80\times20=\tfrac{1600}{2}=200.$$

6. *Example.*

One hundred Eggs are placed every one a yard diſtant from another, and 1 yard diſtant from a Basket; whether might one gather up the Eggs one after another, ſtill returning to the Basket and putting them in, before another doth run 4 Miles or 6040 yards? Anſwer, he that gathered the Eggs went 10000 yards, which exceeds 4 miles by 3960 yards.

Here the firſt, third and fourth Terms are given to find out the fifth, by the ſixth Rule.

$$100\times2=200-2+2=200\times100=\tfrac{20000}{2}=10000.$$

7. *Example.*

7. *Example.*

A Sum of Money is to be paid in 12 Days, paying the first Day 10 *l.* and increasing every payment after 5 *l.* till the Days be expired, what is the whole Debt? Anſ. 168 *l.*

$$12-4=8+20=28$$
$$28\times12=\frac{336}{2}\ (168.$$
$$\frac{12}{336}$$

8. *Example.*

Six hundred eighty five Pounds is to be disburſed by Arithmetical Progreſſion, to how many I know not, but the firſt hath 19 *l.* and the laſt 118 *l.* How many did receive the Money, and how much had each? Anſwer, there were 10 to receive the Money.

The common Difference being 11 *l.* each Man's Sum may eaſily be had by adding 11 to the firſt Man's, who had 19 *l.* &c.

$685 \times 2 = 1370$ (10
$118 + 19 = 137$)

118	19	13924	685
118	19	361	2
944	171	13563	1370
1298	19		137
13924.	361		1233) 13563 (11
			1233
			1233
			1233

After this manner may ſuch Queſtions of Arithmetical continued Proportion be wrought.

5. *Geometrical Propo tion continued*, is when any *Series* of Numbers ſet down, have the ſame or equal Reaſon, that is, the Quotients of each following Term divided by the former, are equal; as 2. 4. 8. 16. *&c.* the *Ratio* being 2, for 16 contains 8 twice, and ſo 8, 4. and 4, 2.

6. In Geometrical Proportionals continued, the firſt and laſt are commonly called the Extremes, and all the middle Terms are called the Means.

7. Becauſe the finding the Means in a Progreſſion Geometrical is done by extracting the Roots, not yet taught, and becauſe the great Uſe of this Proportion is in ſatisfying Queſtions of compound Intereſt, which are all reſolved by Tables made for that Purpoſe, explained in the End of this Book; I refer you to the ſame; only take this Rule for the finding the Sum of any Progreſſion Geometrical.

Multiply

Multiply the last Term by the same *Ratio* found by *Sect.* 5. from the Product take the first Term, divide the Remainer by a Number less by 1 than the *Ratio*, the Quotient is the Sum of that Progression.

1. *Example.*

3. 6. 12. 24. 48. the Sum is 93, for the *Ratio* being 2. $48 \times 2 = 96, - 3 = 93$ divided by 1 equal to 93.

Therefore for resolving of such Questions where nothing but the first Term, *Ratio* and Number is given, you must first find the last, and then the Sum as before, the last Term or any other Term may be discovered by placing Arithmetical Terms over the Heads of the Proportion, and continuing some few of the first of the Progression.

Ex.	1.	2.	3.	4.	5.	6.	7.	8.	*Arith. Pro.*
	2.	4.	8.	16.	32.	64.	128.	256.	*Geo. Pro.*

The lower Numbers 2. 4. *&c.* are in Geometrical Proportion, the higher in Arithmetical Proportion; and note that for the desired Term, if it be greater than any given, add up such two of the Arith. Pro. as will make up the desired Term, and multiply the Geo. Terms, under the Arith. it will satisfie your desire.

As suppose in the former Progression I have only 4 of the first Terms given, and the 10*th.* Term is desired;

I find $2+4+4=10$ *The* 10th. *Term in Arith.*
$4 \times 16 \times 16 = 1024$ *The* 10th. *Term in Geo.*

But if the Term desired be less than a Term given, substract the Arith. Terms from the given,

and divide the Geo. anſwering, you have your Deſire. As in the former Progreſſion the 3 firſt Terms $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ and the eighth Term $\frac{8}{256}$ were given to find the fifth Term.

$8-3=5$
$8)\ 256\ (=32$
$\underline{24}$
16

The ſame manner of Work may be obſerved in the finding any Term propoſed, in what Queſtion ſoever.

1. *Example.*

One bought an Horſe after this manner: The Horſe had 4 Shoes, every Shoe containing 6 Nails, in all 24 Nails, he thinking to have a good Bargain was content to pay a Farthing for the firſt Nail, and double it to the laſt; what ought he to pay for the Horſe? Anſwer.

$$4+5 = 9, + 9 = 18, + 5 =$$
$$16\times32=512\times512=262144\times32=$$
$$=23, +1 = 24$$
$$= 8388608 = 24 \text{ Or laſt Term.}$$

Then by the Rule *Sect.* 7. $8388608\times2=16777216-1=16777215$, which is the Sum of the Progreſſion, and the Number of Farthings he ought to pay for the Horſe, which make 17476 *l.* 5 *s.* 3 *d.* 3 *f.* a very great rate to give for a Horſe.

The End of the First Book.

MOORE'S ARITHMETICK;

BOOK II.

CONTAINING THE

DOCTRINE

OF

Decimal Fractions.

Methodically digeſted, and inlarged

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CHAP. I.

Notation of Decimals.

1. A Decimal Fraction hath always for it's Denominator an Unite with Cyphers, and consequently must be either 10. 100. 1000. 10000. or 100000. *&c.*

2. In writing down of a Decimal Fraction, there is no Necessity of expressing its Denominator, that being always certainly known by Inspection only, for it containeth an Unite with as many Cyphers annex'd to it as there are places in the Numerator; as for example, the Decimal Fraction $\frac{75}{100}$ may be written thus, .75, its Denominator being known to consist of an Unite with 2 Cyphers annexed to it, because the Numerator .75 containeth two places, so likewise $\frac{464}{1000}$ may be thus written .464, and $\frac{6582}{10000}$ thus .6582.

3. Although Cyphers annexed to the End of an Integral Number do increase its Value in a decuple Proportion, yet being annexed to the End of a Decimal Fraction, they do neither increase nor diminish its Value, so $\frac{2600}{10000}$ is equivalent to .26 or $\frac{26}{100}$, and $\frac{90}{100}$ or .90 is equivalent to .9 or $\frac{9}{10}$: And though Cyphers being prefixed to an integral Number, do neither increase nor diminish it's Value, as 45 is equivalent to 045 or 0045, yet being prefixed to a Decimal, they depress its Value in

a

a decuple Proportion; ſo .45 which is $\frac{45}{100}$ by prefixing a Cypher becometh .045 which is $\frac{45}{1000}$, and by prefixing 2 Cyphers, *viz.* .0045 it is $\frac{45}{10000}$.

Wherefore, when you are to write down a Decimal Fraction, whoſe Denominator hath more Cyphers in it than there are places in the Numerator, ſuch Defect muſt be ſupplied by prefixing as many Cyphers to the Numerator as the ſaid Exceſs is: as for Example, ſuppoſe I were to write down this Decimal, *viz.* $\frac{75}{10000}$ without its Denominator; here becauſe there are 4 Cyphers in the Denominator, and but 2 Places in the Numerator, therefore I prefix 2 Cyphers to the Numerator thus, .0075, by which means it becometh $\frac{75}{10000}$, for if I ſhould have written it thus .75 it would have been but $\frac{75}{100}$, or if thus .075, then it would have been but $\frac{075}{1000}$; Likewiſe $\frac{8}{100}$ is thus written .08, and $\frac{98}{100000}$ thus .00098, *&c.*

4. For diſtinguiſhing of the Decimal Fractions from the *Integers*, divers Men have their divers Ways. For ſome call their tenth Parts, Primes; the hundredth Parts, Seconds; the thouſandth Parts, Thirds *&c.* and mark them with equivalent Indices over their Heads; as if they would expreſs 245 whole, and $\frac{1234}{10000}$ parts of an Unite, they would do it thus, $245\ \overset{\prime}{1}.\ \overset{\prime\prime}{2}.\ \overset{\prime\prime\prime}{3}.\ \overset{\prime\prime\prime\prime}{4}.$ or thus, 245, 1234. and would read them thus, *viz.* 245 whole, one Prime, 2 Seconds, 3 Thirds, 4 Fourths. Others do nothing but put a Comma before the Decimal Parts thus, 245,1234, Others draw a Line under them thus, 245,$\underline{1234}$ writing the Parts in ſmaller Figures than the Integers. Others write them

them thus, 245|1234, but the beſt and moſt diſtinct Way of diſtinguiſhing them is, by putting a Point or Prick before the Decimal Fraction, and likewiſe in a mixt Number between the Integer and the Decimal, ſo ſhall .45 be known to be $\frac{45}{100}$, and 75.75 to be 75$\frac{75}{100}$. As in this Example, where the Decimals have their Denominations by 1, and Cyphers annexed thereto, in a contrary order to the Integers.

h	g	f	e	d	c	b	a	U.	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
8	7	6	5	4	3	2	1	0	.1	2	3	4	5	6	7	8	9
100000000	10000000	1000000	100000	10000	1000	100	10	1	10	100	1000	10000	100000	1000000	10000000	100000000	1000000000
								The Unites Place.									

5. Obſerve, that in this Table there are four Progreſſions; thoſe in the Figures above, which are called Indices proceeding both ways from Unity are *Arithmetical*, the Difference from the Unites Place being but 1, both Progreſſions being the ſame from Unity; though, contrarily, the lower is a Geometrical continual Proportion from Unity both Ways, every Degree or Place being ſucceſſive 10 times more or leſs, than the next from Unity.

6. As the *Series* of the *Numbers* from *Unity* are continued and do increaſe a decuple Proportion from the right Hand towards the left, ſo their Value

lue do increaſe in the ſame Proportion from Unity, as the other do increaſe above Unity; for let 3210.123 be a Number given, and ſtand with his Letters thus,

b is equal to 10 a. and a is equal to 10 U.
U is equal to 10. *a*, and *a* is equal to 10. *b*,
and *b* is equal to 10. *c*.

Again,

a is equal to 10 U.
a is equal to $\frac{1}{10}$ U.
b is equal to 100 U.
b is equal to $\frac{1}{100}$ U.

7. Hence it follows that Decimals are ſet down in a Retrograde Order to Integers; for if you were to ſet down in Integers the Number ſeven thouſand and thirty, it would be thus 7030.

But to ſet theſe down in Decimals, it would be in a Retrograde order, as if you were to ſet down $\frac{7}{1000}$ and $\frac{3}{100}$ it would be thus, 0307; for the *Unite* or one *Integer* is always underſtood to be divided into Parts, bearing the Denomination, or Name of the Place of the laſt Figure in the Decimal Fraction. As for example, .1 ſignifieth one tenth Part, as if it were written at length thus, $\frac{1}{10}$, the Denomination of the Place being Tenths, as the firſt Place from the Unite towards the right Hand, and noted in the former Example with 10; ſo .12 ſignifieth 12 hundredth Parts, as if it were written at large thus, $\frac{12}{100}$, the Denomination of the laſt Place being noted with (100) under the laſt Figure 2. and

.123 signifieth 123 thousandth Parts, as if it were written at large thus, $\frac{123}{1000}$: And .1234 are 1234 ten thousandth Parts, as if it were written at large thus, $\frac{1234}{10000}$, and so of the rest.

8. The Index of Unity is put to be 0, as in the former Example, and the Index of any other Place from Unity, is known by its Distance, or the Number of Places from *Unity*, as the Index of 3 in this Number 73921. is 3, of 7 is 4, of 2 is 1. taking their Denomination from their Distance in Places from Unity.

In Decimals the just Number of Decimals from Unity is the Index, as in this Decimal Number .73921, the Index of 9 is 3, of 1 is 5, of 7 is 1, where note that the *Indices* of Integers are expressed with a greater Character, and those of the Decimals with a smaller.

These *Indices* are of great Use in the finding out of the true Values of Numbers in Multiplication and Division, as shall be after shewed.

The learned Mr. *Oughtred*, in his *Clavis Limata*, expresseth the *Indices* of *Integers*, *affirmatively*, and of *Decimals*, *negatively*; and so adds and substracts them, as in Cossick Numbers; but because it would trouble the new Beginner to understand that Way, I shall shew some few Rules, that with small Practice will perform the same.

9. The Reason of these *Indices* (they increasing in Arithmetical Proportion, as the Numbers under increase in a Geometrical Proportion) is plain out of Mr *Oughtred*'s *Clavis Limata*, *Chap.* 6.

Arith. Prop. working that in *Addition* and *Substraction*, which the other doth in *Multiplication* and *Division*.

And

And you may obſerve, that in all other Parts of Arithmetick, as in the *Logarithms*, where, by the Advice of our Learned Country-man Mr. *Briggs*, the Logarithm of 1 or Unite was put to be a Cypher, the Logarithm of 10 to be 1, *&c.* As alſo in the Sexagenary Account, where for Degrees the Index is o, and ſo to increaſe both Ways. Theſe *Indices* have the ſame Work, and are of the ſame Uſe in Decimals.

I am plainer in this decimal Way, for the great Facility it brings with its Practice, in all the Parts of Arithmetick, Geometry, Aſtronomy, and the reſt of the Mathematicks.

In practical Arithmetick, if the firſt Inſtitution had been in Decimals, we had never been troubled with ſo many Fractions; it were yet worthy the Name of Reformation, to cauſe the Fractions of Money and Weight to be altered: And as concerning the Eaſe in Meaſuring, Surveyors and Land-Meters, who uſe the decimal Chain, and thoſe who uſe a Decimal Foot, Yard, or Scale, can beſt certifie upon Experience; if it had not been the Stationers Fault, I had long ſince publiſhed ſome Tables after the moſt eaſy and methodical Way hitherto thought on, towards the ready obtaining the true place of any of the Planets in this decimal Way, which would have been a ſufficient Teſtimony of the Eaſe thereof in Aſtronomical Calculations: Thus much for *Notation* of *Decimal Arithmetick.*

CHAP.

CHAP. II.

Reduction of Decimal Fractions.

1. IN the Reduction of Decimals, I shall first shew how to reduce a vulgar Fraction to a Decimal, and then how to reduce any Decimal given, to the known Parts of Coyn, Weight, Measure, Time, Motion, *&c.* and of these in their Order, with as much Plainness and Perspicuity as may be.

2. When it is required to reduce a Vulgar to a Decimal Fraction, the same may be performed by this Analogy, *viz.*

As the Denominator of the given Fraction is in Proportion to its Numerator,

So is (10000, &c.) *an Unite with Cyphers annexed, to its equivalent Decimal.*

Let it be Required to reduce $\frac{3}{4}$ to a Decimal Fraction, the Operation will be as followeth;

$$4 : 3 :: 100 : .75$$

$$\begin{array}{r} 3 \\ \hline 4)\ 300\ (.75 \\ 28 \\ \hline 20 \\ 20 \\ \hline (0) \end{array}$$

So that .75 ($\frac{75}{100}$) is found to be equivalent to $\frac{3}{4}$.

3. By the foregoing Analogy it is evident that if to the Numerator of any given Vulgar Fraction you annex a competent Number of Cyphers, and then divide it by the Denominator, the Quotient will be the Decimal required; and look how many places you would have the Decimal to consist of, so many Cyphers you are to annex to the given Numerator; as in the following Examples.

2. *Example.*

Reduce $\frac{12}{13}$ to a Decimal consisting of 4 Places, *facit* .9230 as by the following Operation.

$$\begin{array}{r} 13)\ 120000\ (.9230 \\ 117 \\ \hline 30 \\ 26 \\ \hline 40 \\ 39 \\ \hline 10 \end{array}$$

In

In the foregoing Example, becauſe it is required to reduce the given Fraction to a Decimal conſiſting of 4 Places, wherefore to the given Numerator (12) I annex 4 Cyphers and it makes 120000, which being divided by the Denominator (13) the Quotient gives .9230 for the Decimal required, which (becauſe there is a Remainder after Diviſion is ended) is not exactly equivalent to the given Fraction, and yet it is ſo near the true Value, that it wanteth not $\frac{1}{10000}$ Part thereof, and if you proceed yet further to make the Decimal conſiſt of 5 Places it will be .92307, and then it will not want $\frac{1}{100000}$ Part of an Unite of the true Value; and if you proceed to make the Decimal conſiſt of 6 Places it will then be .923076, which yet is not exact, but it wanteth not $\frac{1}{1000000}$ Part of an Unite of the true Value; ſo that if you ſhould make it .923077 it would then exceed the true Value, and thus by increaſing the Number of Places in the Decimal you may come infinitely near the true Value, but never attain it exactly.

3. *Example.*

What is the Decimal equivalent to $\frac{25}{29}$? *facit* .86206.

Here ſuppoſe the Decimal required were to conſiſt of 5 Places, I annex 5 Cyphers to the Numerator and it makes 2500000, which being divided by (29) the Denominator, the Quotient gives .86206 for the Decimal required.

29) 2500000 (.86206
232
180
174
60
58
200
174
(26)

Note. If in the Reducing of a Vulgar Fraction to a Decimal, after Division is ended there be not so many Figures or Places in the Quotient as you anexed Cyphers to the Numerator, such Defect must be supplied by prefixing as many Cyphers to the Quotient, so as that they may take place in the Decimal by prefixing a Point before, as before hath been taught; and as in the following Example.

4. *Example.*

Let it be required to reduce $\frac{1}{15}$ to a Decimal consisting of 4 places: *facit* .0666, as in the Operation following.

15) 1.0000 (.0666

```
15) 1.0000 (.0666
     90
     100
      90
      100
       90
       10
```

To answer the foregoing Question, I first annex 4 Cyphers to the Numerator and it makes 10000 for the Dividend, which being divided by (15) the Denominator, the Quotient is *666*, which consisteth but of 3 Places, when according to the third Rule of this Chapter it should consist of 4 Places; therefore, to supply this Defect, you must prefix a Cypher to *666*, and it makes *0666*, to which prefix a Point, and it makes *.0666*, which is the Decimal required: This will be found to be of excellent Use in the Calculating Tables of Reduction for Coin, Weight, Measure, *&c.* as shall be shewed in the following Rule.

4. From the foregoing third Rule it is manifest that the Parts or Fractions of Money, Weight, Measure, *&c.* may be reduced to Decimal Fractions; for if it were required to find what is the Decimal of a Pound Sterling, equivalent to 7 Shillings; here I consider that 20 Shillings is one Pound, therefore 7 *s.* is $\frac{7}{20}$ of a Pound, which by the foregoing Rule will be found to be .35 *l.* which is the Decimal of 7 Shillings.

In like manner if it were required to find what Decimal of a Pound Sterling is equal to 9 *d.*

Here I consider, that in a Pound Sterling are 240 Pence, therefore 9 *d.* is $\frac{9}{240}$ of a Pound; wherefore to the Numerator 9 I annex 4 Cyphers, (or you may put more or less at pleasure) and it makes 90000, which being divided by 240 gives in the Quotient 375, which indeed ought to consist of 4 Places because 4 Cyphers were annexed to the Numerator (9); wherefore (according to the Note upon the third Rule) I prefix a Cypher thereto, and it makes .0375 for the Decimal of 9 *d.* as was required.

But if it had been required to find what Decimal part of a Shilling is equal to 9 *d.* then you must have considered that 12 Pence is one Shilling, and 9 *d.* is $\frac{9}{12}$ of a Shilling, which in Decimals will be reduced to .75 Parts of a Shilling.

After the same manner the Decimal of a Pound Sterling answerable to 3 Farthings is found to be .003125; for in a Pound Sterling are 960 Farth. therefore 3 Farthings is $\frac{3}{960}$ *l.* which will be reduced as aforesaid; so $9\frac{3}{4}$ *d.* is $\frac{39}{960}$ *l.* which in Decimals is .040625. The like is to be understood of any other.

And if you are to put Compound Fractions, commonly called Fractions of Fractions into Decimals, you must first reduce them to a single Fraction, which is taught in the Doctrine of Vulgar Fractions: and indeed to reduce the famous Parts of Money, Weight, Measure, *&c.* into Decimals, is in Effect the Reducing of a Compound Fraction to a Simple one, and that into a Decimal as before is taught.

The

The like is to be underſtood in Troy Weight, the Denominator of Ounces is 12, of Penny weights is 240, of Grains is 5760, which may be reduced into Decimals, by annexing Cyphers to the Numerators, and then dividing them by the Denominators anſwering; the like is to be underſtood of all other Weights, and Meaſures, and therefore $\frac{1}{4}$ of a Yard, Ell, *&c.* in Decimals is .25, $\frac{1}{2}$ is .5, $\frac{3}{4}$ is .75.

5. But becauſe it will be ſomewhat tedious to convert Fractions into Decimals this Way, therefore it is beſt to prepare Tables of all theſe undenominate Fractions, out of which the Decimal Fractions may be gathered, as in the Tables following.

1. *Table of Coyn.*

Shillings.	Decimals of a Pound.
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05
Pence.	
11	.04584
10	.04166
9	.0375
8	.03333
7	.02917
6	.025
5	.02083
4	.01667
3	.0125
2	.00833
1	.00417
Farthings.	
3	.003125
2	.002083
1	.001041

2. *Table of Coyn, and Troy weight.*

Farthings.	*Decimals of a Shill. or a pound Troy.*
1	.02083
2	.04166
3	.06249
Pence or Ounce with Farthings.	
oz. 1 d	.083333
1	.10416
2	.12499
3	.14582
oz. 2 d	.16667
1	.1875
2	.20833
3	22916
oz. 3 d	25
1	.27083
2	.29166
3	.31249
oz. 4 d	.333333
1	.35416
2	.37409
3	.39582
oz. 5 d	.416667
1	.43749
2	.45832
3	.47915
oz. 6 d	.5
1	.52083
2	.54166
3	.56249
oz 7 d	.58333
1	.60416
2	.62499
3	.64582
oz. 8 d	.666667
1	.68749
2	.70832
3	.72915
oz. 9 d	.75
1	.77083
2	.79166
3	.81249
oz. 10 d	.83333
1	.85416
2	.87499
3	.89582
oz. 11 d	.91665
1	.93749
2	.95832
3	.97915

Penny

Penny weights.	*Decimals of a Pound Troy.*
19	.079167
18	.075
17	.070833
16	.066667
15	.0625
14	.058333
13	.054133
12	.05
11	.045833
10	.041667
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016667
3	.0125
2	.008333
1	.004167
Grains.	
18	.003125
12	.002083
6	.001041
5	.000868
4	.000694
3	.00052
2	.000347
1	.000173

Third Table.
Averdupois great Weight.

	Decimals of a C weight.
$\frac{3}{4}$ *of a* C	.75
$\frac{1}{2}$ *of a* C	.5
$\frac{1}{4}$ *of a* C	.25

Pounds.	*Decimals of a C weight.*
27	.24107
26	.23214
25	.22321
24	.21428
23	.20535
22	.19642
21	.1885
20	.17857
19	.16964
18	.16071
17	.15178
16	.14285
15	.13392
14	.125
13	.11607
12	.10714
11	.09821
10	.08928
9	.08035
8	.07143
7	.0625
6	.05357
5	.04464
4	.03571
3	.02678
2	.01785
1	.00893
Quarters of Pounds.	
3	.0067
2	.00446
1	.00223

Averdupois little Weight.

Ounces.	*Decimals of a Pound.*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.1250
1	.0625
39	.046875
29	.03125
19	.015625

6	.5
5	.41667
4	.33333
3	.25
2	.16667
1	.08333
Days.	
30	.08219
25	.06849
20	.0548
15	.04109
10	.02734
5	.01369
4	.01096
3	.00822
2	.00548
1	.00273
39	.00204
29	.00136
19	.00068

Fourth Table of Time.

Months.	*Decimals of a Year.*
11	.91667
10	.83333
9	.75
8	.66667
7	.58333

Fifth Table of Measure, viz. of Yards, Ells, &c.

Quarters.	*Decimals of a Yard, Ell, &c.*
3	.75
2	.5
1	.25
Nails.	
3	.1875
2	.125
1	.0625

To

To make the Decimal Tables abovesaid, for Shillings, Pence and Farthings.

6. Upon the third and fourth Rules of this Chapter is grounded the Calculation of the foregoing Tables; for if you conceive 1 Pound or 20 Shillings to be divided into 10000 Parts; and then saying, if 20 be equal to 10000, then 10 Shillings (the half of 20) is equal to .5 the half of 10, and so 5 Shillings the half of 10*s.* is equal to .25, the half of .5 or .50; and .05 equals 1 Shilling, the fifth of 25 is 5*s.* add up the intermediate Spaces of Shillings, and you may make the Table for Shillings, which if you conceive a Cypher to be annexed to the Number of any Sum of Shillings and take half thereof, you have the Decimal answering thereto.

s.
20=1.000
10=.5
5=.25
1=.05

Now to make the Table of Pence, you may say, if .05 be equal to 12 Pence, then is .025 equal to 6 Pence, and 1004166, *&c.* equal to one Penny; which is no pure Decimal, but it may be continued at pleasure; now the intermediate places of Pence are made up by Addition or Substraction, thus, take the Decimal answering 1 Penny, and substract it from that answering 3 Pence, and it leaves the Decimal answering 2 Pence to be 00833333, *&c.* so of 4 *d.* to be .0166666, *&c.* and so of the rest,

d.
12=.45
6=.025
3=.0215
1=.004166, *&c.*

rest, always noting thus much, That a perfect Decimal (which is always one that ends with 5) may be one of the two you add or substract together.

Again, if 1 Penny, or 4 Farthings be equal to .0041666, then 2 Farthings must needs be equal to .0020833, and 1 Farthing equal to .0010416, and the Decimal of 3 Farthings not expressed, is equal to .0031249 by adding the Decimals of 2 and 1 Farthings together, and after this manner is the first Table of Decimals of Coin made: where note this, that as all perfect Decimals end with 5, and express the true value of the Fraction; so all other Decimals are imperfect, not expressing the true value, though they may want but very little of Truth; to the helping whereof, you may continue them to what number of Places you please; (as before hath been said) As for Example; the Decimal of 1 Penny is .00416, so likewise is it .00416666, &c. and observe always in the end of every Decimal, if the Figure that should follow it be either 5, or above 5, then augment the last Figure by 1; so if you will use 6 places for the Decimal of 1 Penny, it will be .004167, the which you will observe in all the Tables following.

4	=.0041666
2	=.0020833
1	=.0010466

7. But because pounds are so easily reduced into Shillings by doubling, and Shillings into Pence by halfing; therefore it is far better to make 1 Shill. the Integer, and from that the second decimal Table of English Coin is made, with the Farthings put

put betwixt each Penny, so that this way the Pence and Farthings are suddenly expressed in 3 or 4 Figures: the making of this Table is after the same manner that the last was, conceiving 1 Shill. or 12 Pence to be equal to 100000, and the intermediate places with Farthings are made up by Addition and Substraction, as before.

d.	
12	= 1.00000
6	= .5
3	= .25
1	= .0833
2 *far.*	= .04167
1 *far.*	= .02083

And the Decimals in the second Table signifying Pence, and printed in a greater Character may likewise signifie the Decimals of the Ounces of a Pound Troy Weight, the Pound being 12 Ounces answering the 12 Pence contained in a Shilling. The Decimals of the Penny Weights of the Pound Troy are at the latter end of the second Table, and each sixth Decimal of Grains.

The third Table contains the Decimals of Averdupois great and little Weight, the Integer being an hundred weight in the first; and Quarters, Pounds, and quarters of Pounds are in Decimals; and in the second the Integer is a Pound, and Ounces and quarters of Ounces are in Decimals.

The fourth Table consists of the Decimals of Time, wherein a Year is the Integer, and the Months and each fifth Day are in Decimals.

The fifth Table contains the Decimals of a Yard or Ell: The making of these Tables is as that of Shillings; therefore for your own use you may make Tables

Tables of all other Weights and Measures, but these will serve for our Purpose.

8. The Use of these Tables is to express suddenly the Decimals answering to any of the undenominate Fractions; for if you take the Decimal answering the Parts propounded, or if the Parts be compounded, then adding their Decimals together it shall express the Value of that Fraction; As the Decimal of 15 Shillings is .75; of 9 *d.* is .0375; of 10 *oz.* Troy Weight is .83333; of 7 Shillings is .35; of 2 Farthings is .00208; of 5 Months is .4166; of $\frac{1}{4}$ of an Ell is .25; of 10 Days is .0273.

Also of 15 *s.* 09$\frac{1}{2}$ *d.* is .78958. For,

The Decimal of 15 *s.* is	= .75
The Decimal of 9 *d.* is	= .0375
The Decimal of 2 Farth. is	= .00208
Their Sum is the Decimal of 15 *s.* 9$\frac{1}{2}$ *d.*	= .78958

And the Decimal of 9 *oz.* 16 *p.w.* is = .8166. For

The Decimal of 9 *oz.* is	= .75
The Decimal of 16 *p.w.* is	= .0666
And their Sum is the Decimal of 9 *oz.* 16 *p.w.*	= .8166

But for ready Practice, I would advise the Reader to use the second Table of Coin, where you may have the Pence and Frathings at one Work, and that by Inspection only; only remembring to double the Pounds (if any such be) with a Cypher, and

and add them to the Shillings: As for Example, 15 *s.* 5 *d.* in Decimals is 15.416; and the Decimal of 31 *l.* 12 *s.* 11 *d.* is 632 *s.* .9166; for 31 *l.* 12 *s.* is 632 Shill. and the Decimal of 11 *d.* is .9166, *&c.*

So 152 *l.* 11 *s.* 5½ *d.* in Decimals is 3051 *s.* .458, for 152 *l.* 11 *s.* is 3051 Shillings, and the Decimal of 5½ *d.* is .458, as appears by the Tables.

9. Thus have I shewed how to reduce *Fractions*, whether *denominate* or *undenominate*, into *Decimals*. And now I shall shew you how to know the Fraction, or Value expressed by a Decimal.

When it is required to find the Value of any given Decimal, in the known Parts of Coin, Weight, Measure; This is the Rule, *viz.*

Multiply the given Decimal by the Unites of the next inferiour Denomination which are contained in an Integer or Unite that is of the same Denomination with the given Decimal; and look how many Places there are in the Product more than there are in the given Decimal, so many Figures must be cut off from the left hand with a dash of the Pen for the Answer; and in the same manner proceed till you have found out the Value of the given Decimal as low as you please, and at the last the Figures which remain on the right hand are Decimals of the lowest Denomination that you have reduced the given Decimal to; an Example or two will make this Rule very plain.

1. *Example.*

What is the Value of .845 *l.* Sterling? *Facit* 16 *s.* 10 *d.* 3. *qrs.*.2

First

First I multiply the given Decimal by 20 (the Shillings in a Pound) and the Product is 16900, which consisteth of 5 places, and the given Decimal but of 3, wherefore I cut off 2 Figures on the left Hand,(which here is 16) for Shillings, and then multiply the remaining Figure which is 900 by 12, and the Product is 10800; wherefore (for the reason abovesaid) I cut off 10 for Pence, and multiply the 800 by 4 (the Farthings in a Penny) and the Product is 3200, therefore I cut off the 3 for Farthings, and the 200 that stands on the right Hand is the Decimal of a Farthing; so that the Value of the given Decimal is 16 *s.* 10*d.* 3*qrs.*.200. See the Operation.

```
     845
      20
s. ———
  16|900
      12
d. ———
  10|800
       4
f. ———
   3|200
```

2. *Example.*

What is the Value of .7458 *l.* Troy Weight?

oz. p.w. gr.
facit 8—18—23.808.

Multiply by 12, by 20, and by 24, which are the Subdivisions of a Pound Troy Weight, obser-

ving

ving to cut off the Figure on the left hand of every Product as in the First Example,and you have your desire. See the following Work.

```
      .7458
         12
  oz.------
     8|9496
         20
  ---------
    18|9920
         24
  ---------
      39680
      19840
  ---------
    23|8080
```

10. But if you are to reduce a Decimal Fraction into a low Denomination, that is, into a Denomination having one or more Denominations between it and the Decimal, you had best to do it by the Denominator of the least Parts; as for Example:

Let it be required to find the Value of .0125 *l.* in Pence; here I consider, that in a Pound there are 240 Pence, wherefore multiply .0125 by 240, and you will find its Value to be 3 *d.* See the Work following.

$$
\begin{array}{r}
.0125 \\
240 \\
\hline
5000 \\
250 \\
\hline
\textit{facit}\ 3.0000
\end{array}
$$

11. But if you either make Tables as I have directed, or make uſe of the Tables here made, to expreſs the Value of any Decimal Fraction, you muſt do it thus;

If the Queſtion be in the firſt Table of Shillings, Pence, or Farthings, firſt ſeek out what Shillings are intimated by the 2 firſt Figures of the Decimal, and ſetting down the Shillings expreſſed, ſubſtract the Decimal ſignifying the ſaid Shillings from the Decimal whoſe value you ſeek, and with the Remainder enter the Table containing the Decimals of Pence, *&c.* As for Example,

I find by the firſt Table that .35 ſignifieth 7 *s.* and 0375 ſignifieth 9 *d.*

Alſo .78958 ſignifieth 15 *s.* $9\frac{1}{2}$ *d.*

The Decimal propoſed	.78958
The Decimal of 15 *s.* ſubſt.	.75
	.03958
The Decimal of the ſubſt.——	.0375
Rem. of the Decim. of 2 farth.	00208

Likewiſe

l. *s.* *d.*

Likewise .55312 is the Decimal of 11 $00\frac{3}{4}$.
.55 = 11 Shillings

.00312 = 3 Farthings.

l. *l.* *s.* *d.*

Also 32.5785 is equal to 32 — 11 — $06\frac{3}{4}$
.55 = 11 Shillings.

.0285
.025 = 06 Pence.

.0035 = 3 Farthing.

For the better understanding of Substraction of Decimals consult the fourth Chapter following.

But if you use the second Table, then you are to account the Integers Shillings, reducing them to Pounds, halfing the Sum, and taking out the Pence ard Farthings answering the Decimals.

s. *l.* *s.* *d.*

So 3257.982 is the Decimal of 162—17—$11\frac{3}{4}$
And 13250.416 is the Decim. of 662—10—05
Likewise 3051.458 is the Dec. of 152—11—$05\frac{1}{2}$

If the Question be of Troy Weight, then work as before, first with Ounces, and then with penny Weights, and Grains, as 32.*l.*8166 Troy, is 32 *l.* 09 *oz.* 16 *p. w.*

.8166
79 = 9 *oz.*

.0266 = 16 *p.w.*

CHAP. III.

Addition of Decimals.

1. THere is little or no difference between Addition of Decimals and Addition of Integers, only in setting down of the Numbers; where note, that as in Integers Unites are set in one File, Tens in another, &c. so in Decimals you are to place Primes in one File, Seconds in another, &c. And then the Operation is in every Respect the same with that of Addition in Integers; as in the following Example where it is required to add ·347 to .4865, the given Decimals are to be placed

Thus $\left\{\begin{array}{l}.347\\ .4865\end{array}\right.$ *Not thus* $\left\{\begin{array}{r}.347\\ .4865\end{array}\right.$

Sum .8335

2. If when you have added your Decimals together you are to carry any Tens from the Primes, they are to be added to the Integers; if the Numbers given to be added are mixt Numbers, otherwise they are to be placed for Integers; as in the following Examples.

·7458

.7458	.9	.83574
.634	.74	.9765
.79857	.689	.843
.48	.9847	.79
		.9
Sum 2.65837	3.3137	4 34524

Examples of Addition of mixt Numbers.

3542.12	39.0123	9.4
492.9125	7.51	48.07
91.781	918.925	164.983
5189.8921	32.7891	4390.8634
ſum 9316.7056	998.2364	4613.3164

3. But here you are diligently to obſerve that the Decimals to be added be of one Denomination, or ſo reduced before any Operation can be performed; As for Example, Suppoſe you were to add .759 *l.* and 387 *s.* together, it would be abſurd to offer to perform the Work before the Fractions are reduced to one Denomination, which to do, this is the Rule, *viz.* According to the 5*th.* Rule of Chap. 2. Multiply .759 *l.* by 20, and the Product giveth 15.180 Shillings, and now are the given Fractions both of one Denomination, *viz.* both Decimals of a Shilling and the Work of Addition may be performed as followeth.

15.180
.387
Sum 15.567

So that the Sum of the given Decimals is 15.*l*.569 Shilling. The like is to be obſerved in any other Decimals given to be added.

CHAP. IV.

Subſtraction of Decimal Fractions.

1. THere is no Difference between Subſtraction of Integers, and that of Decimals and mixt Numbers, if you obſerve to place Unites under Unites, *&c.* Primes under Primes, Seconds under Seconds, *&c.* placing the biggeſt Number uppermoſt, as in the following Examples.

From	134.5780	538275.00019
Subſtract	12.5812	4638.0057
Rem.	121.9968	533636.99449

2. If the decimal Fractions have not an equal Number of Places, the Vacancies are to be ſupplied with Cyphers, or to be underſtood to be ſo ſupplied, eſpecially in the upper Number.

Let it be required to ſubſtract 43.6374 from 472, four Cyphers are to be annexed to the greater; as you ſee in the following Work.

from

from	472.0000
ſubſtract	43.6374
Rem.	428.3626

Or to be ſuppoſed to be ſo annexed; thus,

from	472
ſubſtract	43.6374
Remains	428.3626

More Examples follow.

From	65.2	*Or*	65.200	532.64
ſubſtr.	48.375	*thus*	48.375	214
Rem.	16.825		16.825	318.64

3. As in Addition, ſo in Subſtraction the Decimals given muſt be of one Denomination, or ſo reduced before Subſtraction can be made, as in the following Example.

Let it be required to ſubtract .947 *s.* from .4865 *l.*

In this Example the Decimal from whence Subtraction is to be made is the Deçimal of a Pound, and the Number to be ſubtracted is the Decimal of a Shilling, therefore before Subſtraction can be made, you muſt multiply .4865 *l.* by 20 and the Product will be 9.7300 Shillings, and then making Subtraction, the Remainder will be 8.783 Shillings, as by the following Operation appeareth.

```
                 l.
              .4865
                 20
             ------
   from 9|7300
subtract  .947
         ------
remains 8.783 Shillings.
```

CHAP. V.

Multiplication of Decimal Fractions.

1. IN Multiplication of Decimals, the Multiplier is to be placed under the Multiplicand in every respect as if they were all Integers, without any respect to them as Decimals or mixt Numbers, then let the Operation be performed in like manner as in whole Numbers, which being finished the Value of the Product is discovered by this general *Rule.*

Look how many decimal Places there are in the Multiplicand and Multiplier, so many decimal places there must be in the Product.

Examples.

Examples.

238.645	.7538
821.75	.5689
1193225	67842
1670515	60304
238645	45228
477290	37690
1909169	.42883682
Prod. 196106 52875	

In the first of these Examples 5 Decimals are cut off, for so many decimal Places there are in the Multiplicand and Multiplier, the like in the second Example, the Number of Places being 8.

2. But if after Multiplication is ended, the Product consist not of as many Places as there are decimal Places in the Multiplicand and Multiplier, then must you supply the defect of the Product by prefixing Cyphers; as in these following Examples,

Ex. 1.	*Ex.* 2.	*Ex.* 3.
.0023	.005	123.01
.0003	.03	.0021
.00000069	.00015	123.01
		24602
		.258321

In

In the first of these Examples, after I have finished Multiplication I find in the Product 69, but it ought by the general Rule to consist of 8 Places, for so many there are In the Multiplicand and Multiplier, therefore to make them up 8 Places, I prefix to 69 six Cyphers, and then there is .00000069 or $\frac{69}{100000000}$ for the true Product.

The same is to be understood of the second Example.

In the third Example, though the Multiplicand be a mixt Number, yet is not the Product, because in both the *Factors* there are 6 decimal Places, and the *Factus* or Product hath 6 Places, and therefore it is a pure Decimal; and if the Multiplicand had been 23.01 and the Multiplier the same as before, *viz.* .0021 then would the Product have been ,048321.

The Consideration of this Practice will be of great Use in Division of Decimals.

CHAP. VI.

Division of Decimal Fractions.

1. Decimal *Parts*, whether mixt or pure, are to be divided, as to the manner of the Work as if they were whole Numbers, the main Difficulty lying in Discovering the Value of the Quotient, after Division is ended. Now to know what

what Number of *Decimal Parts* there are in the Quotient, observe this *Rule* ;

That there must be as many places of Decimal Parts in the Divisor and the Quotient as there are in the Dividend.

The Dividend being in Effect the Product ; and the Divisor and the Quotient the Multiplicand and Multiplier.

2. Therefore when it is required to divide a pure Decimal, or a mixt Number, by a Decimal or mixt Number, be sure there be more decimal Places in the Dividend (or made so by annexing Cyphers) than in the Divisor, and so much as the Excess is so many decimal Places will there be in the Quotient ; and if the decimal Places in the Dividend and the Divisor are equal, the Quotient will be wholly integral.

The foregoing Rule will easily be understood by a few Examples.

1. *Example.*

Let it be required to divide 774.43692 by 28 64, the Quotient will be 27.040. See the following Work.

28.64)

```
28.64) 774.43692 (27.040
        572 8
        -------
        201 63
        200 48
        -------
         11569
         11456
         -----
        (1132)
```

According to the Rule there being 5 Decimal Places in the Dividend, and 2 in the Divisor, there must be 3 in the Quotient, for you must make up the Places of the Decimals of the Quotient and the Divisor equal to that of the Dividend; and if you had made one decimal Place more in the Dividend by annexing of a Cypher thereto, then the Quotient would have been 27.0404 as you may prove at your Leisure.

And if there be no decimal Places in the Dividend, but only in the Divisor, add as many Cyphers to the Dividend as you please, be sure as many at least, or more than there be decimal Places in the Divisor, and work as before.

2. *Examp.* *Divide* 32 *by* 5.71

```
5.71) 32.00 (5
      28 55
      -----
       345
```

So

So that here the Quotient is 5 Integers because the decimal Places in the Dividend and Divisor were equal; but if you had annexed 3 Cyphers to the Dividend, the Quotient would have been 5.6, &c.

3. *Example.*

Divide 32 by 5.71, the Quotient will be 56.04, &c.

In the Divisor there are 2 decimal Places and none in the Dividend, therefore I annex 5 Cyphers thereto, and then there will be three decimal Places in the Quotient, because the Excess of decimal Places in the Dividend above that of the Divisor is three. See the Operation.

```
5.71) 32.00000 (5.604
          • •:•
      28 55
      ------
       3 450
       3 426
       ------
         2400
         2284
         -----
         (116)
```

3. And if all the decimal Places in the Quotient and the decimal Places in the Divisor will not amount to as many as there are decimal Places in the Dividend, then after Division is made, place Cyphers before the Figures in the Quotient until the Rule be made good, and the decimal Places compleated.

4. *Examp.*

4. *Example.*

What is the Quotient of 7.7443692 being divided by 286.4? *facit* .027040.

In this Example there is but one decimal Place in the Divisor, and 7 in the Dividend, therefore there must be 6 Decimals in the Quotient; but when Division is ended there is but 5 Figures in the Quotient, *viz.* 27040, wherefore to compleat them I prefix a Cypher thereto, and it makes .027040, which is the true Quotient. See the Work.

```
286.4) 7.7443692 (.027040
         5 728
         -----
         2 0163
         2 0048
        -------
          11569
          11456
          -----
          (1132)
```

5. *Example.*

Divide .77443692 by 2864. *facit* .00027040.

In this Example the Dividend and Divisor consist of the same Figures with the fourth Example, but only the decimal places differ, there being 8 Decimals in the Dividend, and not one in the Divisor, therefore must there be 8 decimal Places in the Quotient; but after Division is ended, the Quotient is the same with the Quotient in the last Example,

Example, *viz.* (27040) consisting of 5 Places, wherefore to compleat them 8, I prefix 3 Cyphers thereto and it makes .00027040 for the true Quotient sought. *More Examples of Division in Decimals may be such as follow.*

6. Example.

```
.24) .834696 (3.4779
      72
      114
       96
       186
       168
        189
        168
         216
         216
         (0)
```

7. Example.

.0024) .834696 (347.79

8. Examp.

8. *Example.*

```
.046) 54.07824 (1175.64
      46
      --
       80
       46
       ---
       347
       322
       ---
        258
        230
        ---
         282
         276
         ---
          064
           46
          ---
          (18)
```

And thus much ſhall ſerve for Diviſion of decimal Fractions.

I might proceed to give ſome Examples of Decimals in the *Golden Rule*, *&c.* but theſe being ſufficient for the meaneſt Capacity, I ſhall neglect that as altogether needleſs.

CHAP. VII.

Of Compound Intereſt.

AT the End of this Book there are certain Tables for Reſolving all Queſtions concerning Intereſt of Money, and Annuities, made at the ſeveral Rates *per Centum*, *per Annum* therein mentioned.

Now we ſhall conſider theſe Tables in their Conſtruction, Explication, and Uſe, and firſt of their Conſtruction.

1. The Increaſe of Uſe upon Uſe is after a Geometrical Proportion continued, therefore the firſt Table is nothing elſe but the Continuing of the Terms in a Geometrical Progreſſion for each Year, as 108 for the firſt year, 116'64, for the ſecond, *&c.* But to do it for Half years and Quarters, you muſt find a mean proportial betwixt 100. 108. by taking the ſquare Root of 10800, which is 103'923, and betwixt 100 and 103,923, which is 101,942, *&c.* now having the two firſt Terms 100. 101'942, *&c.* you are to continue it out for quarters of years. What is ſaid here for 8 *per C.* is done for the reſt of 6 *per C.* 7 *per C. &c.* The ſecond Table is a continual Proportion decreaſing. The third Table increaſing: and thus much for the *Conſtruction*, now to the *Explanation*.

2. The firſt Table gives the Intereſt of 1 *l.* after the rates 5, 6, $6\frac{1}{4}$, $6\frac{1}{2}$, 7, $7\frac{1}{7}$, $7\frac{9}{13}$, 8 *per C.* *As for Example*, the Uſe of 1 *l.* forborn for 7 years is

1|7138, that is 1 *l.* 14 *s.* 3 *d.* *per Centum*, for every year to 30 years: *As for Example*, the Use of 100 *l.* for half a year is 103'923, which by the first Table of the Decimal of Pounds (to be used in this Work) is 103 *l.* 18 *s.* 6 *d.* almost, and not 104 *l.* as is usually taken. The same Table gives you the Use of 10 *l.* if you cut off one Figure after the Comma with your separating Line, and if 100 *l.* of two, *&c.* so it will of 1000 *l.* and this is to be observed as well in this as in the rest of the Tables that if the *Principal*, whose *Interest* you seek, be above 1 *l.* you remove the Comma or *separating* Line. Accordingly the second Table sheweth what 1. 10. 100 *l.* due at the end of any year or quarter is worth in ready money. The Uses of the rest of the Tables appear by their Superscription, and by the Uses following.

1. *The Use of the first Table.*

Prop. 1. *To know what the Interest of any Sum of Money put out for any Number of years (according to the Rates in the Tables) will amount unto at the end thereof.*

Multiply the Sum propounded by the Number in the Tables (observing the Rules for Decimals) and the Product is the Answer. For as 1. is to the Tables Number, so is the Sum proposed, to the Answer. And this Rule is to be observed for all the Tables.

Example.

Example.

If 842 *l.* 6 *d.* be put out for 3 years at Compound Intereſt, what is it worth? or what is the Principal and Gain after 8 *l. per Cent.*? Anſwer, 1060 *l.* 14 *s.* 2 *d.*

```
       1|_5971   The Table Number an-
     842|025       ſwering to 3 years.
    ---------
       63.....
      252....
    25194...
   503884..
  10077680.
  ---------
  1060|7073
```

If the Queſtion be for Years and Months, firſt ſeek the Intereſt for Years, and then the Months.

Example.

At 6 *per Cent.* what is the Intereſt of 72 *l.* 12 *s.* after 5 years and 5 months? Anſw. 99 *l.*--10--8.

Firſt I take the Tab. Number for 5 years 1,33822, and multiply it by 72|6 it poduceth the Intereſt 97|1647 for 5 years, the which I multiply by the Tab. Numb. for 5 months, *viz.* 1|014576 it produceth 99|5347.

Oper.		
	1|33822	97|1547
	72|6	1|0245
	802932	485 | . . . .
	267644	3884 | . . .
	936754	19431 | . .
	97|154772	971547 | .
		99|5347

2. *The Use of the second Table:*

Prop. 2. *To know the Value of any Sum of Money which is due after a Number of Years or Months.*

The Work is done as before; as for Example, in rates of 8 *per C.*

Quest. 1. What is the present Worth or Value of 750 *l.* 4 *s.* 3 *d.* due at the end of 7 years? Answ. 430 *l.* 10 *s.*

Quest. 2. What is the present Value of 81 *l.* 2 *s.* due at 9 years hence and $\frac{1}{2}$? Answ. 39 *l.* 0 *s.* 9 *d.*

Quest. 3. What is the present Value of 5 *l.* 11 *s.* 6 *d.* due at the end of two years 3 quarters in ready Money? Answ. 4 *l.* 10 *s.* 0 *d.*

3. *The Use of the third Table.*

Prop. 3. *To know the Value of any Annuity or yearly Payment in ready Money.*

1. *Example.*

There is an Annuity of 6 *l.* 10 *s.* 4 *d.* *per annum*, to endure for 12 years, what is it worth in ready Money?

Money? Anſw. 49 *l.* 2 *s.* 2 *d.* after 8 in the *C.*

2. *Example.*

What is a Leaſe of 142 *l.* 6 *s.* the year, to endure for 7 years, worth in ready Money? Anſwer, 739 *l.* 11 *s.* 3 *d.* after 8 in the *C.*

And in all theſe Propoſitions the Queſtions may put on the Principal or any of the other Terms, for becauſe in the firſt Propoſition it was put thus; If 1 *l.* Principal come to 1|259 in 3 years, what ſhall 842|025 Principal come to? Anſw. 1060|7089 by Multiplication as before.

Or: If 1|259 come of 1 *l.* Principal, what did 1060|7089 come of? Anſ. 842|025 Principal; which Variety may be obſerved in the reſt.

Many other Concluſions ariſe from theſe Tables, but becauſe I work them far more eaſie by the *Logorithms* in the Tract thereof following, I ſhall ſay no more thereof.

I. *Table of* Compound Interest, *shewing the Use*

Year.	5 *in* C.	6 *in* C.	$6\frac{1}{4}$ *in* C.	$6\frac{2}{3}$ *in* C.	7 *in* C.
1	1,05000	1,06000	1,06250	1,06666	1,07000
2	1,10250	1,12360	1,12890	1,13778	1,14490
3	1,15762	1,19016	1,19946	1,21363	1,22504
4	1,21551	1,26247	1,27443	1,29454	1,31078
5	1,27628	1,33822	1,35408	1,38084	1,40255
6	1,34009	1,41852	1,43871	1,47289	1,50073
7	1,40710	1,50363	1,52863	1,57109	1,60578
8	1,47745	1,59385	1,62417	1,67583	1,71818
9	1,55133	1,68948	1,72568	1,78755	1,83845
10	1,62889	1,79085	1,83353	1,90672	1,96715
11	1,71034	1,89830	1,94813	2,03383	2,10485
12	1,79586	2,01219	2,06989	2,16942	2,25219
13	1,88565	2,13293	2,19926	2,31405	2,40984
14	1,97993	2,26090	2,33671	2,46832	2,57853
15	2,07893	2,39656	2,48275	2,63288	2,75903
16	2,18287	2,54035	2,63793	2,80840	2,95216
17	2,29203	2,69277	2,80280	2,99563	3,15881
18	2,40662	2,85434	2,97797	3,19534	3,37993
19	2,52695	3,02559	3,16409	3,40836	3,61652
20	2,65329	3,20713	3,36185	3,63558	3,86968
21	2,78596	3,39956	3,57197	3,87795	4,14056
22	2,92526	3,60353	3,79522	4,13649	4,48040
23	3,07152	3,81975	4,03242	4,41225	4,74052
24	3,22510	4,04893	4,28444	4,70641	5,07236
25	3,38636	4,29187	4,55222	5,02017	5,42743
26	3,55567	4,54938	4,83673	5,35484	5,80735
27	3,73346	4,82234	5,13903	5,71183	6,21376
28	3,92012	5,11168	5,46022	6,0926	6,64883
29	4,11613	5,41839	5,80148	6,49879	7,11425
30	4,32194	5,74349	6,16408	6,93205	7,61225

Year	$7\frac{1}{7}$ in C.	$7\frac{9}{13}$ in C.	8 in C.
1	1,07143	1,0769	1,08000
2	1,14796	1,15976	1,16640
3	1,22995	1,24897	1,25971
4	1,31781	1,34505	1,36049
5	1,41194	1,44851	1,46933
6	1,51279	1,55994	1,58657
7	1,62085	1,6799	1,71282
8	1,73662	1,80916	1,85093
9	1,86066	1,94832	1,99900
10	1,99357	2,09819	2,15893
11	2,13597	2,25959	2,33164
12	2,28854	2,43341	2,51817
13	2,45201	2,62000	2,71962
14	2,62715	2,82218	2,93719
15	2,81480	3,03927	3,17217
16	3,01586	3,27306	3,42594
17	3,23128	3,52484	3,70002
18	3,46201	3,79598	3,99602
19	3,70938	4,08798	4,31570
20	3,97433	4,40244	4,66096
21	4,25821	4,74109	5,03383
22	4,5623	5,10579	5,42654
23	4,88826	5,49854	5,87146
24	5,374 9	5,92151	6,34119
25	5,61152	6,37701	6,84848
26	6,01234	6,86755	7,39635
27	6,44179	7,39582	7,98806
28	6,90192	7,96473	8,62711
29	7,39492	8,57740	9,31727
30	7,92313	9,23720	10 06266

Mon.	5 in C.	6 in C.	$6\frac{1}{4}$ in C
1	1,0040	1,00486	1,00506
2	1,00816	1,06058	1,01015
3	1,01227	1,01467	1,01527
4	1,01639	1,01961	1,02041
5	1,02054	1,02457	1,02558
6	1,02469	1,02956	1,03078
7	1,02887	1,03457	1,03599
8	1,03306	1,03961	1,04124
9	1,03717	1,04467	1,04651
10	1,04149	1,04975	1,05182
11	1,04574	1,05486	1,05714

Mon.	$6\frac{3}{5}$ in C.	7 in C.	$7\frac{1}{7}$ in C
1	1,00539	1,00565	1,00576
2	1,0 081	1,01134	1,01156
3	1,01626	1,01705	1,01739
4	1,02174	1,02281	1,01376
5	1,02725	1,02859	1,02916
6	1,03279	1,03441	1,03509
7	1,03836	1,04025	1,04106
8	1,04396	1,04614	1,04707
9	1,04959	1,05205	1,05311
10	1,05525	1,05800	1,05917
11	1,06094	1,06398	1,06528

Mon.	$7\frac{9}{13}$ in C	8 in C.
1	1,00619	1,00643
2	1,01243	1,01292
3	1,01869	1,01943
4	1,02501	1,02595
5	1,03136	1,03150
6	1,03775	1,03925
7	1,04417	1,0459
8	1,05064	1,05265
9	1,05715	1,0594
10	1,06370	1,06623
11	1,07029	1,07309

II. *Table of* Compound Intereſt diſcounted, or *what any Sum of Money at the end of any*

Year.	5 *in* C	6 *in* C.	$6\frac{1}{4}$ *in* C.	$6\frac{2}{5}$ *in* C.	7 *in* C.
1	,95238	,94339	,94117	,93750	,93458
2	,90703	,88999	,88581	,87890	,87344
3	,86384	,83962	,83371	,82397	,81629
4	,82272	,79209	,78466	,77247	,76289
5	,78353	,74726	,72851	,72419	,71299
6	,74621	,70496	,69507	,67893	,66634
7	,71068	,66505	,65418	,63650	,62274
8	,67684	,62741	,61570	,59672	,58201
9	,64461	,59185	,57948	,55942	,54393
10	,61391	,55839	,54539	,52446	,50835
11	,58468	,52678	,51331	,49168	,47509
12	,55684	,49697	,48312	,46095	,44401
13	,53032	,46884	,45470	,43214	,41496
14	,50506	,44230	,42795	,40513	,38781
15	,48102	,41726	,40278	,37981	,36244
16	,45811	,39364	,37904	,35607	,33874
17	,43629	,37136	,35678	,33382	,31657
18	,41552	,35035	,33580	,31295	,29586
19	,39573	,33051	,31604	,29339	,27650
20	,37680	,31180	,29745	,27506	,25842
21	,35894	,29415	,27995	,25786	,24151
22	,34185	,27750	,26349	,24175	,22571
23	,32557	,26179	,24799	,22664	,21094
24	,31006	,24698	,23340	,21247	,19715
25	,29530	,23290	,21967	,19919	,18424
26	,28124	,21981	,20675	,18674	,17419
27	,26785	,20736	,19459	,17507	,16093
28	,25509	,19563	,18314	,16413	,15040
29	,24295	,18455	,17237	,15387	,14056
30	,23138	,17411	,16223	,14425	,13136

Year.	$7\frac{1}{7}$ in C.	$7\frac{9}{13}$ in C.	8 in C.
1	,93333	,92857	,92593
2	,87111	,86224	,85734
3	,81303	,80065	,79383
4	,75883	,74346	,73503
5	,70824	,69036	,65058
6	,66103	,64105	,63017
7	,61696	,59526	,58349
8	,57583	,55274	,54026
9	,53744	,5[illegible]326	,50025
10	,50161	,47659	,46319
11	,46817	,44255	,42888
12	,43696	,41094	,39771
13	,40784	,38159	,36769
14	,38064	,35433	,34046
15	,35526	,32902	,31524
16	,33158	,30552	,29189
17	,30947	,28370	,27026
18	,28884	,[illegible]6343	,25025
19	,[illegible]6958	,24462	,23171
20	,25161	,22715	,21455
21	,23484	,21092	,19866
22	,21918	,19585	,18394
23	,20457	,18186	,17031
24	,19093	,16887	,15770
25	,17820	,15681	,14602
2[illegible]	,1663[illegible]	,14561	,13530
27	,15523	,13521	,12518
28	,14488	,12555	,11591
29	,13523	,11658	,10733
30	,12621	,10826	,09937

Mon	5 in C.	6 in C.	$6\frac{1}{4}$ in C.
1	,99594	,99515	,99496
2	,99190	,99033	,98995
3	,98787	,98554	,98496
4	,98387	,98076	,97999
5	,97687	,97901	,97505
6	,97194	,966581	,97014
7	,96799	,961898	,96525
8	,96407	,957239	,96039
9	,96016	,952602	,95555
10	,95626	,947988	,95073
11	,	,	,94594

Mon	$6\frac{2}{5}$ in C.	7 in C.	$7\frac{1}{7}$ in C.
1	,99493	,99437	,99426
2	,98936	,98878	,98856
3	,98399	,98322	,98290
4	,97872	,97770	,97766
5	,97347	,97220	,97166
6	,96824	,96674	,96609
7	,96305	,96130	,96055
8	,95788	,95585	,95505
9	,95275	,95051	,94957
10	,94203	,94518	,94412
11	,94255	,93986	,93871

Mon	$7\frac{9}{13}$ in C.	8 in C.
1	,99384	,99361
2	,98772	,90725
3	,98164	,98094
4	,97560	,97467
5	,96959	,96844
6	,96362	,96225
7	,95769	,95609
8	,95179	,94998
9	,94593	,94396
10	,94011	,93788
11	,93432	,93888

III. *Table, shewing what an Annuity is worth*

	5 *in* C.	6 *in* C.	$6\frac{1}{4}$ *in* C.	$6\frac{2}{5}$ *in* C.	7 *in* C.
1	,95238	,94339	,94117	,93790	,93457
2	1,85941	1,83339	1,82698	1,81641	1,80802
3	2,72325	2,67301	2,66069	2,64038	2,62431
4	3,54595	3,46510	3,44536	3,41285	3,38721
5	4,32947	4,21236	4,18387	4,13705	4,10019
6	5,07569	4,91732	4,87894	4,81599	4,76653
7	5,78637	5,58238	5,53311	5,45249	5,38929
8	6,46321	6,20979	6,14881	6,04921	5,97129
9	7,10782	6,80169	6,72829	6,60863	6,51523
10	7,72173	7,36008	7,27369	7,13309	7,02358
11	8,30641	7,88687	7,78700	7,62477	7,49867
12	8,86325	8,38384	8,27012	8,08573	7,94268
13	9,39357	8,85268	8,72482	8,51787	8,35765
14	9,89864	9,29498	9,15277	8,92300	8,74546
15	10,37965	9,71224	9,55555	9,30281	9,10791
16	10,83777	10,10589	9,93463	9,65889	9,44665
17	11,27406	10,47725	10,29142	9,99270	9,76322
18	11,68958	10,82760	10,62722	10,30566	10,05908
19	12,08532	11,15811	10,94326	10,59906	10,33559
20	12,46221	11,46992	11,24072	10,87411	10,59401
21	12,82115	11,76407	11,52068	11,13198	10,83553
22	13,16300	12,04158	11,78417	11,37373	11,06124
23	13,48857	12,30337	12,03216	11,60037	11,27218
24	13,79864	12,55035	12,26556	11,81285	11,46933
25	14,09394	12,78335	12,48523	12,01205	11,65358
26	14,37518	13,00316	12,69199	12,19879	11,82577
27	14,64303	13,21053	12,88657	12,37387	11,98671
28	14,89813	13,40616	13,06971	12,53800	12,13711
29	15,14107	13,59072	13,24208	12,69188	12,27767
30	15,37244	13,76483	13,40431	12,83613	12,40901

in ready Money.

	$7\frac{1}{7}$ *in* C.	$7\frac{9}{13}$ *in* C.	8 *in* C.
1	,93333	,92857	,92592
2	1,80444	1,79082	1,78326
3	2,61748	2,59147	2,57711
4	3,37632	3,33493	3,31212
5	4,08456	4,02530	3,99271
6	4,74559	4,66635	4,62288
7	5,36255	5,26161	5,20637
9	5,93838	5,81435	5,74663
8	6,47582	6,32761	6,24688
10	6,97743	6,80421	6,71008
11	7,44560	7,24676	7,13896
12	7,88256	7,65771	7,53608
13	8,29039	8,03930	7,90377
14	8,67103	8,39364	8,2[illegible]433
15	9,02629	8,72266	8,55948
16	9,35787	9,02819	8,85137
17	9,66735	9,31189	9,12164
18	9,95619	9,57533	9,37188
19	10,22578	9,81994	9,60359
20	10,47739	10,04709	9,818[illegible]5
21	10,71232	10,25801	10,01680
22	10,93142	10,45387	10,20075
23	11,13599	10,63573	10,37106
24	11,32692	10,[illegible]8461	10,52876
25	11,50513	10,96142	10,67[illegible]77
26	11,67145	11,10704	10,80997
27	11,82669	11,24225	10,93516
28	11,97157	11,36780	11,05108
29	12,10681	11,48439	11,15841
30	12,23302	11,59264	11,25778

The End of the Second Book.

MOOR'S ARITHMETICK; BOOK III.

SHEWING the USE of the

Logarithms

In a far more easie Manner than formerly: As also general Rules by them for *Compound Interest*, *Annuities*, *&c.* at any Rate desired.

Revised and Augmented

By *JOHN HAWKINS*, Schoolmaster at St. *George's* Church in *Southwark*.

LONDON,

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Sect. 1. Of the Logarithms.

The Canon of Logarithms *is in every Man's Hand, but their perfect Use in* Decimal *Fractions known to a few: Therefore what Mr.* Oughtred *hath briefly delivered in his* Clavis *concerning them and Compound Interest with Annuities, I have amplified with Examples, being of singular Use and speedy Performance.*

1. THE Logarithm of any Number is found out in the Canon thus; under the Title *N.* seek out the Number proposed, and the Number of Figures answering under the Tittle of *Log.* is the Logarithm desired.

Example.

The Logarithm of 34 is 1.53148, and of 3454 is 3.53832.

The first Figure of every Logarithm which is separated from the rest by a Point, may fitly be termed the *Index*, because it sheweth how many Places the Integer signified by such Logarithm consisteth of, *viz.* it sheweth how far it is distant from Unity: Therefore whether the Number proposed be Whole, Mixt, or Decimal, find out the Logarithm to it as if it were a whole Number, and then according to the Distance from Unity of the first

first Figure prefix an *Index* to the Logarithms found, which always differ as the Nature of your Number doth, though the Logarithms may be the same, as in the Examples following.

Numb.	*Logarithms.*
3571.	3.55279
35'71	1.55279
3'571	0.55279
'003571.	$\overline{3}$.55279.

Note that the same *Logarithms* serve, but the *Index* alters according to the Nature of the first Figure of the Number; for in the first the Figure 3 is the third from Unity, therefore the *Index* of the *Log*. is 3, but in the last it is $\overline{3}$ negative because Decimals.

2. If the Logarithm be given, and the Number answering it be desired, seek among the Logarithms where the Index is greatest (neglecting for the most part the Index of the *Log.* proposed) for it, and set down the Number answering, which must be ordered according to its own Index, what *Integers*, *Decimals*, or *Mixt* must be kept?

As for Example.

Suppose the *Log*. 2.57825 were proposed to find its corresponding *Number*, I look for this *Log*. under the Index 4, if my *Canon* give leave, and find the Number 37866, which according to the Index 2 is 378.66.

Likewise the Number answering 1.64345 is 44000, which according to the Index 1, is 44 Integers.

Likewise

Likewise the Number anſwering this Logarithm

$\bar{2}$72028 is ‘052515
to—$\bar{3}$72028 is ‘0052515
to—$\bar{1}$72028 is ‘52515

Thus much concerning the finding of the *Logarithms* of Numbers.

3. To add two Logarithms together, or ſubtract one from another, is the ſame with Multiplication and Diviſion in vulgar Arithmetick, and in it there is no Difficulty if the Indices be not Negative.

As for Example.

Add	2.57832	Sub.	2.57832
	1.62517		1.62517
	4.20349		0.95315

But if the Indices be Negative, work them as was taught before, changing the Nature of the lower in Subſtraction; but remember, that if in Addition you carry any Tenths they are Affirmative, but in Subſtraction if you borrow one, account the higher one leſs in Value by Negation, as if it be 2 account it 1, if $\bar{2}$ account it $\bar{3}$.

Addition.

	(1st.)	(2d.)	(3d.)
	2.05782	$\bar{1}$.39794	2.15836
	$\bar{3}$.58321	$\bar{1}$.87506	$\bar{1}$.87506
Log. of Product.	$\bar{1}$.64103	$\bar{1}$.27300	2.03342

	(4th.)	(5th.)
	$\bar{1}$.87506	2.23724
	2.69897	$\bar{1}$.87506
		2.69897
Logarithm of the Product.	2.57403	0.81127

Note that when the Indices are homogeneal, *viz.* both Negative, or both Affirmative, they are to be added together, but if heterogeneal the lesser must be subtracted from the greater and the Remainder marked with the Sign of that wherein the Excess lieth.

As in the first of these Examples there is given 2.05782 the Logarithm of 114.24 to be added to $\bar{3}$.58321 which is the Logarithm of .00383, the Characteristick of the first, *viz.* 2. being affirmative, and of the second, *viz.* $\bar{3}$. Negative. The Work is the same as in common Addition, till you come to the Characteristicks; and there, because the Signs are different, substract the lesser from the greater, giving the Remainder the Sign of that Index wherein lieth the Excess; so here, because 2 is subtracted from $\bar{3}$. the Remainder 1 must be made

made $\overline{1}$. (negative.) And so the Sum of these Logarithms is $\overline{1}$.64103, which is the Logarithm of .4375, *&c.* and is the Product of 114.24 multiplied by .00383.

And in the second Example, where both the Indices are $\overline{1}$. negative, they must be added together, and their Sum is $\overline{2}$. but the 1 that is carried for the 10 is affirmative and maketh the said $\overline{2}$. to be $\overline{1}$.

The like is to be understood in the 4*th.* Example where the Sum of the negative Indices is $\overline{3}$. but made less by 1 for the Ten that is carried.

Substraction.

	(1*st.*)	(2*d.*)
	2.03342	$\overline{1}$.87506
	$\overline{1}$.87506	2.03342
Log. of Quot.	2.15836	$\overline{3}$.84164

(3*d.*)	(4*th.*)
$\overline{1}$.87506	1.23578
$\overline{2}$.57403	3.57214
1.30103	$\overline{3}$.66364

In the first of these Examples the Indices are 2. and $\overline{1}$. the uppermost being affirmative and the lowermost negative; wherefore for the 1. that is borrowed in the place next the Indices, the uppermost Index 2. is made 1. and the Sign of the lowermost being changed, *viz.* of $\overline{1}$. being made 1. and then added to the uppermost 1. the Sum is 2. for the Index of the Remainder.

The Indices of the second Example are $\bar{1}$. the uppermost, and 2. the lowermost; wherefore the Sign of the lowermost being changed, and made $\bar{2}$. and then being added to the uppermost gives $\bar{3}$. for the Index of the Remainder.

The Indices of the third and fourth Examples are homogeneal, therefore when the Signs of the lowermost are changed, the lesser is subtracted from the greater, as is taught in Addition.

4. To find the Logarithm of a Fraction, substract the *Log.* of the Denominator, from the *Log.* of the Numerator, changing the Index of the lowermost, and then adding the Indices together as is taught in Substraction.

$\frac{3}{4}$	0.47712	$\frac{3}{341}$	0.47712	$\frac{31}{572}$	1.49136
	0.60206		2.53275		2.75739
.75	$\bar{1}.87506$		$\bar{3}.94537$		$\bar{2}.73395$

5. To multiply a Logarithm whose Index is affirmative is no Difficulty; but if the Index be Negative, observe that in Multiplying the next Figure to the Index, the Tens to be born in mind are affirmative, and so many are to be deducted out of the Product of the negative Indices.

(1*st.*)	(2*d.*)	(3*d.*)
1.54321	$\bar{2}.54321$	$\bar{4}.61432$
3	3	4
4.62963	$\bar{5}.62963$	$\bar{14}.45728$

In the first Example the Index is affirmative, and

and therefore the 1 that is carried is added to the Product of the Index by the Multiplier.

In the second Example the Index is negative, and therefore the 1 that is carried to the Product of the Index is deducted therefrom, for $3\times\overline{2}-1=\overline{5}$. the like in the third Example.

6. To divide a Logarithm that hath a negative Index, observe whether the Divisor will evenly divide the Index, then is there no Difficulty, as in these Examples.

2) $\overline{4}$.32578	3) $\overline{9}$.58321	5) $\overline{15}$.32571
$\overline{2}$.16289	$\overline{3}$.19440	$\overline{3}$.06514

But if it do not evenly divide the Index, add to the Index so many Unites till it may be evenly divided, setting the Quotient down for a new Index, keeping those Unites added in mind: augment them by Ten, which Product add to the next Figure, as in the Examples.

(1st.)	(2d.)	(3d.)
3) $\overline{5}$.32141	3) $\overline{7}$.23215	2) $\overline{5}$.61228
2.44047	3.74405	3.80614

In the first of the foregoing Examples the Index is $\overline{5}$. which will not be evenly divided by the Divisor (3) therefore to the Index I add 1 to make it 6, and then the Quotient is $\overline{2}$. then accounting the 1 I added to the said Index to be 10, I add it to the next Figure 3, and it makes 13, which being divided by 3, the Quotient is 4, *&c.*

The fame is to be obferved in the fecond Example, which is augmented with 2 to make 3 divide it evenly, and therefore 20 is added to the next Figure making it 22, &c.

Sect. 2. *The Use of the Logarithms.*

1. *Multiplication.*

Add the Logarithms of the Multiplicand and Multiplier together, the Sum is the Logarithm of the Product, the Index fhewing its nature.

Md.	3.42	2.53402	75‘8	1.87967	3‘42	0.53402
Mr.	13	1.11394	92	1.96378	‘13	$\bar{1}$.11394
P.	4446	3.64796	6973‘6	3.84345	‘4446	$\bar{1}$.64796

‘75	$\bar{1}$.87506	144.	2.15836
‘65	$\bar{2}$.69897	‘75	$\bar{1}$.87506
‘0375.	$\bar{2}$.57403	108‘00	2.03342

172‘68	2.23724	‘037.	$\bar{2}$.56820
‘75	$\bar{1}$.87506	‘008.	$\bar{3}$.90309
‘05	$\bar{2}$.69897	‘000296.	$\bar{4}$.47129
6‘475.	0.81127		

To fquare, cube, fquare-fquare, &c. any Number is to double, triple, quadruple, &c. its *Log.*

N. 32.

N.32.	1.50515.	1.50514	1.50514
	1 *sq.* 2	*Cub.* 3	*sq.sq.* 4
	3.01030	4.51542	6.02056
square is.	1024	*Cub.* 32764	*sq.sq.* 1048448

N.‘024.	$\bar{2}$.38021		$\bar{2}$.38021
	sq. 2		*Cub.* 3
‘000576.	$\bar{4}$.76042	‘0000013824	$\bar{5}$.14063

2. *Division.*

Substract the *Log.* of the Divisor from the *Log.* of the Dividend, the remaining *Log.* is the *Log.* of the Quotient.

Dd.	4446.	3.64796.	4‘446	0.64796
Dr.	342	2.53402	3‘42.	0.53402
Qt.	13.	1.11394	1‘3.	0.11394

Dd.	‘0375	$\bar{2}$.57403	108	2.03342
Dr.	‘05	$\bar{2}$.69897	‘75	$\bar{1}$.87506
Qt.	‘75	$\bar{1}$.87506	144‘00	2.15836

To extract the square Cube and square-square Root, *&c.* is to take half a third Part, *&c.* of the *Log.*

N. 75832.	2) 4.87985	3) 4.87985
	2.43992.	1.62662

square Root 275‘37. Cube Root 42‘327.

'05213.	2) $\bar{2}$.71708	3) $\bar{2}$71708
	$\bar{1}$.35854	$\bar{1}$.57236
	ſq. Root is '2283	Cube Root is '36506.

Sect. 3. *Of* Compound Uſury, *or* Intereſt *upon* Intereſt.

Though the taking and bargaining after this manner of Intereſt be complained of by many who underſtand not what they ſpeak, yet it is eaſie to make it appear that it is far more reaſonable in all *Bargains*, than that which is called *ſimple Intereſt*, for what is more unjuſt than after 8 *per Cent.* for a year, to take 4 *l.* for the Uſe of 100 *l.* for ½ a year, or 2 *l.* for a quarter, when as that 4 *l.* will give in half a year 3 *s.* 2 *d. ob.* But in Leaſes, Reverſions, Annuities, *&c.* it is far more unreaſonable, as divers can by woful Experience witneſs, who compounding according to the Rules of ſimple Intereſt, have paid more for their Tenements, Annuities, Leaſes in Reverſion, *&c.* than they have been really worth.

1. the Intereſt in this Kingdom is rated after ſo much Gain in the 100 *l.* for a year. Whatſoever therefore it is, whether 108 or 106 find out its *Log.* the which, if the Payments be half yearly, or quarterly, or monthly, take its ½, ¼, or 1/12, accordingly; then will your Queſtion fall upon ſome of theſe ſix fundamental Theorems, invented and ſet down by Mr. *Oughtred*, the practice whereof followeth.

1. To

1. To know what ought to be paid for any Sum forborn for any Number of years, *Interest* upon *Interest*, at any Rate propounded.

2. To know what any Sum due after a certain time is worth in ready Money.

3. To know what ought to be paid for an *Annuity* discontinued for any time, at the end of that time.

4. To know what *Annuity* any sum that is due after a certain time will buy.

5. To know what an *Annuity* due for many Payments is worth in ready Money.

6. To know what *Annuity* a present Sum of Money will buy for any time proposed.

2. For the more speed find out the *Log.* of the Rate, which divide by 2. 4. 12. 36. for the parts of the year.

at 8 *per Cent.*		*at* 6 *per Cent.*	
1.08.Log.	0.0334237.	1.06.Log.	0.0253058
$\frac{1}{2}$ year.	0.0167118.	———	0.0126529
$\frac{1}{4}$ year.	0.0083559.	———	0.0063264
Month.	0.0027853.	Month.	0.0021088
Week.	0.0006963.	Week.	0.0005272
Day.	0.0000994.	Day.	0.0000753

Lastly, find the Interest, or resolve the Question by 1 *l.* and add the *Log.* of the Answer to the *Log.* of the Sum propounded.

Question 1.

To know the Interest of any Sum for any time required.

Multiply the Logarithm of the rate by the years or time propounded.

1. *Example.*

1. *Examp.* To know the use of 25 *l.* 8 *s.* for 7 years at 8 *per cent.*

1.08 Log. of the Rate ——— ———	0.03342
	7
The Log. of 1.7138 ——— ———	0.23394
To which add the Log. of 25'4 *l.* —— ——	1.40483
	1.63877

The Log. of Principal 43.529
and Use ——— *viz.* 43 *l.* 10 *s.* 7 *d.*

2. What is due for 25 *l.* 8 *s.* for three Years and a Quarter.

Log. of the Rate divided by 4. 4)	0.03342
Log. of the quarterly Rate.	0.00835
the Quarters in 3 ¼	13
	0.02505
The Log. of Principal and Use	00835
for 1 *l.* ——— ———	0.10855
To which add the Log. of 25'4	1.40483
	1.51338

The Principal and Use is 32.632, that is 32 *l.* 12 *s.* 7 *d.* 3 *f.*

3. What is due for 13 *l.* 5 *s.* for a month?

Log. of the Rate divided	0.03342
by 12) ———	0.00278
Add the Log. of 13 *l.* 25	1.12221
The Answer is 13'319.	1.12499

that is 13 *l.* 6 *s.* 4 *d.* *ob.*

What will 17 *l.* 15 *s.* amount unto at the end of 3 years and a half, 2 months and 8 dayes, at the Rate of 6 *per cent.*?

Rate.

Rate. 1'06. Log. 0.02530 by 3 is	-	0.07590
$\frac{1}{2}$ year is	—— ——	0.01265
A month is 0.00211.by 2 is		0.00422
A day is 0.00007.by 8 is		0.00056
Intereſt for 1 *l.*	——	0.09333
Log. of 17 *l.* 15 *s.*	——	1.24919
22'005		1.34252

The Anſwer is 22 *l.* 0 *s.* 1 *d.*

The ſame work may be obſerved for any other broken or uneven time.

Queſtion 2.

To know what any Sum due at a time to come is worth in ready money.

Find by the laſt what 1 *l.* would come to in the time propoſed, which ſubſtract out of 0.00000, the Log. of 1 *l.* to this Log. add the Log. of the Sum: You have the Reſolution.

1. What is 30 *l.* Pound to be paid at 7 years end worth in ready Money?

Log. of the Rate 1.08	0.03342
	7
	0.23394
	0.00000
	$\overline{1}$.76606
Log. of 30 *l.* ——	1.47712
Anſ. 17 *l.* 506 *i. e.* 17 *l.* 10 *s.* 1 *d.*	1.24318

2. What is 40 *l.* due at 2 years hence worth in ready money after 6 *l. per cent.*?

Log.

Log. of the Rate	0.02530
	2
	0.05061
	0.00
	1.94939
Log. of 40 *l.*	1.60207
An. 35'6 or 35 *l.* 12 *s.*	1.55146

Question 3.

What any *Annuity* is worth to be paid at the end of the Term.

For resolving of all *Questions* about *Annuities*, you must find out the Log. of the Rate and its Number; as also the Log. of the Principal and Use, and its number by the first *Q.* From both which Numbers subtract an Integer, and then find the correspondent Log. of the remaining Numbers.

Lastly, substract the Log. of the Rate less by 1. from the Log. of the Principal and Use less by 1. the remaining Log. shews the Value of the Annuity for 1 *l.* to which add the Log. of the Sum for the Resolution.

1. There is a yearly Annuity of 15 *l.* 11 *s.* forborn for 7 years, what is to be paid at the end after the Rate of 8 *per Centum*?

1.08

1.08 Log. of the Rate 0.03342 } '08 | $\bar{2}$.90309
7 }
Principal. & use 1.7138 0.23384 } '7138 | $\bar{1}$.85357

1.08 less by 1. is '08 } va. of 1 *l*. 8.9224. 0.95048
1.7138 less by 1. is '7138 } val. of 15 *l*. 11 *s*. 1.19173
} 138'74 ——— 2.14221

The Sum to be paid at the end is 138 *l*. 14 *s*. 10 *d*.

2. There is a Quarterly Annuity of 15 *l*. 11 *s*. forborn for 3 years and a Quarter,

What is to be paid at the end, the Rate after 6 *per centum*?

1.06. the Rate 0.02530.
Quarterly Rate (4) 0.00632. 1.0147
Quarters. 13
0.01896
0.0632
Principal and Use. 1.2083
0.08216
1'2083. less by 1. is '2083 Log. $\bar{1}$.31868
1'0147. less by 1. is '0147 Log. $\bar{2}$.16731
Value for 1 *l*. ——— ——— 1.15137
Log. of 15 *l*. 11 *s*. ——— ——— 1.19173
220'34 2.34310

To be paid at the end 220 *l*. 6 *s*. 10 *d*.

Question 4.

What Annuity any Sum due after a certain time will buy after a rate for any time proposed.

The

The Operation differs from the last, only in this, that the Log. of the Principal and Use less by 1. is to be substracted from the Log. of the rate less by 1.

352 *l.* 10 *s.* is due after 7 years, what yearly Annuity will it buy for that time, the Rate 8 *per centum?*

Log. of the Rate	1'08 0.03342.	'08	$\bar{2}$.90309
	7		
Princip. & Use	1.7138 0.23394	'7138	$\bar{1}$.85357
Value of 1 *l.*	———	———	1.04952
Log. of 352 *l.* 10 *s.*	———	——	2.54716
Log. of	——39'510———	—	1.59668

That is 39 *l.* 10 *s.* 3 *d.* yearly.

Question 5.

To know the Value of an Annuity in ready money due for many Payments at a Rate given.

Obtain the Log. of the Principal and Use less by 1. out of which substract the Logarithms of the Rate less by 1. and of the Principal and Use added together.

1. What is 60 *l.* a year to continue 30 years worth in ready money, at 8 *per centum*? Answer, 675 *l.* 9 *s.* 4 *d.*

1.98 Log. ——— 0.03342
30
10.06 1.00260
'08 $\bar{2}$.90309 Log. of Rate less 1.
$\bar{1}$.90569
Princip. & Use less 1. 0.95713
$\bar{1}$.05144 the Value of 1 *l.*
1.77815 Log. of 60
2.82959 Log. of 675.46

Question 6.

To know what Annuity any present Sum will buy for a certain time, after any Rate propounded.

The Rule differs little from the former, only substract the Log. of the Principal and Use less 1. from the Log of the Rate less 1. and the Principal and Use added together.

1. What quarterly Annuity for 7 years will 132 *l.* buy after the Rate of 8 *per cent.*? An. 6 *l.* 3 *s.*

1.08 0.03342
4) 0.00835 Rate 1.0194
28 Quarters in 7 Years
6680
1.670
0.23380 Principal and Use 1.7132
$\bar{2}$.28780 Rate less 1. is .04194
$\bar{2}$.52160
$\bar{1}$.85327 Principal and Use less 1. is 7132
$\bar{2}$.66833 Value for 1 *l.*
2.12057 Log. of 132 *l.*
0.78890 Log. of 61504

The

The like may be done for half-yearly or monthly Payments or Pensions and at what Rate is desired.

The same Practice may be made by the Log. in the other parts of Arithmetick.

Ex. If 17 *l.* 3 *oz.* of Silver cost 32 *l.* 11 *s.* 8. what will 132 *l.* and 2 *oz.* cost? Answer,

32 *l.* 11 *s.* 8 *d.*	32.583.	Log.	1.51299
132. 2 *oz.*	132.16	Log.	2.12110
		Sum ——	3.63409
17 *l.* 3.	17.25.	Log. —	1.23678
Log. of 249'66			2.39731

The Answer is 249 *l.* 13 *s.* 2 *d.*

Now, having gone through all the ordinary Parts of Arithmetick, by all the easy and facil Ways and means that have been yet known, with divers new Additions and Practices; it seems to me that nothing is wanting in this Part, I will therefore hasten to the fourth, containing the great Rule of *Algebra.*

The End of the Third Book.

ALGEBRA.

BOOK IV.

ARITHMETICK
IN
SPECIES or SYMBOLS.

CONTAINING AN

EASY INTRODUCTION
Into that ART.

Shewing by an *Universal Method*, how all *Difficult Questions* are resolved; and how a *General Rule* may likewise be found, to perform Problems of the same kind.

By JONAS MOORE *Professor of the Mathematicks.*

LONDON,
Printed for *Obadiah Blagrave*, at the *Bear* in St. *Paul's* Church-Yard, 1687.

Arithmetick in Species.

BOOK IV.

CHAP. I.

Of Notation.

1. A *Rithmetick in Species* teacheth from a thing *suppositious* and unknown, to find truly that which is sought; and differeth from the *vulgar Arithmetick*, the quantities to be measured in this being expressed by certain *alphabetical Characters* (as it were in their *Species*;) from the which it draweth its Name: And in this Part we consider, 1. the *Notation*, 2. the *Numeration*, 3. the *Equation*, which we consider also in three parts, 1. the *Composition* of the Question or Intention of the Equation, 2. the Proprieties and *Reduction*, 3. the *Resolution* of the Equation.

2. The *Notes* or *Characters* used in this Art are commonly the Letters of the Alphabet, *A*, *B*, *C*, &c.

a,*b*,*c*,*&c.* α,β,γ,*&c.* and for the more ſhort and quick expreſſing of Words, many other Symbols, ſuch as are uſed by me, follow,

+ More, to be added, the ſign of Addition.

— Leſs, to be ſubſtracted, Subſtraction.

× When this Croſs is uſed it ſignifieth Multiplication, or that the quantities are to be multiplied.

= Equal, the ſign of Equality.

√. A Root, the ſign of the Root of any Quantity.

√: The ſign of an univerſal Root, or of ſo many notes as are contained until the next Comma.

∺ Continual Proportion.

∷ Disjunct Proportion, or the Symbol of the *Golden Rule.*

Multiplication commonly hath no Note or Sign, for when two or more Characters are joyned together without any ſign or note betwixt them, they are ſuppoſed to be multiplied together.

Diviſion is expreſſed either by Characters betwixt two *Parentheſes*, the firſt ſignifying the Diviſor, the ſecond the Quotient, and the Dividend betwixt them; or elſe by a line like a *Fraction*, the higher or *Numerator* ſhewing the *Dividend*, the lower or *Denominator* the *Diviſor.* It will not be amiſs to ſet down a few Examples.

$A+B=D$ to be read thus, *A* added to *B* is equal to *D*; and ſuppoſing *A* to be a line of 12 foot long, or to repreſent the Number 12, and *B* to be a line of 8 foot long, or to repreſent the Number 8, then will *D* be a line of 20 foot long, or the Number 20: For $12+8=20$.

$D-A=B$ to be read thus, *D* leſs by *A* or taking

or

or ſubſtracting A out of D, the Remainer will be equal to B in Numbers as before $20-12=8$.

And this is carefully to be underſtood, that the Character that hath no note or ſign before it, if it be the leading Character, is to be underſtood to have the ſign $+$; as in the two laſt Examples, A and D have no ſigns before them, yet the ſign $+$ is to be underſtood.

3. If A ſignifie a line 3 foot long, or the Number 3. then if it be required to double, treble, &c. that line or note, it is done thus, $2A$, $3A$, which is likewiſe to be underſtood of all figures ſtanding before any character or characters. Or if it were required to take the half, the third part, &c. of A it muſt be ſet down thus, $\frac{1}{2}A$, $\frac{1}{3}A$, or rather thus $\frac{A}{2}$, $\frac{A}{3}$, to be read thus, A divided by 2 or by 3.

4. BA is to be read, B mutiplied by A, which if B be 8 and A 12 will be 96. $BA=C$ 96. Likewiſe $\frac{C}{A}=B$ to be read thus, C divided by A is equal to B in numbers $\frac{96}{12}=8$. Or Diviſion is ſet thus $A)C(B$, where C is imagined the Dividend, A the Diviſor and B the Quotient.

5. That the Uſe of theſe Symbols may the better be underſtood, take the firſt four Propoſitions of the ſecond Book of *Euclid*, expreſſed by them.

Prop. 1. If there be two right lines or numbers, Z 12 and A 8, and ſuppoſe the one Z be cut into 3 parts, D 2, E 4, F 6, the Rectangle or Multiplication of the two lines, Z and A together 96, is equal to the ſeveral Rectangles made of the line A 8, and

the parts of the line Z, *viz.* D, E, F, expressed in species thus:

$$ZA = AD + AE + AF$$
$$96 = 16 + 32 + 48$$

Prop. 2. If a line or number Z 12, be cut into two unequal parts A and E, 8 and 4, the Rectangle or Multiplication made of the whole line or number Z, and of the Segments A and E, will be equal to the square of the whole line.

$$QZ = ZA + ZE$$
$$144 = 96 + 48$$

Prop. 3. If a line be cut into two unequal parts, the Rectangle made of the whole line and one of the parts, will be equal to the Rectangle made of the parts, and the square of that part added together.

That is if $Z = A + E$ then
$$ZA = AE + AA$$
$$ZE = AE + EE$$

Prop. 4. If a line or number be divided into two unequal parts, the square of the whole line, will be equal to the two squares made of the parts each severally and to two Rectangles made of the parts: to be expressed in Symbols thus,

if $Z = A + E$ then
$$ZZ = AA + EE + 2AE$$

Where note, that the meaning of a square is the Multiplica-

Multiplication, Rectangle or Power of a line or number: as for Examp. suppose Z 12, A 8, E 4. Then

$$ZZ = AA + EE + 2AE$$
$$144 = 64 + 16 + 64$$

But of this more plainly in the next Section.

6. In this Arithmetick the Species of the same kind, which by the Power of Multiplication ascend, are called *species ascending*, or by the ancient Algebrists *Numbers Cossick*. As for Example, suppose A to be 2, this is called or termed the Root, and marked by the Ancients with ℞ for Radix, in this way only by A, or any other single Figure. Again, if 2 be multiplied by it self it will make 4, which is called a Square, or if ℞ be multiplied by ℞ it would make Q for a Square, or if A be multiplied by A it will be AA, or A^2, or Aq, for any of these will signifie the Square.

Again, if the square 4 be multiplied by the Root 2 it will be 8, the Cube of 2; or if Q be multiplied by ℞ it will be C for the Cube; or if AA, or Aq, or A^2 be to be multiplied by A it will be AAA, or Ac, or A^3, for any of these will indifferently signifie the Cube of A: But this will more evidently appear by the Table of Powers following, which hath been the Ground and Rise of all the ease and excellency of this kind of Arithmetick.

6. It is observed by Dr. *Wallis* in his most learned Book of Arithmetick, that all Nations have used the decuple proportion; and now all decimal Arithmetick is brought to that Scale or Degree, whether ascending or descending, as appears by the Table in

the beginning of my other Book. Now let the Proportions be Continual, however the same shall be, the Degrees of the ascention and descention shall be alike: *viz.*

Whether it proceed by 0. 10. 100. 1000. 10000. *&c.* which is decuple or 10 times more.

Or whether it proceed by 0. 100. 10000. 1000000, *&c.* which is 100 times more.

Or whether it proceed by 0. 60. 360. 2160, *&c.* which is 60 times more.

Or whether it proceed by 0. 2. 4. 8. 16. 32. 64, *&c.* which is double, and contains in it those numbers called Cossick Numbers, 2 being the Root, 4 the Square, 8 the Cube, as appears by the Table.

1	2	3	4	5	6	7	8	9
8.	11^{viii}	12^{viii}	$\frac{1}{256}$	ſq.ſq.ſquare.	QQQ	*Aqcc.*		a^8.
7.	75^{vii}	18^{vii}	$\frac{1}{128}$	2.ſurſolid	2S.	*Aqqc.*		a^7.
6.	89^{vi}	9^{vi}	$\frac{1}{64}$	ſq.cube	QC.	*Acc.*		a^6.
5.	32^{v}	12^{v}	$\frac{1}{32}$	ſurſolid	S.	*Aqc.*	&c.	a^5.
4.	$91''''$	$38''''$	$\frac{1}{16}$	ſq.ſquare	QQ	*Aqq.*	*aaaa.*	a^4.
3.	$75'''$	$56'''$	$\frac{1}{8}$	cube	C.	*Ac.*	*aaa.*	a^3.
2.	$15''$	$39''$	$\frac{1}{4}$	ſquare	Q	*Aq.*	*aa.*	*aa.*
1.	$12'$	$15'$	$\frac{1}{2}$	root	R.	*A.*	*a.*	*a.*
o	1	1	o		o	o	o	o
1.	$12'$	$15'$	2.	root	R	*A.*	*a.*	*a.*
2.	$15''$	$39''$	4.	ſquare	Q.	*Aq.*	*aa.*	*aa.*
3.	$75'''$	$56'''$	8.	cube	C.	*Ac.*	*aaa.*	a^3.
4.	$91''''$	$38''''$	16.	ſquared ſquare	QQ.	*Aqq.*	*aaaa.*	a^4.
5.	32^{v}	12^{v}	32.	ſurſolid	S.	*Aqc.*	*aaaaa.*	a^5.
6.	89^{vi}	9^{vi}	64.	ſquare cube	QC.	*Acc.*	*aaaaaa.*	a^6.
7.	75^{vii}	18^{vii}	128.	2.ſurſolid	2S.	*Aqqc.*	&c.	a^7.
8.	11^{viii}	12^{viii}	256.	ſquared ſq.ſquare	QQQ.	*Aqcc.*		a^8.

This

This proportional Table hath in it 9 Divisions, as may appear in the Margin, and it is divided in the middle with a black line, being the Division betwixt the affirmative and negative Proportions.

The uppermost Division are the Indices or Exponents of the lower Proportions, being a progression of arithmetical Numbers proceeding from 0 both ways towards the left hand affirmatively, and towards the right hand negatively; and these are of great use in finding the places and Value of the other.

The next row are a Centesimal progession of Numbers, every one being 100 times increased towards the left Hand from the line, and the other way decreased Centesimally: These differ not from the ordinary decimal Arithmetick. I place it here to please those that are so much taken with those Sexagena's, and ordinary accounting by minutes and seconds, *&c.* in Astronomy, and will not come willingly to embrace the decimal or this centesimal way, whereas in this way the old manner of accompting by minutes and seconds may be retained; for here every degree is supposed to be divided into 100 minutes, every minute into 100 seconds, *&c.* *Examp.* 32 Degrees and |1753 decimal parts, they are to be read thus, 32 degrees, 17 minutes, and 53 seconds, and set thus, 32°. 17′. 53″. and so likewise 153 degrees is 1′. centene prime and 53 degrees. And to the end that this centesm way may be rendred facile and without trouble, at the charge and first contrivance of *Nich. Shuttleworth* Esquire of *Forcet* in *Yorkshire*, I have caused Rods to be made after the manner of *Nepair*'s Bones, that will most

easily

easily multiply or divide with 2 Figures at once unto 100, as the little ones do to 10.

The third Column or division is that Progression of 60 both ways, in ordinary use in Astronomy, proceeding both ways; Ascending called Sexagena's, Primes, Seconds, *&c.* Descending called Sexagesm's, Primes, Seconds, *&c.*

The fourth Column containeth a Geometrical progression from a Cypher, and may begin as well with any other Figure as here it doth with the Figure 2. In the Ascending side of the Scale every one is twice bigger than the last, and in the descending is $\frac{1}{2}$ the former; and this is the Scale from whence *Algebra* had its Rise or Beginning.

For in the ascending side 2 is accounted the Root, and 4 is the square, because it is the power of 2, $2 \times 2 = 4$, and 8 is the Cube of three Dimensions, because the square 4 multiplied by 2 makes 8, or $2 \times 2 \times 2 = 8$, the same being to be conceived of the rest upon the ascending side; but upon the descending Part, $\frac{1}{2}$ being the Root, the square, cube, *&c.* are every one diminished in the former Proportion.

The fifth Column contains the old Cossick Names of the Powers, and against every Name or Power you have the Character of it in the sixth Column. The other three Columns have every one the correspondent power of *A* the Root, the first after Mr. *Oughtreds* way, the second after Mr. *Harriots* way, and the third after *Des-Chartes.*

7. The uppermost or highest Numbers being the *Indices* or *Exponents* of the Powers, shewing the Powers themselves, that is, how far distant from the Root, or what Proportional it is in the Progression.

For

For how many unites there are in the Exponent, so many are the powers or proportionals from unity; as if the Exponent be 5, the Power is *aqc*.

1. 2. 3. 4. 5.
for 1. *a*. *aq*. *ac*. *aqq*. *aqc*. or a surfolid.

8. Every Power is made of the multiplication of the Root so many times into it self, as there are Unites in the Exponent; as if the Index be 4, then $A \times A = Aq \times A = Ac \times A = Aqq$ the Power under the Index 4, or $A \times A \times A \times A = Aq\,Aq = Aqq$: but more plainly of this in Multiplication.

9. The Sum of any two Exponents do intimate the Plane made by the multiplication of the Powers shewed by them.

	$2+5=7$		$3+4=7$
As	$4 \times 32 = 128$	Likewise	$8 \times 16 = 128$
	$Aq \times Aqc = Aqqc$.		$Ac \times Aqq = Aqqc$.

10. The Difference of any two Exponents do intimate the Plane made by the Division of the greater Power by the less shewed by them.

	$5-2=3$		$4-3=1$
As	32. 4) 32 (8	Likewise	16. 8) 16 (2
	Aqc. *Aq*) *Aqc* (*Ac*.		*Aqq*. *Ac*) *Aqq* (*A*.

11. By *Sect*. 6. the *composition* of the Powers, and by that also the definition of the *Root* of any Cossick number appears, which is the side or first *species* whereof the Power was made: As the root square of *Aq* or $\sqrt{Aq}$ is *A*. the $\sqrt{c}$ of *Ac* is *A*. the $\sqrt{qq}$ of *Aqq* is *A*. &c.

A

A

• • • • $4 * 4 = 16\ \sqrt{4}$

Aq • • • • A $A * A = Aq \sqrt{A}$

• • • •

• • • •

$5 * 5 * 5 = 125 \sqrt{c5}$

A. $A * A * A = Ac \sqrt{cA}$

12. But if the Root be unexpreſſible in Numbers or quantity, as if the *Ratio* betwixt 1, and the √ cannot be expreſſed, the Root thereof is called *Surde*, becauſe properly it cannot be expreſſed; and it is noted as was ſaid before by √. As ſuppoſe *B* to ſignifie 10, then the Root of *B* is no other way to be expreſſed than thus, $\sqrt{B}$, ſo the cube, *ſqq.ſqc.* are thus expreſſed, $\sqrt{cB}$. $\sqrt{qqB}$. $\sqrt{qcB}$, &c. Sometimes alſo it is deſired to expreſs the roots or powers of a ſurde root, as the ſquare root of $\sqrt{q}\,5$. the cube root of the $\sqrt{qq}\,74$, *&c.* to do which you muſt multiply the Indices of the two powers named for a new Index; as $\sqrt{q}$. of $\sqrt{q}\,5$. is $\sqrt{qq}\,5$. *viz.* $2 * 2 = 4$. ſo $\sqrt{c}$ of $\sqrt{qq}\,74 = \sqrt{cccc}\,74$, *viz.* $3 * 4 = 12$, and ſo of any other.

13. Sometimes the root of many ſpecies is expreſſed after the ſame manner; but then after the laſt ſpecies we joyn a *Colon* thus, $\sqrt{\overline{b + D:}}$ $\sqrt{\overline{cC + E - D:}}$

14. If there be more ſimple ſpecies expreſſed beſides the √, then is ſuch a ſpecies if the ſign be affirmative, called a *Binomial*, if negative a *Reſidual*, if many, then is ſuch a *Trinomial* or *Polynomial*; and theſe are accounted Compound Surds.

15. But

15. But if the compounded ſpecies be ſuch a *Binomial*, *Trinomial*, *&c.* and yet the root of the whole be to be expreſſed, then is ſuch a ſpecies called an univerſal Surd, and may be noted thus, $\sqrt{\overline{B+\sqrt{c}}}$: or thus $\sqrt{:}B+\sqrt{c}$.

16. Sometimes it falls out that we muſt expreſs the root of a Coſſick *ſpecies*, as the cube root of *Aqc*. to be thus expreſſed, $\sqrt{c}\,Aqc$, and ſo of any others.

17. Thus are all rooted Magnitudes explained: now he that deſires to enjoy the benefit hereof, muſt accuſtom himſelf to expreſs all quantities, whether of Number or Magnitude by their ſpecies, which if they conſiſt of one letter are called Simple, if of many, Binomial or Polynomial, the Signification wherof is firſt to be learned: As if *A* and *E* ſignifie two Numbers: let *Z* ſignifie the Sum of theſe Numbers, *X* the Difference, *Æ* the Rectangle or Multiplication, $\frac{r}{s}$ the Proportion, *K* the Sum of the ſquares, *L* the Difference of the ſquares, *M* the Sum of the cubes, and *N* the Difference of the cubes of them.

And if you appoint *A* to be 3, and *E* to be 2; then will *Z* be 5, *X* 1, Æ 6, *S* 4, *K* 13, *L* 5, *M* 35, *N* 19.

Now if you would expreſs *AqE* in words, it ſignifieth that (if you appoint *A* for the greater and *E* for the leſſer number) the greater number ſquared is to be multiplied by the leſs, in Numbers $9 \times 2 = 18$.

ZA-Aq in words, that the Sum of the two numbers multiplied by the greater is to be leſſened by the ſquare of the greater, which in Numbers is $5 \times 3 - 9 = 6$.

Zq

$Zq-2ZE+Eq.$ in words, requireth that you substract the Sum of the two Numbers multiplied by the lesser Number doubled from the Sum of the two Numbers squared, added to the square of the lesser, in Numbers $25-20+4=1$.

$\frac{Sq\,Aq}{Rq.}$ In Words signifieth that the Square of the lesser Proportional is to be multiplied by the square of the greater Number, and that the Product is to be divided by the square of the greater proportional, in Numbers $\frac{16\times 9}{36}=\frac{144}{36}=4.$

$\sqrt{K-Aq}.$ Signifieth that the square of the greater is to be substracted from the Sum of the squares of the two Numbers, and that the square root thereof is to be taken, in Numbers $\sqrt{13-9}=\sqrt{4}=2.$

$\sqrt{\frac{KRq-RqAq.}{Aq}}$ Signifieth that you must take the square root of the Quotient of these two Quantities, *viz.* the Sum of the squares multiplied by the square of the greater Prop. and made less by the square of the greater Prop. in the square of the greater Number, the Remainer is the Dividend, and the square of the greater, the Divisor in Numbers.

$$\frac{468-324}{9}=\frac{144}{9}=\sqrt{\frac{144}{9}}=\frac{12}{3}=4.$$

$\frac{Z}{2}+\sqrt{\frac{Zq-4Æ}{4}}$ Signifieth that you must add the half of the Sum of the two Numbers, unto the square root

of

of the Sum ſquared, made leſs by 4 times the rectangle, and divided by 4, in Numbers,

$$\tfrac{5}{2}+\sqrt{\frac{25-24}{4}}=2\tfrac{1}{2}+\sqrt{\tfrac{1}{4}}=\tfrac{6}{2}=3.$$

$\sqrt{:\frac{K}{2}-\sqrt{\frac{Kq-4Pq}{4}}:}$ Signifieth that you muſt take the univerſal ſquare root of half the Sum of the ſquares, made leſs by the ſquare root of the Sum of the ſquares ſquared, leſſened by 4 rectangles of the Numbers ſquared, and divided by 4 in Numbers,

$$\sqrt{:\tfrac{13}{2}-\sqrt{\frac{169-144}{4}}:}=\sqrt{:6\tfrac{1}{2}-\tfrac{25}{4}}=\sqrt{:6\tfrac{1}{2}-2\tfrac{1}{2}}=\sqrt{4:}=2$$

But if the ſign of Equality come in, then in words it will not much differ; as for Example,

$E=\frac{Zq-L}{2Z}$ Signifieth that the leſſer Number is equal to the Quotient of the Sum ſquared, leſſen'd by the Difference of the ſquares, and the remain divided by the Sum of the Numbers doubled, in Numbers,

$$2=\frac{25-5}{10}=\frac{20}{10}=2.$$

$2Aq-2XA=K-Xq$, Signifieth that the ſquare of the greater doubled, made leſs by the greater multiplied by the Difference doubled, is equal to the Sum of the ſquares, made leſs by the Difference ſquared and ſo of any other; and after the ſame manner you may expreſs any of the forementioned words

words in characters, as to expreſs thus much, that the greater Number, is equal to the Difference of the ſquares, made leſs by the difference of the Numbers ſquared and divided by the ſame Difference doubled, it would be

$$A = \frac{L + Xq}{2X}$$

Now for that the Conſideration and varieties of the Operations of two Numbers are of great Uſe in the Practice of this kind of Arithmetick, many Queſtions in *Diaphantus*, *Vieta*, *Ghetaldus*, *&c.* being of ſome of their Parts, I have in the following Book made ſeveral Operations thereof, for the practice of what was taught before.

18. But I muſt here give notice to the Reader, that although I in this Practice appoint theſe letters to ſignifie the Numbers propoſed, yet may thoſe letters ſignifie any other figures or quantities, according to the will of the Arithmetician; as in Mr. *Oughtred's Clavis*, Chap. 19. Prop. 6. where he appoints for the ſolution of Problems in an Arithmetical Progreſſion, α for the firſt term, ω the laſt, T the number of terms, X the common difference, Z the Sum of all the terms; and therefore his 1. *Prop.* $T\omega + T\alpha = 2Z$ to be read thus: If you add the Product of the Number of the terms multiplied by the laſt term, to the Product of the Number of the terms multiplied by the firſt, the Sum of theſe two Products is equal to the Sum of the terms doubled.

19. For diſtinction ſake, note always the given quantities or numbers with Conſonants, and thoſe which are ſought with Vowels, or elſe the given quantities with the former letters in the Alphabet,

and the ſought with the laſt ſort of letters, as *z*, *y*, *x*, *&c.* leſt you make a confuſion in your work.

20. All magnitudes that are under the Power propoſed, are called *Parodical to the Power*: as for example, the *parodical degrees* to *Aqq* are the root *A*, the ſquare *Aq*, and the cube *Ac*.

21. And if the *parodical* degree have a known magnitude joyned with it, it is called the *Coefficient*, as *Aq-AB*, *B* is the Coefficient, and *A* joyned with it is the Parodical degree under *Aq*.

22. Thus have we gone through *Notation*, with a deſire to make the beginning plain, whereof, as of that which ſhall follow, take this little Table.

Arithmetick

Arithmetick in *ſpecies* is either

- *Simple* conſiſting in
  - *Notation* by alphabetical Symbols and Characters.
  - *Numeration* which is
    - Addition, Subſtraction, Multiplicatiō, Diviſion, and that in
      - Rational ſpecies,
      - Surde,
        - And theſe either ſimple or compound or Univerſals.
- *Comparative* in the great Rule of
  - *Algebra* or *Æquation*, which we conſider in the
    - *Invention* of the Æquation.
      - The work preceding it.
    - *Reduction* of the Æquation.
      - The work concomitant with the Æquation.
    - *Reſolution* of the Æquation.
      - The work following it, or ſubſequent, which is done either by Diviſion or extraction of the roots.

CHAP. II.

Addition of Rational species both Simple and Compound.

ADdition joyneth together the Quantities given, keeping the signs; that is, sets into one line both or all the species that are given to be added, and then contracts them, as in the following Examples.

	(1)	(2)	(3)	(4)	(5)
Quantities to	$3a$	a	$6b$	$9e$	$15d$
be added—	a	$-a$	$-7b$	$-5e$	$-18d$
Collected	$3a+a$	$a-a$	$6b-7b$	$9e-5e$	$15d-18d$
The Sum	$4a$	0	$-b$	$4e$	$-3d$.

	(6)	(7)	(8)	(9)
Quantities to	a	a	aa	$a+b$
be added—	e	$-5e$	$a+a^3$	$a+b$
Collected	$a+e$	$a-5e$	$a+aa+a^3$	$a+b+a+b$
The Sum	$a+e$	$a-5e$	$a+aa+a^3$	$2a+2b$

	(10)	(11)	(12)
Quantities to	$a+d$	$a-b$	$13a+15b$
be added—	$a-5d$	$-a-b+c$	$-18a-15b$
Collected	$a+d+a-5d$	$a-b-a-b+c$	$13a+15b-18a-15b$
The Sum	$2a-4d$	$c-2b$	$-5a$

Quan-

Quantities to be added	$5a-10b+7d$ / $-2b-5c$	$a+b-c$ / $a-b+2c$
Collected	$5a-10b+7d-2b-5c$	$a+b-c+a-b+2c$
Sum	$5a-12b+7d-5c$	$2a+c$

2. Here is to be noted, that if in the collection of any quantity consisting of many species or parts annexed with the signs + or - there be two species noted with the same Letter, that then they must be made up into one species, and noted with one Letter, by prefixing their common sign before their Sum, if their signs be both alike; or by prefixing their difference before the sign of that letter wherein the Excess did lie, if their signs be unlike.

As in the first Examp. $3a+a$. Here these two species are made one, *viz.* $4a$ as the Sum of both, because the signs are both alike, *viz.* both +; for + is understood before $3a$ though it be not expressed.

In the second Example $a-a$ the signs are divers or unlike, for + is understood before the first a, and therefore their difference is nothing, and no more they come to, the latter taking away the former, and so the Sum is noted with a Cypher 0.

In the third Examp. $6b-7b$, the signs are unlike, + being understood before $6b$, and therefore they are made up into $-b$, which is their Difference of $7b$ and $6b$, having the sign - prefixed before it, which is the sign where the excess lay, *viz.* upon $7b$.

In the fourth the Sum $4e$ being the difference of $9e$ and $5e$, and the sign +, because the Excess was on $+9e$.

In the sixth because the species are unlike, they are

joyned with the ſign of Addition $+$, *viz.* $a+e$, and ſo in the ſeventh $a-5e$.

In the ninth the ſigns are made up $2a+2b$.

In the tenth I make up $+d-5d$ into $-4d$, and $a+a$ into $2a$, which is $2a-4d$.

In the eleventh, finding $+a-a$, I cancel them both and make $-b-b$ to be $-2b$, and then the Sum is $c-2b$, the ſame order is obſerved in the reſt of the Examples.

Yet note that after ſome practice the ſetting down the collection may be omitted, and the Sum may be written down at once, as in theſe Examples.

	$\frac{1}{2}ab+\frac{2}{3}bb$	$2b+a$	$a^3+5dc-aag+abc$
	$\frac{1}{2}ab+\frac{1}{3}bb$	$3b-a$	$a^3+aag-2dc+b^3$
Sum	$ab+bb$	$5b$	$2a^3+3dc+abc+b^3$

Addition of Indices or Exponents are made after the very ſame manner, conceiving all the affirmative ones to have the ſign $+$ and the deſcending part $-$; all which you will find out of the firſt row of the Table.

2	3	3	$\overline{3}$	$\overline{8}$	$\overline{4}$
3	0	$\overline{3}$	5	3	5
5	3	0	2	$\overline{5}$	1

CHAP.

C H A P. III.

Of Subſtraction.

1. Subſtraction in ſpecies joyneth together both the quantities given, changing all the ſigns of the quantity to be ſubſtracted, *viz.* ſet the higher ſpecies firſt down as you did in Addition, and in ſetting the lower ſpecies down changing all the notes ſet before each, making + to be - and - to be +.

	(1)	(2)	(3)	(4)	(5)
From	a	$5a$	$5a$	$2a$	a
Subſtract	a	$2a$	$-2a$	$5a$	b
The Collection	$a-a$	$5a-2a$	$5a+2a$	$2a-5a$	$a-b$
The Remain	0	$3a$	$7a$	$-3a$	$a-b$

From	a	aa	$3a+5b$	a^3
Subſtract	$b-c$	a	$4a+b$	$aa-a^3$
The Collect.	$a-b+c$	$aa-a$	$3a+5b-4a-b$	a^3-aa+a^3
The Remain	$a-b+c$	$aa-a$	$4b-a$	$2a^3-aa$

From	$5aa-2ba-cd$	$\frac{1}{2}ab+\frac{2}{3}bb$
Take	$cd-3dd-4aa$	$\frac{1}{4}ab+\frac{1}{3}bb$
Collect.	$5aa-2ba-cd-cd+3dd+4aa$	$\frac{1}{2}ab+\frac{2}{3}bb-\frac{1}{4}ab-\frac{1}{3}bb$
Rem.	$9aa-2ba-2cd+3dd$	$\frac{1}{4}ab+\frac{1}{3}bb$

Obſerving the Rule to alter the ſigns of the lower, as you ſee in the Examples, then muſt you collect all into the Remain, as you did in Addition. As in the ſecond Example, ſetting down $5a-2a$ by changing the ſign of the lower, $+2a$, I collect it into one note, which is $3a$, as before.

2 But if any of the Species be in power above the Root, as a Q, a C, or a QQ, or any ſuch Coſſick power, then cannot ſuch be added, ſubſtracted, or collected, but with another of the ſame power or kind, as in the ninth Example a_3 and aa could not be joyned, but a^3 and a_3 might, becauſe both Cubes.

And note, that after ſome Practice the Collection ſhall not need to be made, but upon view the Remain may be ſet down, as in the following Examples.

From	$3a-2d$	$5a^3+\frac{1}{6}dc$	$3aa-2b+6$
take	$2a+5d$	$2a^3-\frac{2}{6}dc$	$-2aa-3b+9$
Reſts	$a-7d$	$3a^3+\frac{3}{6}dc$	$5aa+b-3$

From	$a^3+aad-add-d^3$	$18aa-17ad$
take	$aad-2a^3+c^3-add$	$15a^3+15ad$
Reſts	$3a^3-d^3-c^3$	$18aa-32ad-15a^3$

3. As in Numbers ſo in Species, if you add together the Remainer and the Species to be ſubtracted, they will make up the quantity of the Species from which the ſubſtraction was made. As in the third Example, if you add $7a$ and $-2a$ it will be $5a$, and ſo of any other.

Subſtraction of Indices or Exponents is done after the former manner by changing the Nature of the lower.

3	$\overline{3}$	3	$\overline{5}$	$\overline{7}$	8	$\overline{9}$
2	2	$\overline{2}$	$\overline{7}$	$\overline{5}$	9	$\overline{8}$
1	$\overline{5}$	5	2	$\overline{2}$	$\overline{1}$	$\overline{1}$

Thus

Thus have I done with Addition and Substraction, neither is it any matter in setting down the Species which go first or last, for if $b+c-d=f$, then $c-d+b=f$, and $-d+c+b=f$.

CHAP. IV.

Of Multiplication.

1. MUltiplication *in Species* connecteth both the quantities propounded without any note at all; but if the quantity be noted with two Letters, then you must use the sign ×.

2. In Multiplication, if the signs of the quantities to be multiplied together be alike, that is, both + or both - then the sign of the Product is affirmative or +, but if they be unlike, then it is negative or -.

3. If Numbers be annexed to the species that are to be multiplied, then you must multiply those numbers together, and joyn the Product to the Product of the species multiplied with the correspondent sign.

Examples of Multiplication.

Md.	A	Aq	Ac	Aqq	Aqc		Aqc	$5+$	Aqc	5
Mr.	A	A	A	A	A	&c.	Aq	2	$Aqq+$	4
Pd.	Aq	Ac	Aqq	Aqc	$Acc.$		$Aqqc$	7	$Accc$	9

Or

Or by the leſſer Characters.

$$\begin{array}{ccccccc} a & aa & a^3 & a^4 & a^5 & a^5 & a^5 \\ a & a & a & a & a & aa & a^4 \\ \hline aa & a^3 & a^4 & a^5 & a^6 & a^7 & a^9 \end{array}$$

For $aa = a^2\ aaa = a^3\ aaaa = a^4\ aaaaa = a^5$

Theſe are of one kind, as if a multiply aa, it will be aaa or a^3, as is clear in the former Table, but if the power ariſe from two Names or Letters, then they ariſe as followeth.

$$\begin{array}{ll} a+b & \text{The Multiplicand.} \\ a+b & \text{The Multiplier.} \\ \hline aa+ba & \text{The Product by } a. \\ \quad +ba+bb & \text{The Product by } b. \\ \hline aa+2ba+bb & \text{The square of the Root } a+b \\ \qquad a+b & \text{The Root.} \\ \hline a^3+2baa+bba & \\ \quad +baa+2bba+b^3 & \\ \hline a^3+3baa+3bba+b^3 & \text{\&c.} \end{array}$$

In which Example when I begin to multiply $a+b$ by a, firſt $a \times a$ is aa, and $a \times b$ is ba, both which I ſet down; then I multiply $a+b$ by b thus, $b \times a$ will be ba, and $b \times b = bb$, both which Products added will be $aa+2ab+bb$, the which more fully in the great letters.

$A+E$

$$\begin{array}{l}
A+E \\
\underline{A+E} \quad \text{The Root.} \\
Aq+AE \\
\quad \underline{+AE+Eq} \\
Aq+2AE+Eq = Q:A+E. \\
\qquad \underline{A+E} \\
Ac+2AqE+AEq \\
\quad \underline{+AqE+2AEq+Ec} \\
Ac+3AqE+3AEq+Ec = C:A+E \\
\qquad\qquad \underline{A+E} \\
Aqq+3AcE+3AqEq+AEc \\
\quad \underline{+AcE+3AqEq+3AEc+Eqq} \\
Aqq+4AcE+6AqEq+4AEc+Eqq: \ QQ:A+E \\
\qquad\qquad\qquad A+E \ \&c.
\end{array}$$

This is the Genesis of all *Binomial* Powers, by means whereof not only the Genesis but Analysis of all Powers or extraction of Roots are demonstrated and taught, and that most excellent Posterior Table in Mr. *Oughtred* s Table is composed.

Examples of Multiplication.

	(1)	(2)	(3)	(4)	(5)
Multiplicand	$a-c$	ac	dbb	$2dd$	$\frac{1}{2}f$
Multiplier	a	a	b	$3\,c$	$2f$
Product	$aa-ca$	aac	db^3	$6ddc$	$1ff$

Explanation of these Examples.

First I multiply $a-c$ by a, saying a by a is aa keeping the sign; again, c by a is ca, and make the sign - because

because it was $-c+a$ unlike, according to *Sect.* 2.

In the second I am to multiply a in c by a, I joyn them together, saying aac.

So in the fourth Example, where $2dd$ are to be multiplied by $3c$, first multiply the Figures 3 and 2 makes 6, and set them down $6ddc$, as in the Example.

In the last Example I multiply $\frac{1}{2}$ by 2 makes 1, and f into f makes $1ff$.

$$\begin{array}{lr|r}
\text{Md.} & a-b & 3cc+12g^2d+5h^4e \\
\text{Mr.} & a-b & 3cc-3gd \\
\hline
 & aa-ba & 9c^4+36ccg^2d+15cch^4e \\
 & -ba+bb & -9ccgd-36g^3d^2-15gdh^4e \\
\hline
\text{Prod.} & aa-2ba+bb &
\end{array}$$

$$\begin{array}{lr}
\text{Md.} & aa-ab+bb \\
\text{Mr.} & a+b \\
\hline
 & a^3-aab+bba \\
 & +aab-bba+b^3 \\
\hline
\text{Prod.} & a^3+b^3
\end{array}
\qquad
\begin{array}{r}
3a^3+\frac{1}{2}bb \\
-7d \\
\hline
21a^3d-\frac{7}{2}bbd
\end{array}$$

$$\begin{array}{lr}
\text{Md.} & 5b^3+4bb-2b+5 \\
\text{Mr.} & bb-3b+6 \\
\hline
 & 5b^5+4b^4-2b^3+5bb \\
 & -15b^4\ 12b^3+6bb-15b \\
 & +30b^3+24bb-12b+30 \\
\hline
\text{Prod.} & 5b^5-11b^4+16b^3+35bb-27b+30.
\end{array}$$

Sometimes Multiplication may be only set down with the sign $\times$ or *in* betwixt the multiplicand and multiplier

multiplier for some special uses thus, if $3a$-$15d$ be to be multiplied by $18d$-$\frac{1}{2}a$, express it thus,

$$3a\text{-}15d \text{ in } 18d\text{-}\tfrac{1}{2}a.$$

Certain Propositions for the use of Multiplication.

Prop. 1. If a number, suppose Z, be cut into two unequal parts, that is, $Z=A+E$, to find unto what Plane the multiplication of $A+E$ in it self shall be equal to:

$$\begin{array}{l} Z=A+E \\ Z=A+E \\ \hline Zq=Aq+2AE+Eq \end{array}$$

By which I find that the square of the whole line, is equal to the squares of either Segment added to the double Rectangle of the unequal parts, which is the 4*e.* 2.

Prop. 2. If a number be divided into two unequal Parts, that is, $Z=A+E$, to find out unto what Planes (having relation thereto) the square of the difference of these parts shall be equal.

$$\begin{array}{l} A-E \\ A-E \\ \hline Aq-AE \\ \quad -AE+Eq \\ \hline Aq-2AE+Eq. \end{array}$$

Wherefore I conclude that the square of the difference of two numbers is equal to the squares of the parts made less by the doubled Rectangle of the Parts.

Prop. 3. If a number be divided into two unequal Parts, *viz.* $Z=A+E$, to find out unto what Planes, the Rectangle of the whole line, *viz.* $A+E$, and the difference of the parts, *viz.* $A-E$ is equal.

$A+E$

$A+E$ $A-E$ $Aq+AE$ $-AE-Eq$ $Aq-Eq$	I find by multiplying $A+E$ by $A-E$. That the Rectangle made of the whole Number, and of the Difference of the parts is equal to the Difference of the squares of those parts.

Prop. 4. To find unto what Plane, having relation to the former parts, the square of the whole Number made less by the square of the Difference of the parts, is equal.

By the first *Prop.* the square of the whole Number $= Aq+2AE+Eq$.

By the second *Prop.* the square of the Difference of the parts $= Aq-2AE+Eq$.

The lower being subtracted from the higher, resteth $4AE$.

Therefore I conclude that the square of the whole Number made less by the square of the Difference of the parts, is equal to four Rectangles made of the Parts.

Prop. 5. To find unto what Plane the addition of the square of the whole line, and the Difference of the parts will be equal.

The square of the whole Number	$Aq+2AE+Eq$
The square of the Difference ——	$Aq-2AE+Eq$
The Addition will be ——	$2Aq+2Eq$

Wherefore I conclude that the square of the whole Number added to the square of the difference of the parts, is double the square of the parts added together.

4. All

4. All the Parts of the Power of any Binomial ſimply taken without Unity, are in continual Proportion.

$$Q: A+E = Aq + AE + Eq \ddot{\div}$$

$$\text{for } \frac{AqEq}{Aq} = Eq \text{ the third Prop.}$$

$$C: A+E = Ac.\ AqE.\ AEq.\ Ec \ddot{\div}$$

$$\text{for } Ac.\ AqE.\ \frac{AqqEq}{Ac}.\ \frac{AqEqq}{AqE} \ddot{\div}$$

$$QQ: A+E = Aqq.\ AcE.\ AqEq.\ AEc.\ Eqq \ddot{\div}$$

$$\text{for } Aqq.\ AcE.\ \frac{AccEq}{Aqq}.\ \frac{AqqEqq}{AcE}.\ \frac{AqEcc}{AqEq} \ddot{\div}$$

5. Out of the conſideration of *Chap.* 12. *Sect.* 5. The Powers of any Binomial are ſuddenly made, by ſetting down all the Parodical degrees under the higheſt Poteſtas to the root both ways, and joyning them together contrarily, and at the laſt prefixing the Unity as in this little Table, which are alſo made and continued by Arithmetical Progreſſion on either ſide in form of a Triangle, and adding up every two numbers above for middle intermediate under, as you may ſee in the Example.

					A						
					2						*Aq*
				3		3					*Ac*
			4		6		4				*Aqq*
		5		10		10		5			*Aqc*
	6		15		20		15		6		*Acc*
7		21		35		35		21		7	*Aqqc.*

As

As if it be desired to produce the QC of $A+E$, which is the fifth Power; I first set down all the Parodical degrees of them both to the Root, then I couple them contrarily sparing the highest Potestas of either, and then prefix the right numbers to them as in the Table, and therefore conclude that $Aqc+5AqqE+10AcEq+10AqEc+5AEqq+Eqc$ is the fifth Power, or QC: of AE.

Aqc	Eqc	Aqc	Aqc
Aqq	Eqq	$AqqE$	$5\,AqqE$
Ac	Ec	$AcEq$	$10\,AcEq$
Aq	Eq	$AqEc$	$10\,AqEc$
A	E	$AEqq$	$5\,AEqq$
		Eqc	Eqc

6. If any Power be to be multiplied by another species, if that species be negative, then it makes all the other power if before affirmative to be negative.

$$A+E \text{ by } A+E \text{ by } -B = -BAq-2BAE+BEq.$$

CHAP. V.

Of Division.

1. If the Dividend be made or compounded of the Divisor, as the one Factor (which is easily seen by marking well the order of Multiplication) then the other Factor is the Quotient, (by the eighth Section of Division in the former Part.)

2. But if the Dividend be not so made (as commonly it falls out in this kind of Arithmetick), then you

you must set the Dividend over the Divisor in form of a Fraction with a little line betwixt them;(by the last part of the second Section of the said Chapter of Division)

3. If you observe the Proof of Division, which is, that the Product of the Quotient into the Divisor, be equal to the Dividend; it will further you to this kind of Division, by taking such a number for your Quotient, as being multiplied by the Divisor will be equal to the Dividend.

4. Like signs, or both +, or both -, make +, and unlike -, as in Multiplication.

Examples of Division in species.

Divisor. Dividend.

B) *BA* (*A* the Quotient, for *BA* is compounded of *B* and *A*, and if *B* be the Divisor *A* is the Quotient, & *contra*, and for proof multiply the Divisor and Quotient *B* and *A*, that is, $BA = BA$ the Dividend.

$$3aa)\,6a^3\,(2a \qquad a-e)\,ba-be\,(b \qquad a)\,ca+da\,(c+d$$

$$\begin{array}{ccc} & ba-be & ca \\ \hline & 0\ \ 0 & c+da \\ a^3)\,a^7\,(a^4 & & da \\ \hline & & 0 \end{array}$$

In the two last Examples I have set down the progress of Division: for Example, I demand (in the last) how many times *a* first in *ca*, it will be *c*, then I multiply my Quotient and Divisor *ca*, and subtract from *ca* above rests nothing, I then set down my

next ſpecies *da*, and again work as before, and ſubſtracting there reſts o. Which ſhews the Diviſor will evenly divide the Dividend, which Operation muſt carefully be obſerved.

Now to the intent you may be very perfect in this manner of Diviſion, it is the beſt way to multiply ſome ſpecies together, and then divide the Product by either the multiplicand or multiplier, the Quotient will be the other. *Examp.*

$$\begin{array}{lll}
 & 2a^2-3bc & \\
 & a+2bc & \\
\hline
 & 2a^3-3bca & \\
 & \quad +4bca^2-6bbcc. & \\
\hline
\text{Product is} & 2a^3+4bca^2-3bca-6bbcc. & Q. \\
\text{Dr. } a+2bc) & 2a^3+4bca^2 & (2a^2-3bc \\
\hline
 & \quad o-3bca-6bbcc. & \\
 & \quad -3bca-6bbcc. & \\
\hline
 & \quad o &
\end{array}$$

Note that the Quotient $2a^2-3bc$ is got at two Operations and nothing remains, which Work being often repeated will make perfect.

But if the Diviſor will not evenly divide the Dividend, as if you were to divide *B* by *A*, then muſt they be ſet in form of a Fraction, and thus $\frac{B}{A}$ or $\frac{b}{a}$ and ſo of any other.

More

More Examples of Division.

$aa)\,a^6\,(a^4$ $\qquad$ $a)\,ca+a\,(c+1$ $\qquad$ $b-c)\,ba-be-ca+ce\,(a-e$

$$\begin{array}{l} ca \\ \hline 0+a \end{array} \qquad \begin{array}{r} ba-ca \\ \hline 0\ be+ce \\ be+ce \\ \hline 0 \end{array}$$

Divisor $2a-3b$) Dividend ($3a^4-2ba^3+5bbaa-13$ Qt.

$$\begin{array}{ll}
6a^5-13ba^4+16bba^3-15b^3aa-26a+39b. & \\
6a^5-9ba^4 & (1)\ 3a^4 \\ \hline
4ba^4+16bba^3 & (3)\ -2b^3 \\
4ba^4+16bba^3 & \\ \hline
+10bba^3-15b^3aa & (3)\ +5bbaa \\
+10bba^3-15b^3aa & \\ \hline
00-26a+39b & \\
-26a+39b & \\ \hline
00 &
\end{array}$$

So the Quotient is $3a^4-2ba^3+5bbaa-13$ which if it be multiplied by the Divisor $2a-3b$, will bring forth the Dividend.

The same would follow if the Operation begun from the right hand; and note that sometimes Division is to be wrought, though the Divisor do not evenly divide the Dividend, and the Remainder is to be set over the Divisor in form of a Fraction.

Of Multiplication and Division in Centesm's and Sexagesm's, and how the Table of Powers in the first Chapter is appliable to the Logarithms.

For Multiplication add together the Indices of the Centesm's, *&c.* to be multiplied, it shews the nature of the Product.

For Division substract them.

Multiply 35' by 15''' $\begin{array}{c} 1 \\ \hline 3 \\ = \\ 2 \end{array}$ The Product will be thirds'''.

Multiply 5° by 15'' The Product will be '', for $c + \overline{2} = \overline{2}$.

Divide 45''' by 15''' The Quotient will be degrees, for $3 - 3 = 0$.

Divide 15' by 5''' The Quotient will be '', for $+1 - 3 = \overline{2}$.

Now these Indices are the Characteristicks of the Logarithms, or the first figures, and are appliable as I have shewed in my first Book; all which Rules there must be made clear by this Table of Powers, the true understanding whereof is needful to those Rules, the Powers and Cossick Numbers being indeed the numbers proposed to be multiplied or divided, and the Exponents or Indices the Logarithms.

CHAP.

CHAP. VI.

Of the four Parts of Numeration of Fractions in Species.

IT is very requisite for him who desires to be a perfect Arithmetician to be very ready in the Work of Fractions, and therefore to be cunning in the four first Sections of Chap. 16. of the first Book; for whosoever shall understand them, will easily understand the Work of Species following, having a Dependance thereon and according to the parts of those Sections expressed.

Sect. 1. *Of the greatest common Measure.*

5.6. $\frac{ba}{bc}$ the greatest common Measure is $b)\frac{ba}{bc}\frac{(a}{(c}$.

$\frac{bcd}{bda}$ the greatest common Measure is $bd)\frac{bcd}{bda}\frac{(c}{(a}$.

Sect. 2. *Of the Reduction of Fractions.*

6. Let b be reduced to an improper Fraction whose Denomination is a.

It will be thus $\frac{ba}{a}$ the Fraction.

Again let $b+c$ be reduced to an improper Fraction whose Denominator is a.

It will be thus $\begin{array}{c|} b+c \\ a \\ \hline ba+ca \end{array}$ $\frac{ba+ca}{a}$ the Fraction.

7. Let $b+\frac{d}{a}$ be reduced to an improper Fraction:

It will be thus $\begin{array}{c|} b \\ a \\ \hline ba \end{array}$ $\frac{ba+d}{a}$ the improper Fraction.

Let $b-c$ & $\frac{r+d}{s}$ be reduced to an improper Fraction:

It will be thus $\begin{array}{c|} b-c \\ s \\ \hline bs-cs \end{array}$ $\frac{bs-cs+r+d}{s}$ the improp. Fract.

8. On the contrary, let the improper Fraction $\frac{ba}{a}$ be reduced to the Integer:

It will be thus $a)\ ba\ (b$ the Integer.

So $\left.\frac{ba+ca}{a}\right|$ $a)\ ba+ca\ (b+c$ the Integer:

So likewise $\frac{bs-cs+r+d}{s}$ $s)\ bs-cs+r+d\ (b-c\frac{r+d}{s}$

9. Fractions of unlike denomination ought to be reduced, keeping still the same value.

As $\frac{b}{ca}$ $\frac{r}{da}$ reduced would be thus.

bd

$$\begin{array}{l} \quad bd \quad rc \\ \quad \cdots\cdots\cdots \\ a)\ \frac{b}{ca}\ \frac{r}{da} \quad \frac{bd}{dca}\ \frac{rc}{dca} \\ \quad \cdots\cdots\cdots \\ \quad \underbrace{d \quad c}_{dca} \end{array}$$

The Fraction reduced.

$$ba \quad dc$$

So $\frac{b}{c}\ \frac{d}{a}$ reduced thus 1) $\frac{b}{c}\ \frac{d}{a}\ \frac{ba}{ca}\ \frac{dc}{ca}$

$$\underbrace{a \quad c}_{c \quad a}$$

The Fraction reduced.

$$bae \quad cde \quad fda$$

10. $\frac{b}{d}\ \frac{c}{a}\ \frac{f}{e}$ reduced $\frac{b}{d}\ \frac{c}{a}\ \frac{f}{e}$ / dae

$\frac{bae}{dae}\ \frac{cde}{dae}\ \frac{fda}{dae}$ The Fractions reduced to one Denomination.

Sect. 3. *Of Addition and Subftraction of Fractions.*

If the Fractions have the fame Denomination, the Sum or Difference of the Numerators muft be fet over the common Denominator.

$\frac{b}{c}\ \frac{d}{c}$ Sum or difference is $\frac{b\pm d}{c}$ | $\frac{bd}{ac}\ \frac{cd}{ac}$ Sum or difference is $\frac{bd\pm cd}{ac}$

If the Fractions have not the ſame baſe, then they muſt be reduced as in the laſt Section, and then added or ſubſtracted.

As $\frac{b}{ca}\ \frac{r}{da}$ reduced, added, or ſubſtracted, are $\frac{bd\pm rc}{dca}$

Sect. 4. *Of Multiplication of Fractions.*

$\frac{b}{d}\times\frac{c}{a}$ Prod. $\frac{bc}{da}$ | $d\times\frac{ca}{f}$ or $\frac{d}{1}\times\frac{ca}{f}$ Prod. $\frac{dca}{f}$ |

$b\frac{c}{a}\times\frac{c}{d}$ Prod. $\frac{bacq}{da}$ | $\frac{b}{a}\times\frac{a}{c}$ Prod. $\frac{b}{c}$ |

$$\begin{array}{cccc} & b & & d \\ a) & \frac{ba}{rc} & \times & \frac{de}{ca} \\ c) & r & & c \end{array}$$ Product is $\frac{bd}{rc}$

Sect. 5. *Of Diviſion of Fractions.*

$\frac{b)}{1)}\ \frac{aa}{d}\ \frac{(aa}{(bd}$ | $\frac{b^3)}{c)}\ \frac{e^3}{a}\ \frac{(e^3c}{(b^3a}$ | $\frac{a)}{s)}\ \frac{\overset{b}{ba)}}{s)}\ \frac{\overset{r}{ra}}{c}\ \frac{(sr}{(bc}$

$\frac{a)}{f)}\ \frac{\overset{c}{ca)}}{\underset{d}{df)}}\ \frac{\overset{b}{ba}}{\underset{r}{rf}}\ \frac{(bd}{(cr}$ $\frac{a)}{r)}\ \frac{\overset{b}{ba)}}{\underset{c}{cr)}}\ \frac{\overset{1}{a}}{\underset{1}{r}}\ \frac{(c}{(b}$

Sect. 6.

Sect. 6. *To add or substract any Part from a Fraction, or to find a Part named, in 6 Propositions.*

Prop. 1. *To add any Part or Parts of a Number given to the same Number.*

As for Example, to this Fraction $\frac{3a}{5}$ add his half or $\frac{1}{2}$, add 1 to $\frac{1}{2}$ it maketh $\frac{3}{2}$, then multiply $\frac{3a}{5} \times \frac{3}{2} = \frac{9a}{10}$ for proof, if I suppose $a = 10$, then $\frac{30}{5} = 6$, the half whereof $3 + 6 = 9 = \frac{9a}{10} = \frac{90}{10}$.

Ex. 2. If to $\frac{3a+d}{e}$ I add his $\frac{2}{5}$, I add 1 to $\frac{2}{5}$ maketh $\frac{7}{5}$, by which I multiply $\frac{3a+d}{e} \times \frac{7}{5} = \frac{21a+7d}{5e}$.

Ex. 3. Of a pure Fraction to $\frac{3}{5}$ add his $\frac{1}{2}$, I multiply

$$\frac{4}{\cancel{3}} \text{ by } \frac{\cancel{3}}{5} = \frac{4}{5}.$$

Prop. 2. *To add the Part or Parts of a given Magnitude to any other of his Parts.*

Ex. 1. If the given Quantity was $5b$, to the $\frac{1}{3}$ thereof I desire to add the $\frac{2}{5}$. I add the Parts together, $\frac{1}{3}$ $\frac{2}{5}$ makes $\frac{11}{15}$, this Fraction I am to multiply in the given Number

$$\frac{\cancel{5}b}{1}\ \frac{11}{\cancel{15}} = \frac{11b}{3}.$$

Ex. 2. If the Number $\frac{15a-12}{8}$ be proposed, and it be desired to add its half to his third part, I add $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ then $\frac{5}{6} \times \frac{15a-12}{8} = \frac{75a-60}{48}$.

Ex.

Ex. 3. I would add the $\frac{2}{5}$ of $7a$ to the $\frac{4}{7}$ of it: I add $\frac{2}{5}$ to $\frac{4}{7} = \frac{34}{35}$, and $\frac{34}{35} \times \frac{7a}{1} = \frac{238a}{35}$ for proof I suppose a 5, then $7a = 35$, the $\frac{2}{5}$ is 14, the $\frac{4}{7}$ is $20 = 34 = \frac{1190}{35} = 34$.

Prop. 3. *To ſubſtract any part or parts from any Number whatſoever.*

Ex. 1. Let the given quantity be $\frac{b+8a}{c}$ it is deſired to ſubſtract the $\frac{1}{3}$ and the $\frac{1}{5}$ from it. I firſt add together the Parts $\frac{1}{5}\,\frac{1}{3} = \frac{8}{15}$, I ſubſt. that Fraction from $\frac{15}{15}$, or 1 that is $\frac{7}{15}$, which I multiply by the given Quantity $\frac{b+8a}{c} \times \frac{7}{15} = \frac{7b+56a}{15c}$

Ex. 2. From $\frac{9}{20}$ take his $\frac{4}{5}$, firſt I ſubduct $\frac{4}{5}$ from 1 or $\frac{5}{5}$, reſts $\frac{1}{5}$, by which I multiply $\frac{9}{20} = \frac{9}{100}$.

Prop. 4. *A Quantity being given to ſubſtract any part or parts from any other of his parts.*

Ex. 1. Let the given Quantity be $\frac{b+8a}{c}$ from the $\frac{2}{3}$, whereof it is deſired to ſubtract $\frac{1}{4}$, firſt I take $\frac{1}{4}$ from $\frac{2}{3}$ reſteth $\frac{1}{12}$, by which I multiply the given Quantity, *facit* $\frac{b+8a}{12c}$.

Ex. 2. Alſo take the $\frac{3}{4}$ of b from the $\frac{8}{9}$ of it, firſt I take $\frac{3}{4}$ from $\frac{8}{9}$, reſteth $\frac{5}{36}$, I multiply that by the given quantity, it makes $\frac{5b}{36}$, the Proof is plain: for if you put $b = 36$. $\frac{8}{9}$ of $36 = 32$; $\frac{3}{4} = 27$ and $32 - 27 = 5 = \frac{5b}{35} = \frac{180}{36} = 5$.

Prop. 5. *To find the part or parts of any given Number.*

Ex. 1. Let the given quantity be $\frac{b+8a}{c}$ I would find the $\frac{1}{3}$ and $\frac{1}{4}$ thereof, I add together the $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$, by this I multiply $\frac{b+8a}{c}\times\frac{7}{12}=\frac{7b+56a}{12c}$

Ex. 2. I would find the $\frac{3}{4}$ of b, I multiply $\frac{3}{4}\times\frac{b}{1}=\frac{3b}{4}$, if you suppose $b=20$ the $\frac{3}{4}$ of it is $15=\frac{3b}{4}=\frac{60}{4}=15$.

Prop. 6. *To find out the principal Number if any of his parts be given.*

Ex. 1. Suppose that $\frac{b+8a}{c}$ be the two thirds of any other Number, I would know that Number, divide $\frac{b+8a}{c}$ by $\frac{2}{3}$, the Quotient is $\frac{3b+24a}{2c}$ which is the principal Number desired.

Ex. 2. It is desired that I should find the Principal whereof b is the $\frac{4}{5}$, I divide $\frac{b}{1}$ by $\frac{4}{5}=\frac{5b}{4}$, now suppose you set $b=8$, then $\frac{40}{4}=10$, for 8 is the $\frac{4}{5}$ of 10.

The 6 former Propositions, if well practised, will be of great help in the solution of many intricate Questions proposed to be resolved, when as part or parts of any quantity are to be added or substracted.

CHAP.

CHAP. VII.

The Parts of Numeration in simple Surds or Surds Cossick.

Sect. 1. *Of Addition and Substraction.*

1. AS in Fractions, so in all simple Surds, you are to consider whether they be Commensurable or no; if they be commensurable, and when they are brought into their least Terms, they then be truly figurative.

They are added or substracted thus: For Addition square the Sum of the Roots, and multiply that square Quantity by the common Divisor, prefixing before the last Product the first Root: In Substraction square the Difference and work as before.

Examp. If $\sqrt{rpp}$ and $\sqrt{rss}$ be given to be added or substracted, I find $\sqrt{r}$ to be the common Divisor, and would leave in the Quotients pp and ss truly figurative, whose Roots p and s added and substracted, *viz.* $p+s$ or $p-s$, and then multiplied by it self, by $\sqrt{r}$, will produce $\sqrt{ppr+2prs+ssr}$: for Addition, and $\sqrt{ppr-2prs+ssr}$ for Substraction.

$\sqrt{R})$	$\sqrt{RPq}$.	$(Pq$.	P
	$\sqrt{RSq}$.	$(Sq$.	S
$\sqrt{PqR+2PRS+SqR}$.	$\sqrt{Pq+2PS+Sq}$:		$P+S$ Add.
$\sqrt{PqR-2PRS+SqR}$.	$\sqrt{Pq-2PS+Sq}$:		$P-S$ Sub.

The Surds added are $\sqrt{PqR+2PRS+SqR}$:

substracted $\sqrt{PqR-2PRS+SqR}$:

$\sqrt{3})$

$$\sqrt{3})\ \begin{matrix}\sqrt{27Aq.} & (9Aq. & 3A\\ \sqrt{\ 3Bq.} & (1Bq. & 1B\end{matrix}$$

Add.$=\sqrt{27Aq+18BA+Bq}:\sqrt{9Aq+6BA+Bq}:3A+B$

Sub.$=\sqrt{27Aq-18BA+Bq}:\sqrt{9Aq-6BA+Bq}:3A-B$

The Work according to the Rule.

$$\begin{array}{r} 3A+B \\ 3A+B \\ \hline 9Aq+3BA \\ +3BA+Bq \\ \hline 9Aq+6BA+Bq \end{array}$$

I conclude $\sqrt{27Aq}+\sqrt{3Bq}=\sqrt{27Aq+18BA+Bq}$ and $\sqrt{27Aq}-\sqrt{3Bq}$ resteth $\sqrt{27Aq-18BA+Bq}$.

2. But if the Surds be *Asymmetra*, or have not such a common measure as before, then are they added with + and -.

As if $\sqrt{B}$ be to be added to $\sqrt{C}$ it would be $\sqrt{B}+\sqrt{C}$, if substracted $\sqrt{B}-\sqrt{C}$.

So likewise $\sqrt{B-D}$: $\sqrt{B-C}$ added $\sqrt{B-D}+\sqrt{B-C}$: substracted $\sqrt{B-D}:-\sqrt{B-C}$:

Sect. 2. *Of Multiplication and Division.*

1. If your Surds be all of one kind, then multiply and divide them as is taught before, and prefix before them their former Root.

As $\sqrt{qA}*\sqrt{qB}\sqrt{=}qBA$, and $\sqrt{qA})\ \overset{\sqrt{qBA}}{\quad}\ (\sqrt{qB}.$

2. But

2. But if they be not of the same kind or power, then must they be reduced thus, set the $\surd$ as in the Example, and multiply each Quantity as his altern Power or Root doth specifie. *Examp.* I would multiply $\surd qB$ by $\surd cC$.

$$\frac{B}{\surd q} \times \frac{C}{\surd c} = \frac{Cq}{\surd qc} = Bc$$

Reduced to one Denomination, are $\surd qcBc + \surd qcCq$ mult. $= \surd qcBcCq$.

3. If the Characters be compounded, then let the one be reduced to that species, by which it exceeds the other.

$\frac{Aq}{\surd q} \times \frac{Bq}{\surd qq}$ are made $\surd qqAqq$, and $\surd qqBq$.

4. If in Multiplication the one be an absolute Number, reduce it by Multiplication to the same Root with the other: As $2 \times \surd qB$, that is, $\surd q4 \times \surd qB = \surd q4B$ $2 \times \surd cC = \surd c8 \times \surd cC = \surd c8C$.

So $\surd qqD \times 2$, that is, $\surd qqD \times \surd qq16 = \surd qq16D$. and so of 2, 3, 4, 5, &c.

Hence it is apparent how any Surd may be added to himself, that is, multiplied by 2. For if I were to add $\surd qB + 9D$: to $\surd qB + 9D$: it would be $\surd q4B + 36D$. So $\surd q3D$ in $2 = \surd q12D$. So $\surd c3D$ added to it self $= \surd c24D$.

5. If you be to multiply any Root or Roots by themselves, if the Powers be alike, *viz.* to square the Root $q5$, then cancel the Root, and set down 5 as a plain Number; so QQ. of $\surd qqB$ is B. But if they be not of the same kind, divide the Index of the greater

by

by the Index of the leſs, and ſet down the power anſwering the remain with your ſpecies as to Q: the $\sqrt{qqB}$ it will be $\sqrt{qB}$, becauſe 2)4(2. but this only, if the greater Root be compounded.

CHAP. VIII.

The Parts of Numeration in Compound Surds.

Sect. 1. *Of Addition and Subſtraction.*

1. THE Addition and Subſtraction of theſe compound Surds is the ſame with the Simple, having reſpect to the ſigns.

Examp. of Addition.

Examp. 1.	$\sqrt{q27}+\sqrt{q8}$		$\frac{z}{2}+\sqrt{\frac{zz-4p}{4}}$
Numbers	$\sqrt{q12}+\sqrt{q2}$		$\frac{z}{2}+\sqrt{\frac{zz-4p}{4}}$
Sum	$\sqrt{q75}+\sqrt{q18}.$		z

$z+\sqrt{sr}$	$\sqrt{qzz-4p:}$	$\sqrt{czz:+5p:}$
$p+\sqrt{ze}$	$\sqrt{qzz-4p:}$	$\sqrt{czz:-5p:}$
ſum $z+p+\sqrt{ſr}+\sqrt{ze}$	*ſum* $\sqrt{q4zz-16p:}$	*ſum* $\sqrt{c8zz}.$

Examp.

Examp. of Substraction.

1. $\frac{z}{2}+\sqrt{\frac{zz-4p}{4}}$

$\frac{z}{2}-\sqrt{\frac{zz-4p}{4}}$

$\sqrt{\frac{4zz-16p}{4}}$

that is $\sqrt{zz-4p}$.

in Numbers $\sqrt{90}+16$
$\sqrt{9320}-8$
$24-\sqrt{9320}$.

$z+\sqrt{sr}$
$p+\sqrt{ze}$
$z-p+\sqrt{sr}-\sqrt{ze}$

2. Addition and Substraction in universal Surds, if they be of divers Denominations, is as in Compounds, setting before the last the universal sign: But if they be of like denomination, then they are added and substracted after this manner; add your two Surds together, then multiply them, double the Product, or multiply it by Root 4, set the first Addition and last Product down with the sign of Addition if you add, but with - if you substract.

Examp. in Numbers and Species.

$\left.\begin{array}{l}\sqrt{:12+\sqrt{6}}\\ \sqrt{:12-\sqrt{6}}\end{array}\right\}$ 24 the Addition.

$144-6=\sqrt{138}$ the Mul.

and m $138\times\sqrt{4}=\sqrt{552}$

therefore $\sqrt{.24+\sqrt{552}}$ the Add.

$$\left.\begin{array}{l} \sqrt{:\frac{z}{2}+\sqrt{\frac{zq-4pq:}{4}}} \\ \sqrt{:\frac{z}{2}-\sqrt{\frac{zq-4pq:}{4}}} \end{array}\right\} \begin{array}{l} \text{to be added.} \\ \frac{z}{2}\text{ the Addition.} \end{array}$$

$$\frac{zq}{4}-\frac{zq-4pq}{4}=pq \text{ the Mul.}$$

$\sqrt{pq}$ in $\sqrt{}$ is $\sqrt{4pq}$

$\sqrt{:z+\sqrt{4pq}}=\sqrt{z+2p:}=$ the Add.

$\sqrt{:z-\sqrt{4pq}}=\sqrt{z-2p}$ the Subſtract.

$$\left.\begin{array}{l} \sqrt{:\frac{2zq-4pq}{4}+\sqrt{\frac{4zqq-16pqzq}{16}}} \\ \frac{2zq-4pq}{4}-\sqrt{\frac{4zqq-16pqzq}{16}} \end{array}\right\} \text{to be added.}$$

$zq-2pq$ the Addition.

$$\frac{4zqq-16zqpq+16pqq}{16}-\frac{4zqq-16pqzq}{16}$$

That is $\frac{4zqq-16zqpq+16pqq-4zqq+16pqz}{16}$

And by cancelling the contrary ſpecies, reſteth only $pqq\times 4=\sqrt{4pqq}=2pq$.

The Addition thereof is $\sqrt{:zq-2pq+2pq}=\sqrt{:zq}=z$.

The Subſtraction is $\sqrt{zq-2pq-2pq}=zq-4pq$.

Another Example.

$$\left.\begin{array}{l} \surd{:}\surd\dfrac{X,q+4Pq:}{4}+\dfrac{X,}{2} \\ \surd{:}\surd\dfrac{X,q+4Pq}{4}\ \dfrac{X,}{2} \\ \hline \surd X,q+4Pq \text{ the Ad.} \end{array}\right\} \begin{array}{l} \dfrac{X,q\cdot 4PqX,q}{4\quad 4}=Pq \text{ the Mul.} \\ \\ \text{and } Pq\times 4=\surd 4Pq,=2P. \end{array}$$

Therefore the Addition is $= \surd{:}\surd X,qq\times Pq{:}+2P$.
The Substraction is $= \surd{:}\surd X,qq+Pq{:}-2P$.

Sect. 2. *Of Multiplication and Division.*

1. There is no great Difficulty in this, observing the signs, and in universals keeping the right sign for the Product.

Examples of Multiplication.

$$\surd{:}\frac{Z}{2}+\surd\frac{Zq\text{-}4P:}{4} \quad \text{Multiplicand.}$$

$$\surd{:}\frac{Z}{2}+\surd\frac{Zq\text{-}4P:}{4} \quad \text{Multiplier.}$$

$$\frac{Zq}{4}+\frac{Zq\text{-}4P}{4}+\frac{2Zq\text{-}4P:}{4} \quad \text{The Squares.}$$

$$\left.\begin{array}{l} \surd\dfrac{Zqq\text{-}4ZqP}{16}\times 4= \\ =\dfrac{4Zqq\text{-}16ZqP}{16} \end{array}\right\} \text{The Rectangles.}$$

$$\surd{:}\frac{2Zq\ 4P}{4}+\surd\frac{4Zqq\text{-}16ZqP:}{16} \quad \text{The Product.}$$

$\surd{:}$

$\sqrt{:}\frac{Z,}{2}+\sqrt{\frac{Z,q-4Pq:}{4}}$ Multiplicand.

$\sqrt{:}\frac{Z,}{2}-\sqrt{\frac{Z,q-4Pq:}{4}}$ Multiplier.

$\sqrt{:}\frac{Z,q}{4}-\frac{Z,q-4Pq}{4}=\sqrt{:}Pq=P.$

$\sqrt{:}\frac{Z,}{2}+\sqrt{\frac{Z,q-4Pq:}{4}}$

$\sqrt{:}\frac{Z,}{2}-\sqrt{\frac{Z,q-4Pq:}{4}}$

$\frac{Z,q}{4}+\frac{Z,q-4Pq}{4}=\frac{Z,q-4Pq:}{4}$ The Squares.

$\frac{4Z,qq-16Z,-4Pq}{16}=2$ Rect.

$\sqrt{:}\frac{2Z,q-4Pq}{4}+\sqrt{\frac{4Z,qq-Z,16Pq:}{16}}$ Product.

2. Now as in ſimple Surds, if you be to multiply an univerſal Surd by himſelf, you are to blot out the Root, and make it a Compound.

3. If you be to multiply an univerſal Surd by a whole quantity or other Root, you muſt reduce it to the ſame denomination, as if you be to multiply $\sqrt{:}\frac{Z,}{2}+\sqrt{\frac{Z,q-4Pq}{4}}$ by R, you muſt reduce R to Rq, and then to Rqq as followeth.

$\sqrt{:}\frac{Z,Rq}{2}+\sqrt{\frac{Z,qRqq-4PqRqq}{4}}$

$\sqrt{q}:23+\sqrt{44}\times$ $\quad\quad$ $\sqrt{q}16+\sqrt{q}9$
$\sqrt{q}:16+\sqrt{q}9$ $\quad\quad$ $\sqrt{q}16+\sqrt{q}9$

$\sqrt{:}25+\sqrt{576}$ $\quad\quad$ $16+9$

$\sqrt{:}23+\sqrt{4}$ $\quad\quad$ $\sqrt{144}\times=576$

$\sqrt{:}25+\sqrt{576}$

$\sqrt{:}575+\sqrt{q}2304+\sqrt{q}2500+\sqrt{q}304704$
that is $575+48+50+552=1225$
$\sqrt{:}1225=35.$

Examp. of Division in Numbers.

$\sqrt{q}3+\sqrt{q}5+\sqrt{q}6)\ 100\ (\sqrt{q}60+2$
$(\sqrt{q}60-2$

that is 56.

$\sqrt{q}3+\sqrt{5}+\sqrt{q}6$
$\sqrt{q}3+\sqrt{q}5-\sqrt{q}6$

$-\sqrt{q}18-\sqrt{q}30-6$
$-+\sqrt{q}15+5+\sqrt{q}30$
$+3+\sqrt{q}15+\sqrt{q}18$

Sum $\sqrt{q}60+2.$
$\sqrt{q}5)\ \sqrt{q}:13+\sqrt{q}7\ (2\frac{3}{5}+\sqrt{\frac{7}{25}}$

The Squares being taken and universal sign cleared.

Sect. 3. *Of the Extraction of the Roots of* Binomials *and* Residuals *Universal.*

1. Take the difference of the squares of the parts of the Surd to be multipled in it self, the Root whereof add to the greatest Particle, and take it from it: Then the Roots of half those Products being carefully set down according to the due sign, expresseth the true Roots.

Examp. To take the Root of $\sqrt{:8+\sqrt{48}}$: 64 and 48 are particular Squares, 16 is the Difference, 4 the Root of the Difference,

4 being added to the greater 8 = 12, $\frac{1}{2}$ 6.
4 being substracted = 4 the $\frac{1}{2}$ is 2.
} Therefore I say $\sqrt{q6}+\sqrt{2}$ is the Root of this *Binomial*.

and $\sqrt{q6}-\sqrt{2}$ the Root of the *Residual*.

The square root of $\sqrt{:\frac{Z,}{2}+\sqrt{q\frac{Zq-4Pq}{4}}}$ is gotten thus

$\frac{Z,q.}{4}\ \frac{Z,q-4Pq}{4}$ the sq.
Pq the Difference, Root is P,
} $P+\frac{Z,}{2}=\frac{Z,+2P}{2}\ \frac{1}{2}$ is $\frac{Z,+2P}{4}$
$P-\frac{Z,}{2}=\frac{Z,-2P}{2}\ \frac{1}{2}$ is $\frac{Z,-2P}{4}$

The Root of the *Binomial* is $\sqrt{q\frac{Z,+2P}{4}}+\sqrt{\frac{Z,-2P}{4}}$

The Root of the *Residual* is $\sqrt{q\frac{Z,+2P}{4}}-\sqrt{\frac{Z,-2P}{4}}$

Thus have we gone through all the simple Elements in Species, now come we to the comparative Part, it consisting in the great Rule of *Algebra* or

Æquation, the Myſtery of the whole Art of Arithmetick, by help whereof obſcure and dark queſtions are reſolved, which without it would never by the ordinary Arithmetick be ſatisfied; and therefore for the true underſtanding hereof, it will be requiſite to diſtinguiſh it into theſe Parts: Firſt the *Definition* of the Rule, *Invention* of the Æquation, and *Compoſition* of the Queſtion, the *Reduction* and ſeveral qualities of it, and its *Reſolution*; and then we will come to ſhew the uſe of it in the eaſie Reſolution of any Arithmetical Queſtion that is poſſible to be reſolved.

CHAP. IX.

Of Æquation, *and firſt of the* Invention *or finding of it out.* Sect. 1.

1. *ÆQuation* is a proportion of Æquality betwixt two or more Quantities, Numbers or things of divers denomination: As $A\text{-}C=D$ ſuppoſing $A=12.\ C=4.\ D=8.$ for $12\text{-}4=8$: The Invention and bringing the queſtion propoſed to an *Æquation* is the great difficulty, (for after we have once certainly found the Æquation, then by Reduction and reſolution the true anſwer will be found) For the eaſy finding whereof we are to conſider theſe 3 Sections following.

Sect. 1. It is very requiſite that we are provided with abundance of Analitical ſtore, to be uſed as Occaſion may offer, ſuch as Mr. *Oughtred* hath recorded

ded in the 18 Chap. of his *Clavis*, *Bachetus* before *Diaphantus*, *Billy* before his continual Proportionals; for how is it possible to resolve a Question in Proportion, unless I know, that in 4 Proportionals, the Rectangle of the Means are equal to the Rectangle of the Extremes, and such are these that follow:

1. If $A.B.C.D.E.$ be in Arith. Prop. $A\text{-}E = A\text{-}B + B\text{-}C + C\text{-}D + D\text{-}E$.

2. If $A.\ M.\ N.\ L.\ E$ ∺ $Q{:}\overline{A+E} = Q{:}\overline{M+L} = Mq + Lq + 2Nq$. For $Q{:}\overline{M+L} = Mq + 2ML\ (=2Nq) + Lq = Mq + 2Nq + Lq$.

3. If $A.\ M.\ E$ ∺ $Q{:}\overline{A+E} = Aq + 2Mq + Eq$. For $Q{:}\overline{A+E} = Aq + 2AE(=2Mq) + Eq = Aq + 2Mq + Eq$.

4. If $A.M.E$ ∺ $Q{:}\overline{A+M+N+E} = \overline{Aq+Mq+Eq} + 2\square$ ex. $\overline{A+M+E}$ in M.

5. If $A.M.N.E$ ∺ $Q{:}\overline{A+M+N+E} = \overline{Aq+Mq+Nq+Eq} + 2\square$ ex. $\overline{A+M+N}$ in $\overline{M+E}$.

6. If $A.\ M.\ E$ ∺ $Aq\text{-}Eq = Q{:}\overline{A\text{-}E} + 2qM\text{-}A$. 7. $A \times X = Mq\text{-}Nq$. 8. $E \times X = Eq\text{-}Mq = Q{:}\overline{A\text{-}E} + \overline{Mq\text{-}Aq}$. 9. $X, \text{-}Q{:}\overline{A\text{-}E} = 2Mq\text{-}2Aq$. 10. $Eq\text{-}Mq + Aq = Q{:}\overline{A\text{-}E}$.

11. $A.\ M.\ N.\ E$ ∺ let Z = Sum of them all, and let $Z = B + C$ and $B \times C = Q{:}\overline{M+N}$. Then $B = A + M$ and $C = N + E$. For $Mq\ (NA) + 3MN\ (MN + AE) + Nq(ME) = NA + MN + AE + ME$.

12. If $A.M.E$ ∺ $A \times \overline{Mq+Eq} = M \times \overline{Aq+Eq} = E \times \overline{Aq+Mq}$. For $AMq + AEq = MAq + MEq = AqE + MqE$.
13 $AE\text{-}Aq = \overline{E\text{-}A} \times A$ and $AE\text{-}Eq = \overline{A\text{-}E} \times E$.

14 $Aq + 2A + 1 = Q\,\overline{A+1}$. 15. $Aq\text{-}2A + 1 = Q{:}\overline{A\text{-}1}$. 16. $Ac + 3Aq + 3A + 1$ = Cube $\overline{A+1}$. 17. If $Aq.\ AE.\ Eq$ ∺ I say $Aq.\ AE\text{-}E.\ Q{:}\overline{A\text{-}E}$ ∺ and $Eq.\ AE\text{-}E.\ Q{:}\overline{A\text{-}E}$ ∺ 18. $Aq \times Eq = Q{:}\overline{A \times E}$. 19. $Q{:}\overline{Ac+Bc} = Q{:}\overline{Ac\text{-}Bc} + 4AcBc$. 20. $Ac\text{-}Ec$ = Cube $\overline{A\text{-}E} + \square\ 3AE \times \overline{A\text{-}E}$. 21. $Ac\text{-}Ec = Ccc{:}\overline{A+E}\ \square\text{-}3AE \times X =$ 'X.

Sect. 2. When a queſtion in Arithmetick is asked, you may place for the quantity or Number ſought *A*, (or *A* joyned with ſome number for the eaſe in working the queſtion) or any other vowel, expreſſing thoſe things that are certainly known by Conſonants, and the unknown or what is ſought by vowels; but after *Des-Chartes* if you practice with the ſmall letters, then mark all the given quantities with the beginning of the Alphabet, *a*, *b*, *c*, *d*, &c. and the unknown with the laſt letters *x*, *y*, *z*, &c. Which according to the tenor of the Queſtion, it muſt be examined, (as though you had it given, and were proving the truth of it) by ſome of theſe Means following, till you can bring it to an *Æquation*.

1. Sometimes the ſought numbers or quantities are many, and that by one Root put they cannot be known, therefore in this caſe we put as many vowels or letters for the firſt magnitude as ſhall be needful, and then as many letters as you uſe for the Roots, ſo many ſeveral Æquations to find out the values of theſe letters in relation to *A*: and herein great care is to be had, that after you have found out the values of thoſe ſecond Roots, you never let them again enter the queſtion.

2. Sometimes you muſt make uſe of ſome known Problem you have in ſtore, which will expedite your Work, and eaſe you very much.

3. And ſometimes you muſt canvaſe your queſtion by Addition, Subſtraction, Multiplication and Diviſion, or by ſome proportion or other artifice, till you have at length brought out your Æquation, which muſt be ſo ordered and reduced, that the Species of the quantity ſought, with all his parodical degrees,

may

may make up one ſide,and the given magnitudes the other ; the which is taught in the Chapter following.

Sect. 3. It will be no little help to the Invention, to obſerve this Rule, That whenſoever any Arithmetical queſtion (wherein numbers are joyned with material things, as Men,Money,Time,Place,*&c.*)is propounded, that is a practical queſtion, the ſame be ſeparated from ſuch things, and reſolved in the pure conſideration of Numbers. The Examples following will more plainly ſet out the meaning of this Section.

Queſtion 1. A certain Man went 9 miles a day, another Man follows him from the ſame place, but ſet forth 10 days after, and went 14 miles each day, in how many days will he overtake him? The queſtion *abſtractly* put, I demand what number is that which being multiplied by *B* (9) and the Product added to *C* (90) is equal to the ſame number multiplied by *D* (14.)

2. A certain man hired a ſervant for a year upon this condition, That for every day that his ſervant laboured he ſhould have a ſhilling,and for every day he was idle or played, he ſhould loſe or diſcount eight pence : Now at the years end the Maſter was to give the Servant nothing, nor the ſervant any thing in the maſters debt: It is deſired to know what days the ſervant laboured, and what he played?

Abſtractly put thus ; divide the number *B*(365) into two parts, that the one multiplied by *C* (12) ſhall make ſo much as the other drawn into *D* (8.)

3. A Vintner ſold 30 Bottles of Wine for 210 ſhillings, whereof ſome of the bottles were White-wine,

wine, others Claret; but he sold one bottle of White-wine for 5 *s.* and a bottle of Claret for 8 *s.* it is demanded what Bottles there were of White-wine, and what of Claret?

Abstractly in numbers thus; divide *B* (30) into two such parts, that the one part drawn into *C* (5) the other part into *D* (8) shall make *F* (210.)

4. There are two sorts of Monies in number 1000, worth 80 pound, whereof 10 of the one kind, and 20 of the other are worth 1 pound; it is demanded what number there were of both these sorts?

Abstractly in numbers thus; divide the number *B* (1000) into two such parts, that the one divided by *C* (10) the other by *D* (20) the two Quotients shall be *F* (80.)

5. Suppose *Barwick* and *London* are distant 228 miles out of which two Foot-Posts take their journies and meet the 12 day, but the one went each day one mile further than the other: it is desired to know what miles each went every day?

Abstractly thus; two numbers are sought whose difference is *X* (1) That if both of them severally be drawn into *B* (12) the Sum of the Product will be equal to *C* (228.)

6. Among two Regiments of Soldiers, whereof the one being Footmen exceeds the other being Horsemen by 300 persons, there was distributed 4000 Crowns, and every Soldier of the Company of Horse had 3 Crowns more than a Foot Soldier: it is demanded how many Foot Soldiers and how many Horse there were in each Regiment?

Abstractly thus; divide the number *B* (4000) by two numbers, so that the lesser shall be exceeded by the

the greater by *C* (300,) and let the Quotient of the firſt work be greater than the Quotient of the ſecond by the number *D* (3:)

7. One that had bought 100 yards of Cloth, another demanded of him what a yard coſt him? He anſwered, for how much leſs I bought 4 yards than for 80 ſhillings, by ſo much leſs did I buy 50 yards than 95 ſhillings.

Abſtractly thus; a number is ſought that being drawn into *B* (40) and *C* (50,) and from the Products if you ſubtract *D* (80) and *F* (95) the Remains will be equal.

And from hence it is apparent converſly how to propound a queſtion, out of numbers ſimply joyned with material ſubſtances. And further it is diligently to be noted, that when divers numbers are propoſed to be found, that you ſeek out the firſt, for the which you placed *A*; as if you did put it for the 1, 2, or 3, then in the reſolution of the Æquation it may ſtill ſignifie the firſt, ſecond or third, according to the Poſition.

And note, that of all the parts of Æquation, this only of the *Invention* is moſt difficult; for beſides the neceſſity of the knowledge of the principles of Geometry and Arithmetick, together with a good judgment, it is very requiſite, that he that deſires to be a learned *Analiſt*, ſhall practiſe the finding out of many hard queſtions, which he may both have from others, and propoſe to himſelf, as occaſion may ſerve.

Sect. 2. *Of the Reduction of the Æquation, the Work Concomitant with it.*

1. To reduce an Æquation, is ſo to order and diſpoſe it (keeping it ſtill to an equality) that it may be fit for reſolution; and this is done five manner of ways: 1. By Tranſpoſition. 2. By Reduction of the Fractions into an Æquation (if there be any) into whole ſpecies. 3. By common Depreſſion. 4. By a common Diviſion of both ſides of the Æquation. 5. By Reduction of either ſide of the Æquation to the ſame power, and cancelling the Coſſick figures.

Reduction by Addition or Subſtraction.

1. That the magnitude or magnitudes ſought may make up the one ſide of the Æquation by Tranſpoſition, you muſt convert the given magnitudes, and they ſought from the one ſide to the other, changing the ſigns.

1. $ac+bc=d$. $ac=d-bc$. 2. $aee+dce=gce$ and $aee=gce-dce$.

3. $dcga+bg=bgdc+5dca$ and $dcga-5dca=bgdc-bg$.

4. $aa+bc=db+af$ and $aa+bc-af=bd$ and $aa-af=db-bc$.

5. $m=2aa-2xa+xx$ and $2aa-2xa=m-xx$.

Reduction by Multiplication.

2. If every or any magnitude in either ſide of the Æquation be a Fraction or Fractions, multiply all the

the other ſpecies in each other denominator but his own; and this is nothing but the work of Fractions, and therefore as occaſion ſerves the greateſt common meaſure may likewiſe be taken. *Examples.*

(1) $a+b=\frac{d}{c}$ and $ac+bc=d$.

3. $a+\frac{b}{dc}=b+\frac{5a}{g}$ and $dcga+bg=bgdc+5dca$.

4. $\frac{acbb}{c}+bbga=\frac{2ddba}{3}+\frac{cbbaa}{d}$

and $3acbbd+3bbgdca=2d^3cba.+3cbaaac$.

Reduction by Diviſion 1.

3. If the magnitude ſought be found in all the magnitudes on either ſide of the Æquation; then you muſt by depreſſion or dividing the magnitudes ſought by ſome Parodical degree, clear ſome ſpecies of the Æquation, that the one ſide may be known.

$ac+baa=da$ depreſſed becomes $aa+ba=d$.

$a^4b+5dcaa=bcaa+daa$ and $aab+5dc=bc+d$ and $aab=bc+d-5dc$.

$a^5d-5baa=dca^4$ and $a^3-5b=dcaa$ and $a^3-dcaa=5b$.

Reduction by Diviſion 2.

4. If the Magnitude ſought or his higheſt Power be joyned with another ſpecies, then muſt the reſt of the ſpecies be divided by that ſpecies, and the higheſt power cleared thereof.

1. $aab = d + c$ and $aa = \frac{d+c}{b}$

2. $aab - da = r$ and $aa - \frac{d}{b} * a = \frac{r}{b}$

5. If any ſpecies be expreſſed in the Æquation by a Coſſick Root, then muſt the reſt be exalted to that Power.

(1) $b + c = \sqrt{a}$ and $bb + 2bc + cc = a$.

(2) $b = \sqrt{ab} - d$ and $b + d = \sqrt{ab}$. Then $bb + 2bd + dd = ab$ and $a = \frac{b + 2d + dd}{b}$

(3) $b + d = \sqrt{:b} * \sqrt{d}$: Then $bb * 2bd + dd = b + \sqrt{d}$ and $bb + 2bd + dd - b = \sqrt{d}$ to be reduced as before.

Examples where all the Rules of Reduction are uſed.

If this Æquation $Zq = \frac{PRq + PSq + 2PRS}{RS}$ were given, and ſuppoſe the ſought Magnitude were S. Then $ZqRS = PRq + PSq + 2PRS$ and $ZqRS - 2PRS - PSq = PRq$ and $\frac{ZqR - 2PR * S - Sq}{P} = Rq$.

Supppoſe R were ſought in the ſame Æquation, Then $ZqRS - 2PRS - PRq = PSq$ and $\frac{ZqS - 2PS * R - Rq}{P} = Sq$.

Suppoſe P were ſought, Then $RqP + SqP + 2RSP = ZqRS$ and $P = \frac{ZqRS}{Rq + Sq + 2RS}$

If

If the given magnitude be equal to 2 Surds, as $Z = \sqrt{AP} + \sqrt{+AS}$. Then $Zq = AP + AS + \sqrt{4AqPS}$. Then $Zq - AP - AS = \sqrt{4AqPS}$ to be resolved as before.

2. These 5 several parts of Reduction do not change the equality.

The 1. doth not alter the equality, the same magnitude being added to both sides or substracted.

The 2. doth not change the equality, for let $\frac{2}{4} = \frac{3}{6}$ Then 2. 4 :: 3. 6. and because the Product of the Means are equal to the Extremes, therefore $4 \times 3 = 2 \times 6$. in species $\frac{A}{B} = \frac{C}{D}$ Then $A. B :: C. D.$ and $BC = AD$.

The 3. and 4. do not alter the equality, both sides of the Æquation being divided by one magnitude.

The 5. doth not alter the equality, both sides being squared or multiplied into themselves.

3. But if the Æquation fall out contrary to the definition, then it is either *Nugatory* or *Impossible*: (1) As if there be an Identical Æquation thus, $3Aq = 3Aq$. Then may it be resolved by any number whatsoever and therefore it is a vain question: 2. If the Æquation fall out to be $5Aq = 4Aq$. Then is the question propounded utterly impossible and cannot be resolved: Likewise if the question be $Aq = 12A - 40$.

Because 40 cannot be taken from 36 the square 6; $\frac{12}{2}$: therefore likewise is this question impossible to be answered: and thus much for the work accompanying the Æquation.

Sect. 3.

Sect. 3. *Of the Resolution of all sorts of Æquations in Numbers.*

1. After your Æquation is invented and reduced, then the sought magnitude is either pure, As $A = B+C$, or secondly it is some Poteſtas, *Aq. Ac. Aqc.* &c. or thirdly, it is an adfected Æquation, of which laſt ſort there are alſo two kinds: 1. The Indices of the Æquation are in Arithmetical Proportion, As $Aq+BA=C$, that is, 0.1.2. $Aqq-DAq=A$, that is 4.2.0. or 6.3.0. or 8.4.0, *&c.* Or ſecondly, they are not aſcending or deſcending in ſuch Coſſick ſigns as will make up any part of Arithmetical Proportionals.

1. If the ſought magnitudes be a pure Root, as *A*, then according as the other ſide of the Æquation doth intimate, *viz.* by Addition, Subſtraction, Multiplication or Diviſion, you may find the value thereof: As for Example, $A=\frac{X,+Xq}{2X}$ ſuppoſing $X,=5$, and $X=1$ as in the Chapter following $\frac{5+1}{2}=\frac{6}{2}=3=A$. In Words you muſt add the number intimated by X, to the ſquare of the Number intimated by *X*, and divide that Sum by *X* doubled, the Quotient is equal to *A*.

2. If the ſought magnitude be ſome *Poteſtas*, then after you have added, ſubſtracted, multiplied or divided, the other ſide of the Æquation as the ſpecies inform you, then muſt you extract the Root of that Reſult according to the Coſſick Sign annexed to the ſought ſpecies; As for *Aq* take the ſquare Root, *Ac* the Cube Root, &c.

The

The Extracting of these Roots, as also the Roots of all adfected Æquations in numbers are most learnedly taught by the oft before mentioned Mr. *Oughtred* in his *Clavis*, and though by some handled, yet never before so plainly, methodically and succinctly explained, not only in the true nature of the Roots and their adfections, but in the easie invention of the first and second Figures of the Root, which before was a business of very great difficulty, as those who make experiments therein may very readily perceive.

For the more easie and speedy Extracting of all sorts of Roots, I shall commend to the Reader, and especially to all Gentlemen that are willing to study this Art (who as they ought to be encouraged to proceed with as much ease as can be, so commonly being deterred by the tediousness of practice and difficulty, do not only leave it off but deter others from studying it) these two ways, *viz. Nepair's* Bones, and the use of the Table at the end hereof joyned both together, by help whereof I dare affirm to extract a simple or adfected Æquation, and that without the ascribing any thing to memory, in half the time, if not less than any who shall do it without.

3. The Table annexed to the end of this Book, contains the Squares and Cubes of all numbers under 1000, the squared Squares of all numbers under 500, and the squared Cubes and cubed Cubes under 250: There is also all the Multiplees of single Squares, Cubes, square Squares, and square Cubes annexed, which are of great use likewise in Extraction.

4. If the ſquare Root of any Number under 1000000, the cube Root of any Number under 1000000000, the *qq* Root of any Number under 6250000000000, the *qc* Root of any Number under 97656250000, or cube Cube Root under 244140625000000 be demanded, under each reſpective Power ſeek out for your number given, and under *A* if you hit juſt of that Number you have the true Root; as if it be demanded what is the $\surd q$ of 243049, I find it to be 493; the $\surd qq$ of 1121513121, I find it to be 183, &c.

But if your number fall out betwixt other two numbers that are not truly radical, then you may be ſure the higher Root is the Root of Integers, and then you may continue it to get a *decimal Fractional* Root by joyning 3 Cyphers to the difference; but by what eaſe and ſpeedy practice both this and other the many Myſteries of theſe Numbers may be wrought, neither the ſtinted bigneſs of this Book, nor other my imployments will give me leave to ſet down.

The Extraction of the ſquare Root by the Bones.

$$\left.\begin{array}{r} Aq: \\ \ldots\ldots \\ \text{Diviſor } 2A:E \\ :Eq \end{array}\right\}\text{Gnom.}$$

Firſt, point every other Figure from your right hand, beginning with the laſt; and finding upon the ſquare Bone, the next leſs or equal ſquare to the Figures of the firſt point to your left hand, ſet the Index of the Quotient, and ſubſtracting the true ſquare from the Number above, to the Remain add the next two figures, and for a Diviſor being (2*A*) double

double the Quotient, and ſet the doubled Quotient on the Bones, and place them before the ſquare Bone, which done, work as in Diviſion, by ſeeking a Quotient, and ſubſtracting the Number on the Bones, (which are equivalent to (2*AE*+*Eq*) the Gnomon) from the Numbers above; only obſerving this difference, that you muſt ſtill draw down two figures, and that every time you muſt double your Quotient for a new Diviſor.

```
            .  .  .  .
          10601536  (3256
     A) 09 Aq
 2A6)    160 R:
         124 (2AE+Eq.
 2A64)   3615 R:
         3225 (2AE+Eq.
 650)    39036 (R:
         39036 (2AE+Eq
         000000
```

```
        .  .  .  .  .
          34'00000000 (5'8308 &c.
      A) 25 Aq
 2A10)     900  R:
           864 (2AE+Eq
 2A116)    3600 R:
           3489 (2AE+Eq
 2A1166)   11100 R:
 2A11660)  1110000 R:
           932864 (2AE+Eq
           177136 &c.
```

To extract the Cube Root.

You are to obſerve the ſame work as in the ſquare Root with theſe Differences.

$$\begin{array}{l} \qquad\qquad\quad Ac \\ \text{Diviſor}\left\{\begin{array}{l} \cdot\ \cdot\ \cdot\ \cdot \\ 3\,Aq{:}E \\ 3\ \ A{:}Eq \end{array}\right. \left.\begin{array}{l} \\ \\ \\ :Ec \end{array}\right\} \text{Gnom.} \end{array}$$

1. That you point every third Figure.

2. That you triple the ſquare of the Quotient (which is nothing but to find the ſquare of the Quotient in the Table of Squares, and to multiply it by 3) for every new Diviſor, which muſt be ſet before the Cube Bone; and that after you have found out a Quotient, you muſt to the Numbers on the Bones add the triple of the Quotient multiplied by the ſquare of the laſt figure found (which by another ſet of Bones had in readineſs may eaſily be done) placing it in one place towards the left hand, that is, ($3\,AEq$) A ſignifying all the Quotient, and E the laſt figure and if then the Gnomon, that is, ($3\,AqE+3\,AEq+Ec$) do exceed the figures, you muſt abate one from the laſt Quotient: The which by obſerving the work may eaſily be avoided.

The

The Analysis of the Cube.

146363183 (527.
125 Ac.
3 *Aq* ... 75) 21363 Resid.
15008 .. 3 *AqE*+*Ec*
60 .. 3 *AEq*
15608
3 *Aq* .. 8112) 5755183 Resid.
5678743 .. 3 *AqE*+*Ec*
7644 .. 3 *AEq*
5755183
00 Resid.

To extract the squared square Root.

The Extraction only alters in the Points, which must be every fourth Figure, the Divisor which must now stand of two setts of Bones, and upon the one the Cube of the Quotient drawn into 4, and the square of the Quotient drawn into 6: and lastly, that after you have estimated a new Quotient, you must add to the number on the Bones these two Sums, *viz.* (6 *Aq Eq*) the square of the Quotient in 6, in the square of the last Figure set two places short from the sum of the Bones, and the Quotient in 4 in the Cube of the last Figure set one place short. An Example will make the matter more plain.

Aqq

: . . . . .

Divisor { 4 *Ac* : *E* / 6 *Aq* : *Eq* / 4 *A* : *Ec* } Gnomon.

: *Eqq*

The Analysis of the Biquadratick Power.

	1071	1675	9488	0904	1761	(57209
	625	*aqq*				
	446	1675				
500 (1))	. . . . .	. . . . .				*aqq*
150 (2))	350	2401	4*ace*+*eqq*			. . . . . .
	73	50	6*aqeq*			4*ac* : *e*
	6	860	4*aec*			6*aq* : *eq*
	~~430~~	~~6001~~	Gno.			4*a* : *ec*
	15	5674	9488			: *eqq*
740772 (1))	14	8154	4016	4*ace*+	*eqq*	
19494 (2))		779	76	6*aqeq*		
		1	824	4*aec*		
	~~14~~	~~8935~~	~~9856~~	Gno.		
74879332528 (1))		6738	9632	0904		
1963104 (2))		6738	9632	0904	1761	
		6737	3729	2800	6561	4*ace*+*eqq*
74879332528000)		1	5901	1424	00	6*aqeq*
			1	6679	520	4*aec*
		~~6738~~	~~9632~~	~~0904~~	~~1761~~	

And

And note that all along the Operation I make uſe of the Table annexed at the end hereof, as when I had 572 to be cubed and multiplied by 4, I find Cube of 572, which is 186149348, and multiply it by 4=744596992, which I do with the Squares, *&c.* which is no ſmall eaſe in the Work.

The Roots of the other Powers are after the ſame manner extracted, obſerving their Points and intermediate Species, which by *Chap.* 4. *Sect.* 5. are eaſily made up, annexing the *Unciæ* to the *A*, and note that all the *Unciæ* and *A* are ſtill the Diviſor, the *Unciæ* *A* and *E* the *Gnomon*: and that further the numbers belonging to the higheſt Powers will fall directly under the Points, and the intermediate Powers will fall ſo far ſhort of either, as they are in diſtance from them.

Aqc:
. . . .
Diviſor *Aqq* : *E*
Ac : *Eq*
Aq : *Ec* Gnomon.
A : *Eqq*
Eqc

5. If the one ſide of the Æquation be an adfected Æquation, and yet the Indices thereof are in Arithmetical Proportion, then it being ſome of theſe that follow, *ZA-Aq*=*Æ*. *Aq-XA*=*Æ*. Or *Eq*+*XE*=*Æ*.

The Resolutions by Chap. 16. Sect. 9. *are*

$$1.\ \frac{Z}{2} + \sqrt{q}:\frac{Zq - 4Æ}{4}: = \frac{A}{E}$$

$$2.\ \sqrt{q}:\frac{Xq + 4Æ}{4}: + \frac{X}{2} = \frac{A}{E}$$

But If it happen the middle Species to be a Square or a Cube, then must it be the universal square Root or cube Root As: if it be $Z, Aq - Aqq = Æq$ the resolution is

$$\sqrt{q}:\frac{Z}{2} + \sqrt{q}\frac{Zq - 4Pq}{4}: = \frac{A}{E}$$

The resolution into numbers is as easie, for if the highest Species be negative, then the Rule is, take half of the Coefficient and add to it, or substract from it the square Root of the square of the Coefficient, lessened by the other side of the Æquation in 4, and divided by 4, the Sum or difference is the greater or lesser Root, and so of the other Rules; only if the middle Species be a square number, then you must exstract the square Root, *&c.*

6. If the one side of the Æquation be adfected, and the Indices not in Geometrical Proportion, you must extract the Root thereof after the Rules delivered in the *Clavis Math.* I shall only give you a Breviate how to make the Canon for extracting the Root: If this Æquation $Aqc - DAq + CA = B$ were proposed.

Aqc	$-DAq$	CA
$.....$	$.....$	$....$
$\left\{\begin{array}{l} Aqq : E \\ Ac : Eq \\ Aq : Ec \\ A : Eqq \\ \quad Eqc \end{array}\right\}$	$-D2A:E$ $-D\,..:Eq$	$C:E$

The first Divisor is $C-D$.

The first Ab. $Aq+CA-DAq$.

The second Divisor is $5Aqq+10Ac+10Aq+5A+C-D2A-D$.

The second Ab. $5AqqE+10AcEq+10AqEc+5AEqq+Eqc+CE-D2AE-DEq$.

The Pricks and intermediate Species follow the difference, as was before set down in the pure Roots.

And if you observe in the Extraction the Powers, you may out of the Table at the end hereof, take out the numbers answering with great ease: And thus have we gone through the parts of Æquation, now come we to practise them and the former Rules by Examples.

CHAP.

CHAP. X.

Containing several considerations of two Numbers, and Questions deduced from them; wherein all the former Rules in this Book are practised, being very useful for the managing of an Æquation.

THe general Varieties go upon these Questions, that having any two of the Numbers mentioned, *Chap.* 1. *Sect* 16. thereof, $a, b, z, x, p, \frac{r}{s}, m, n$, to find out any of the rest.

Quest. Having two Numbers given, whereof a the greater and b the lesser, to find their Sum, Difference, Rectangle, Proportion, Sum of Squares, and Differences of Squares, $z, x, p, \frac{r}{s}\, aa, bb, m$ & n.

The same order being observed in all the Questions, one Instruction will serve for all; on the left hand the streight line stand the Symbols, generally but seven, because one Symbol is given, which only save the wording of it, as what is the greater number, what is the less, what is the difference, &c. instead of which words are placed a for the greater, b for the less number, z the Sum, x the difference, p for the multiplication, $\frac{r}{s}$ the proportion that a hath to b, aa the square of the greater, bb the square of the less, m the Sum of the Squares: there rests therefore only two things to be considered; First, how the Quantities in the place of a and b are had; Secondly, how the rest of the Species are found, both which shall be delivered in 2 Sections immediately after the Questions are set down. (1)

(1) *D*: *a* & *b* qr. *z*, *x*, $\frac{r}{s}$ &c.

a	a
b	b
z	$a+b$
x	$a-b$
p	ab
$\frac{r}{s}$	$\frac{rb}{a}$
aa	aa
bb	bb
m	$aa+bb$
n	$aa-bb$

(2) *D*: *a* & *z* qr. *b*, *x*, $\frac{r}{s}$ *paa*, &c. Æqu. (per 1) $a+b=z$ & $b=z-a$.

a	a
b	$z-a$
x	$2a-z$
p	$za-aa$
$\frac{r}{s}$	$\frac{zr-ra}{a}$
aa	aa
bb	$zz-2za+aa$
m	$zz-2za+2aa$
n	$2za-zz$

(3) *D*: *b* & *z* qr. *a*, *x*, *p*, &c. Æq. (per 1) $z=a+b$ & $a=z-b$.

a	$z-b$
b	b
x	$z-2b$
p	$zb-bb$
$\frac{r}{s}$	$\frac{rb}{z-b}$
aa	$zz-2zb+bb$
bb	bb
m	$zz-2zb+2bb$
n	$zz-2zb$

(4) *D*: *x* & *a* qr. *b*, *z*, *p*, &c. Æq. $x=a-b$ & $b=a-x$.

a	a
b	$a-x$
z	$2a-x$
p	$aa-xa$
$\frac{r}{s}$	$\frac{ra-xa}{a}$
aa	aa
bb	$aa-2xa+xx$
m	$2aa-2xa+xx$
n	$2xa-xx$

(5) *D*: x & b qr. a, z, p, &c. Æq. (per 1) $x=a-b$ & $a=x+b$.

a	$x+b$
b	b
z	$x+2b$
p	$xb+bb$
$\frac{r}{s}$	$\frac{rb}{x+b}$
aa	$xx+2xb+bb$
bb	bb
m	$xx+2xb+2bb$
n	$xx+2xb$

(6) *D*: p & a qr. a, b, z, &c. Æq. $p=ab$ & $b=\frac{p}{a}$

a	a
b	$\frac{p}{a}$
z	$\frac{aaa+p}{a}$
x	$\frac{aa-p}{a}$
$\frac{r}{s}$	$\frac{rp}{aa}$
aa	aa
bb	$\frac{pp}{aa}$
m	$\frac{aaaa+pp}{aa}$
n	$\frac{a^4-pp}{aa}$

(7) *D*: p & b qr. a, b, z, x, &c. Æq. (per 1) $p=ab$ & $a=\frac{p}{b}$

a	$\frac{p}{b}$
b	b
z	$\frac{bb+p}{b}$
x	$\frac{bb-p}{b}$
$\frac{r}{s}$	$\frac{rbb}{p}$
aa	$\frac{pp}{bb}$
bb	bb
m	$\frac{pp+b^4}{bb}$
n	$\frac{pp-b^4}{bb}$

(8) *D*: $\frac{r}{s}$ & a qr. b, z, x, p, &c. Æq. (per 1) $\frac{r}{s}=\frac{a}{b}$ & $sa=rb$ & $b=\frac{sa}{r}$ & $a=\frac{rb}{s}$

a	a
b	$\frac{sa}{r}$
z	$\frac{ra+sa}{r}$
x	$\frac{ra-sa}{r}$
p	$\frac{saa}{r}$
aa	aa
bb	$\frac{ssaa}{rr}$
m	$\frac{rraa+ssaa}{rr}$
n	$\frac{rraa-ssaa}{rr}$

(9)

(9) D: $\frac{r}{s}$ & b qr. a, z, x, p, &c. Æq. per ult. $a = \frac{rb}{s}$.

a	$\frac{rb}{s}$
b	$\ldots b$
z	$\frac{rb+sb}{s}$
x	$\frac{rb-sb}{s}$
p	$\frac{rbb}{s}$
aa	$\frac{rrbb}{ss}$
bb	bb
m	$\frac{rrbb+ssbb}{ss}$
n	$\frac{rrbb-ssbb}{ss}$

(10) D: m & a qr. b, z, x, p, &c. Æq. (per 1) $m = aa+bb$. $b = \sqrt{m-aa}$.

a	a
b	$\sqrt{m-aa}$
z	$a+\sqrt{m-aa}$
x	$a-\sqrt{m-aa}$
p	$\sqrt{maa-a^4}$
$\frac{r}{s}$	$\frac{\sqrt{mrr-rraa}}{aa}$
aa	aa
bb	$m-aa$
n	$2aa-m$

(11) D: b & m qr. a, z, x, p, &c. Æq. $m = aa+bb$ & $a = \sqrt{m-bb}$.

a	$\sqrt{m-bb}:$
b	b
x	$\sqrt{m-bb}:+b$
z	$\sqrt{m-bb}:-b$
p	$\sqrt{mbb-b^4}:$
$\frac{r}{s}$	$\sqrt{\frac{rrbb}{m-bb}}$
aa	$m-bb$
bb	bb
n	$m-2bb$

(12) D: n & a qr. b, z, x, &c. Æq. $n = aa-bb$ & $b = \sqrt{aa-n}$.

a	a
b	$\sqrt{aa-n}:$
z	$a+\sqrt{aa-n}:$
x	$a-\sqrt{aa-n}:$
p	$\sqrt{a^4-naa}:$
$\frac{r}{s}$	$\sqrt{\frac{aarr-nrr}{aa}}$
aa	aa
bb	$aa-n$
m	$2aa-n$

(13)

(13) D: n & b qr. a, z, x, p, &c. Æq. $n = aa - bb$ & $a = \sqrt{n+bb}$.

$$\begin{array}{r|l}
a & \sqrt{n+bb} \\
b & b \\
z & \sqrt{n+bb}:+b \\
x & \sqrt{n+bb}:-b \\
p & \sqrt{nbb+b^4}: \\
\frac{r}{s} & \sqrt{\frac{rrbb}{n+bb}} \\
aa & n+bb \\
bb & bb \\
n & n+2bb
\end{array}$$

(14) D: z & x qr. a, b, p, &c. Æq. (per 2 & 3) $x = 2a - z$ & $a = \frac{z+x}{2}$ & $x = z - 2b$ & $b = \frac{z-x}{2}$

$$\begin{array}{r|l}
a & \frac{z+a}{2} \\
b & \frac{z-x}{2} \\
p & \frac{zz-xx}{4} \\
\frac{r}{s} & \frac{zr-xr}{zx} \\
aa & \frac{zz+2zx+xx}{4} \\
bb & \frac{zz-2zx+xx}{4} \\
m & \frac{zz+xx}{2} \\
n & zx
\end{array}$$

(15) D: z & p qr. a, b, x, $\frac{r}{s}$, aa, &c. Æq. (per 6) $z = \frac{aa+p}{a}$ & $za = aa + p$ & $za - aa = p$ & $\frac{z}{2} \pm \sqrt{\frac{zz-4p}{4}} = \frac{a}{b}$

$$\begin{array}{r|l}
a & \frac{z}{2} + \sqrt{\frac{zz-4p}{4}} \\
b & \frac{z}{2} - \sqrt{\frac{zz-4p}{4}} \\
x & \sqrt{zz-4p}: \\
\frac{r}{s} & \frac{2zr - \sqrt{zzrr-4prr}:}{2zr + \sqrt{4zz-16p}:} \\
aa & \frac{2zz-4p}{4} + \sqrt{\frac{4z^4-16zzp}{16}}: \\
bb & \frac{2zz-4p}{4} - \sqrt{\frac{4z^4-16zzp}{16}}: \\
m & zzx-2p \\
n & \sqrt{z^4-4zzp}:
\end{array}$$

(16) D: z & $\frac{r}{s}$ qr. a, b, z, x, &c. Æq. (per 8 & 9) $z = \frac{ra+sa}{r}$ & $zr = ra + sa$ & $a = \frac{zr}{r+s}$ & $b = \frac{zs}{r+s}$

$$\begin{array}{r|l}
a & \frac{zr}{r+s} \\
b & \frac{zs}{r+s} \\
x & \frac{zr-zs}{r+s} \\
p & \frac{zzrs}{rr+2rs+ss} \\
aa & \frac{zzrr}{rr+2rs+ss} \\
bb & \frac{zzss}{rr+2rs+ss} \\
m & \frac{zzrr+zzss}{rr+2rs+ss} \\
n & \frac{zzrr-zzss}{rr+2rs+ss}
\end{array}$$

(17)

(17) D: z & m qr. a, b, x, p, &c. Æq. (per 10. 11) $z=a+\sqrt{m-aa}$, &c. by Red.

a	$\frac{z}{2}+\sqrt{\frac{2m-zz:}{4}}$
b	$\frac{z}{2}-\sqrt{\frac{2m-zz:}{4}}$
x	$\sqrt{2m-zz:}$
p	$\frac{zz}{4}-\sqrt{\frac{2m-zz:}{4}}$
$\frac{r}{s}$	$\frac{2zr-\sqrt{8mrr-8zzrr}}{2z+\sqrt{8m-8zz}}$
aa	$\frac{m}{2}+\sqrt{\frac{2mzz-z^4}{4}}$
bb	$\frac{m}{2}-\sqrt{\frac{2mzz-z^4}{4}}$
n	$\sqrt{2mzz-z^4}$

(18) D: x & p qr. $a, b, z, \frac{r}{s}$ &c. Æq. (per 4) $p=aa-xa$, &c. by Red.

a	$\sqrt{:\frac{xx+4p}{4}:}+\frac{x}{2}$
b	$\sqrt{:\frac{xx+4p}{4}:}-\frac{x}{2}$
z	$\sqrt{xx+4p:}$
$\frac{r}{s}$	$\frac{\sqrt{4xxrr+16prr:}-2xr}{\sqrt{4xx+16p:}+2x}$
aa	$\sqrt{\frac{x^4+4xp}{4}}:+\frac{xx+2p}{2}$
bb	$\sqrt{\frac{x^4+4xp}{4}}:-\frac{xx+2p}{2}$
m	$xx+2p$
n	$\sqrt{x^4+4xp:}$

(19) D: z & n qr. a, b, x, p, &c. Æq. (per 3) $n=2za-zz$, &c. by Red.

a	$\frac{zz+n}{2z}$
b	$\frac{zz-n}{2z}$
x	$\frac{n}{z}$
p	$\frac{z^4-nn}{4zz}$
$\frac{r}{s}$	$\frac{zzr-nr}{zz+n}$
aa	$\frac{z^4+2zzn+nn}{4zz}$
bb	$\frac{z^4-2zzn+nn}{4zz}$
m	$\frac{1z^4+2nn}{4zz}$

(20) D: x & $\frac{r}{s}$ qr. a, b, z, x, &c. Æq. (per 8) $s=\frac{ra+ar}{a}$ &c. by Red.

a	$\frac{rx}{r-s}$
b	$\frac{sx}{r-s}$
z	$\frac{rx+sx}{r-s}$
p	$\frac{rxxs}{rr-2rs+ss}$
aa	$\frac{rrxx}{rr-2rs+ss}$
bb	$\frac{ssxx}{rr-2rs+ss}$
m	$\frac{rrxx+ssxx}{rr-2rs+ss}$
n	$\frac{rrxx-ssxx}{rr+2rs+ss}$

(21)

(21) *D*: x & m qr. a,b,z,x, &c. Æq. (per 4) $m=2aa-2xa+xx$ & by Red.

a	$\sqrt{\frac{2m-xx}{4}}:+\frac{x}{2}$
b	$\sqrt{\frac{2m-xx}{4}}:-\frac{x}{2}$
z	$\sqrt{2m\,xx}$
p	$\frac{m-xx}{2}$
$\frac{r}{s}$	$\frac{\sqrt{4mrr-2xxrr}:-xr}{\sqrt{4m-2xx}:+x}$
aa	$\frac{m}{2}+\sqrt{\frac{2mxx-x^4}{4}}:$
bb	$\frac{m}{2}-\sqrt{\frac{2mxx-x^4}{4}}:$
n	$\sqrt{2mxx-x^4}$

(22) *D*: x & n qr. a,b,z,x, &c. Æq. (per 4) $n=2xa-xx$ & by Red.

a	$\frac{n+xx}{2x}$
b	$\frac{n-xx}{2x}$
z	$\frac{2n}{2x}$
p	$\frac{nn-x^3}{4xx}$
$\frac{r}{s}$	$\frac{nr+xxr}{n+xx}$
aa	$\frac{nn+2nx+x^a}{4xx}$
bb	$\frac{nn-2nxx+x^a}{4xx}$
m	$\frac{nn4x^4}{2xx}$

(23) *D*: p & $\frac{r}{s}$ qr. a,b,z,x, &c. Æq. (per 7) $s=\frac{rp}{aa}$ & by Red.

a	$\sqrt{\frac{pr}{s}}$
b	$\sqrt{\frac{ps}{r}}$
z	$\sqrt{\frac{pr}{s}}+\sqrt{\frac{ps}{r}}$
x	$\sqrt{\frac{pr}{s}}-\sqrt{\frac{ps}{r}}$
aa	$\frac{pr}{s}$
bb	$\frac{ps}{r}$
m	$\frac{prr+ps_s}{rs}$
n	$\frac{prr-ps_s}{rs}$

(24) *D*: p & m qr. a,b,z,x, Æq. (per 6) $m=\frac{a4+pp}{aa}$ & by Red.

a	$\sqrt{:\frac{m}{2}:+\sqrt{\frac{mm-4pp}{4}}}$
b	$\sqrt{:\frac{m}{2}:-\sqrt{\frac{mm-4pp}{4}}}$
z	$\sqrt{m+2p}:$
x	$\sqrt{m-2p}:$
$\frac{r}{s}$	$\frac{\sqrt{:2mrr-\sqrt{4mmr4-16p2r4}}}{\sqrt{:2m,+\sqrt{mm-16pp}}}$
aa	$\sqrt{:\frac{mm-2pp}{2}+\sqrt{\frac{m4-4m2pp}{4}}}$
bb	$\sqrt{:\frac{mm-2pp}{2},-\sqrt{\frac{mr-4m2p2}{4}}}$
n	$\sqrt{mm-4pp}:$

(25) *D*: p & n qr. a, b, z, x, p, &c. Æq. (per 6) $n = \frac{a4-pp}{aa}$ & by Red.

a	$\surd:\surd\frac{nn+4pp}{4}:+\frac{n}{2}$
b	$\surd:\surd\frac{nn+4pp}{4}:-\frac{n}{2}$
z	$\surd:\surd nn+4pp:+2p:$
x	$\surd:\surd nn+4pp:-2p:$
$\frac{r}{s}$	$\frac{\surd:\surd nnr4+4ppr4:-2nnrr}{\surd:\surd nn+4pp:+2nn}$
aa	$\surd:\surd\frac{n4+4nnpp}{4}:+\frac{nn+2pp}{2}$
bb	$\surd:\surd\frac{n4+4nnpp}{4}:-\frac{nn+2pp}{2}$
m	$\surd nn+4pp$

(26) *D*: $\frac{r}{s}$ & m qr. a, b, z, x, &c. Æq. (per 8) $m = \frac{rraa+ssaa}{rr}$ & by Red.

a	$\surd\frac{mrr}{rr+ss}$
b	$\surd\frac{mss}{rr+ss}$
z	$\surd\frac{mrr}{rr+ss}+\surd\frac{mss}{rr+ss}$
x	$\surd\frac{mrr}{rr+ss}-\surd\frac{mss}{rr+ss}$
p	$\surd\frac{mmrrss}{r4+2rrss+s4}$
aa	$\frac{mrr}{rr+ss}$
bb	$\frac{mss}{rr+ss}$
n	$\frac{mrr-mss}{rr+ss}$

(27) *D*: $\frac{r}{s}$ & n qr. a, b, z, x, &c. Æq. (per 9) $n = \frac{rraa-ssaa}{rr}$ &c.

a	$\surd\frac{nrr}{rr-ss}$
b	$\surd\frac{nss}{rr-ss}$
z	$\surd\frac{nrr}{rr-ss}+\surd\frac{nss}{rr-ss}$
x	$\surd\frac{nrr}{rr-ss}-\surd\frac{nss}{rr-ss}$
p	$\surd\frac{nnrrss}{r4-2rrss+s4}$
m	$\frac{nrr+nss}{rr-ss}$

(28) *D*: m & n qr. a, b, z, x, &c. Æq. (per 10) $n = 2aa-m$ & by Red.

a	$\surd\frac{m+n}{2}$
b	$\surd\frac{m-n}{2}$
z	$\surd\frac{m+n}{2}+\surd\frac{m-n}{2}$
x	$\surd\frac{m+n}{2}-\surd\frac{m-n}{2}$
p	$\surd\frac{mm+nn}{4}$
$\frac{r}{s}$	$\surd\frac{mrr-nrr}{m+n}$
aa	$\frac{m+n}{2}$
bb	$\frac{m-n}{2}$

(29) *D*: a & y $(=a^3+b^3)$ qr. a, b, &c. Æq. $y=a^3+b^3$ & $b=\sqrt{c}:y-a^3:$

a	a
b	$\sqrt{(3)}\,y-a^3:$
z	$a+\sqrt{(3)}\,y-a^3:$
x	$a-\sqrt{(3)}\,y-a^3:$
p	$\sqrt{(3)}\,ya^3-a^6:$
$\frac{r}{s}$	$\frac{\sqrt{yr^3-r^3a^3}}{a^3}$
aa	aa
bb	$\sqrt{(3)}\,yy-2ya^3+a^6:$
m	$aa+\sqrt{(3)}\,yy-2ya^3+a^6:$
n	$aa-\sqrt{(3)}\,yy-2ya^3+a^6:$

(31) *D*: z & $\frac{r}{s}$ that $\frac{x}{m}$ qr. a & b. Æq. (per 2) $r.s::2a.z.\,zz-2za+aa$ wherefore $2sa-zs=zzr-2zra+2raa$. & $2sa+2zrr=zzr+zs$. & $\frac{2s+2zrr}{zr}\times a-aa=\frac{zzr+zs}{2r}$.

(32) *D*: $\frac{r}{s}$ & $\frac{k}{t}$ as $\frac{p}{y}$ qr. a,b,k,t: $\frac{saa}{r}$ $\frac{r^3a^3+s^3a^3}{r^3}$ & $tsraa=kr^3a^3+s^3a^3$ & $\frac{rst}{r^3k+s^3}=a$.

(30) *D*: p & y qr. a, b, z, x, &c. Æq. $p=\sqrt{(3)}\,ya^3-a^6$ & by Red.

a	$\sqrt{(3)}:\frac{y}{2}+\sqrt{\frac{yy-4p^3}{4}}$
q	$\sqrt{(3)}:\frac{y}{2}-\sqrt{\frac{yy-4p^3}{4}}$
z	$\sqrt{(3)}\,y+2pp$
x	$\sqrt{(3)}\,y-2pp$
$\frac{r}{s}$	$\frac{\sqrt{(3)}:2yrr-\sqrt{4yyr^4-16p^3r^4}}{\sqrt{(3)}\,2y+\sqrt{yy-16p^3}}$
aa	$\sqrt{(3)}\,\frac{yy-2p^3}{2}+\sqrt{\frac{y^4-4y^2p}{4}}$
bb	$\sqrt{(3)}:\frac{yy-2p^3}{2}-\sqrt{y^4-y^2p^3}$
m	$\sqrt{c}:yy+4p^3$
n	$\sqrt{c}:yy-4p^3$

(33) *D*: Æq. $a=z-b=x+b=\frac{p}{b}=\frac{rb}{s}=\sqrt{m-bb}:=\sqrt{n+bb}=\sqrt{(3)}\,y+b^3=\frac{z+x}{2}=\frac{z}{2}+\sqrt{\frac{zz-4p}{4}}=\frac{zr}{r+s}=\frac{z}{2}+\sqrt{\frac{2m-zz}{4}}=\frac{zz+n}{2z}=\sqrt{\frac{2m+xx}{4}}:+\frac{x}{2}=\sqrt{\frac{pr}{s}}=\frac{rx}{r-s}=\frac{n+xx}{2x}=\sqrt{\frac{mrr}{rr+ss}}=\frac{m+n}{2}$ and infinitely, which may be pronounced of the rest.

Diaphanti lib. 1. pr. 1, 2, 3, 4, 5, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, &c. and lib. 2. pr. 1, 2, 3, 4, 5, &c.

The

The Explanation of the former Examples.

Sect. 1. Having the greater of two numbers given, either to be a or any other Species equal to it, as z-b, $x+b$, $\frac{p}{b}$ &c. and the lesser b, or that which is equal to it, as z-a, a-x, $\frac{p}{a}$ &c. Then the Sum, Difference, Plane, Proportion, Sum of the Squares, expressed by z, x, p, &c. are gotten, as in the second Question, where a is given, and $b = z$-a, if you add a and z-$a = z$; if you substract z-a out of a it will be $x = 2a$-z, if you multiply a by z-a it will be $p = za$-aa. If you say by the Golden Rule, as a. to z-a:: so is r, or any given Species, to a fourth, then multiplying the second and third Terms, viz. z-a by $r = zr$-ra, and divide that Plane by a, then will the fourth Proportion be $s = \frac{zr-ra}{a}$; Square $a = aa$ and Square $b = zz$-$2za+aa$, add the two Squares together, it will be $m = zz$-$2za+2aa$, and substracting the lesser Square out of the greater it will be $n = 2za$-zz, all which are so expressed in that second Question.

Likewise in the eighth Question, where a is given, and $b = \frac{sa}{r}$, then by addition of those two $z = \frac{ra+sa}{r}$ and by substration of the lesser out of the greater it will be $x = \frac{ra-sa}{r}$, and by multiplication of the greater by the lesser $p = \frac{saa}{r}$, the Squares of the greater $a = aa$, of the lesser $\frac{sa}{r} = \frac{ssaa}{rr}$, being added, the

Sum of the Squares m, will be $\frac{rraa+ssaa}{rr}$ and the difference of the Squares will be $\frac{rraa-ssaa}{rr}$ the which kind of Work is obſerved in all the reſt of the Queſtions.

Sect. 2. It reſts now to ſhew plainly how by the Rules of Æquation the ſeveral Species anſwering to a and b are gotten, which in ſome meaſure is expreſſed at the beginning of every Queſtion, but ſome few I ſhall now more fully expreſs, as in the eighth, where by the firſt Queſtion it is as $a.b::r.s.$ and multiplying the Means and Extremes together, it will be $sa=br$, therefore by Reduction and Diviſion it will be $a=\frac{br}{s}$ and $b=\frac{sa}{r}$ according to the ſaid eighth and to the ninth Queſtions. Likewiſe for the tenth, by the firſt it was $m=aa+bb$, and by Reduction and Subſtraction it will be $m-bb=aa$. Extract the Root of both ſides it will be $a=\sqrt{m-bb}$: and $b=\sqrt{m-aa}$: the ſame way is obſerved in all the reſt, due regard being had to all the Rules of Æquation.

However, that the young Learner may more aſſuredly underſtand all what hath been delivered, it will be the beſt to put the Queſtions into Numbers, as ſuppoſing $a=3$ & $b=2$. Then $z=5$, $x=1$, $p=6$. If $r=6$, then $s=4$, $aa=9$, $bb=4$, $m=13$, $n=5$, the which being obſerved to follow in the Examples will make out the truth, and give ſatisfaction, as in the former eight Examples thus, $a=3.\ b=\frac{4\times 3}{6}=\frac{12}{6}=2.$

$z=\frac{18+12}{6}=\frac{30}{6}=5.$ and ſo of the reſt.

Many other varieties than what are expreſſed in the

the former Queſtions might have been found, as for Example, ſuppoſe the Sum of two Numbers z and the proportion that the difference of them beareth to the Sum of the Squares, be d to f, then imagine the greater Number a, the leſſer will be $z-a$, the Sum of the Squares will be $zz-2za+2aa$, and the difference will be $2a-z$, therefore by the Golden Rule $d.f :: 2a-z. zz-2za+2aa$, and by multiplying the Means together and the Extremes $2fa-fz=zzd-2zda+2daa$, and by Reduction $2daa+2fa+a=zzd+fz$, and $aa+\frac{2f+2z}{2d}\times a=zzd+fz$, which by the Rules aforeſaid (the Indices being 2.1.0) may be reſolved.

Another Example. Having the proportion of two numbers r to s, and the proportion that the Sum of them beareth, to the Sum of their Squares k to t, to find out the numbers themſelves. Let the greater be a, then by the eighth Queſtion, if $\frac{r}{s}$ be given it will be $z=\frac{ra+sa}{r}$ and $m=\frac{rraa+ssaa}{rr}$ wherefore by the Queſtion $\frac{ra+sa}{r}.\frac{rraa+ssaa}{rr} :: k.t$ and by Multiplication $\frac{rta+sta}{r}=\frac{rrkaa+sskaa}{rr}$ and by Reduction $rrta+rsta=rrkaa+sskaa$, and laſtly, dividing all by a it will be $rrt+rst=rrka+sska$, laſtly, $\frac{rrt+rst}{rrk+ssk}=a$.

By which it will eaſily appear that the Queſtions might have been continued infinitely, as taking what proportion you pleaſe, but I left that of purpoſe for the Learners Experiments and further Practice.

Questions concerning mean Proportionals; and first of three mean Proportionals.

The Propositions following resolve all Questions about three mean Proportionals; having two quantities or the proportion, and one quantity of any of those specified, *Chap.* 9. *Sect.* 2.

Ψ	$= A + M + E$	1.2.3.
Σ	$= A + M$	1.2.
Θ	$= M + E$	3 2.
♀	$= M - E$	2.3.
Z	$= A + E$	1.3.
X	$= A - E$	1.3.
Υ	$= Aq + Mq$	1.2.
Λ	$= Aq - Mq$	1,2.
Ω	$= Mq + Eq$	2.3.
℧	$= Mq - Eq$	2.3.
Z,	$= Aq + Eq$	1.3.
X,	$= Aq - Eq$	1.3.
P,	$= A \times E$	1.3.
Φ	$= M \times E$	2.3.
Z	$= A + E$	1.3.
♃	$= aq + mq + eq$	1.2.3.

Prop. 1. Having M and X to find A and E, I put α for the first, then $\alpha - X = E$. Therefore $\alpha . M . \alpha - X$ ∺ and $Mq = \alpha q - X\alpha$ and $\alpha = \sqrt{\frac{Xq + 4Mq}{4}} : + \frac{X}{2}$.

Prop. 2. Having M and Z to find A and E, let α be the first number, then $\alpha . M . Z - \alpha$ ∺. $Mq = Z\alpha - \alpha q$ and $\frac{Z}{2} + \sqrt{\frac{Zq + 4Mq}{4}} = \frac{A}{E}$

Prop. 3.

Prop. 3. Having A and Θ to find M and E, let α be put for $ME = \Theta - \alpha$ and $A{:}\alpha.\,\Theta-\alpha \div .\alpha q = \Theta A + A\alpha$ and $A\alpha + \alpha q = \Theta A$.

Prop. 4. Having E and Σ to find A and M, let $\alpha = M$ and $\Sigma - \alpha.\alpha E \div$ wherefore $\alpha q = \Sigma E - \alpha E$ and $\alpha E + \alpha q = \Sigma E$.

Prop. 5. Having X and Θ to find $A.M.E \div$, let α be put for E, then $\Theta - \alpha$ is M, and $\alpha + X = A$, therefore $\alpha + X.\ \Theta - \alpha.\ \alpha \div$ and $\Theta q - 2\Theta\alpha + \alpha q = \alpha q + X\alpha$, and $\Theta q = X\alpha + 2\Theta\alpha$ and $\alpha = \frac{\Theta q}{2\Theta + X}$.

Prop. 6. Having Ψ and W given to find A, M, $E \div$, let α be put for M, therefore $A + E = \Psi - \alpha$, therefore $\alpha q = \Psi q - 2\Psi\alpha = W$.

Prop. 7. Having the proportion of Aq to $W.R.S.$ to find A,M,E, let α be put for A, then let S be put for E, and $\alpha S = Q.M.$ and $\alpha q.\ S\alpha + Sq :: R.S.$ and $S\alpha q = \alpha RS + RSQ.$ & $S\alpha q - \alpha RS = RSq.$ & $\alpha q - \alpha R = RS$, or contrarily, $S.\alpha S + \alpha q :: R.S.$ then $Sq = \alpha RS + R\alpha q$ and $\alpha S + Sq = \frac{Sq}{R}$.

Prop. 8. Having X and Ω given to find $A,M,E ::$ let α be put for Z, then $\alpha + X = 2A$ and $\alpha - X = 2E$ and $A + E = \frac{\alpha + X}{2} + \frac{\alpha - X}{2}$ and $\frac{\alpha q - Xq}{4} = Q.M.$ and $\frac{\alpha q - Xq}{4} + \frac{\alpha q - 2\alpha X + Xq}{4} = \Omega$ and $2\alpha q - 2\alpha X = 4\Omega$ and $\alpha q - X\alpha = 2\Omega$.

Prop. 9. Having X and W given to find $A,M,E \div$ let α be put for Z, therefore $\alpha + X = 2A$ and $\alpha - X = E$ wherefore $\frac{\alpha + X}{2} + \frac{\alpha - X}{2} = A + E$ and $\frac{\alpha q - Xq}{4} = Mq$

and $\frac{2\alpha q+2Xq}{4}=Z$, and $\frac{\alpha q-Xq}{4}+\frac{2\alpha q+2Xq}{4}=W$

and $3\alpha q=4W-Xq$ and $\alpha=\sqrt{\frac{4W-Xq}{3}}$

Prop. 10. Let $A,M,N,E \div$ and having $A-E=\Sigma$ and $M-N=X$ given to find out the four Prop.

Let α be put for N, then $\alpha+X=M$,
therefore $\frac{\alpha q+2\alpha X+Xq}{\alpha}=A$ and $\frac{\alpha q}{\alpha+X}=E$
and $\frac{3\alpha qX+3\alpha Xq+Xc}{\alpha q+\alpha X}=Z$
and $\alpha qZ+\alpha ZX-3\alpha qX-3\alpha Xq=Xc$
and $\frac{ZX-3Xq}{Z-3X}\times\alpha+\alpha q=\frac{Xc}{Z-3X}$
or $\frac{Z-X}{}\times\alpha+\alpha q\ \frac{Xc}{Z-3X}$

These and such like may be put and resolved infinitely.

CHAP. XI.

Containing many Questions of several Subjects.

PRop. 1. To find a Number that multiplied by B (6) and the Product added to C (8) doth make D (48,) let the Number sought be A, then $\overline{B\times A}+C=D$ and $A=\frac{D-C}{B}$ which is equal to $6\frac{2}{3}$.

Prop. 2,

Prop. 2. To Divide *B* (100) into *A* and *E*, that $\frac{A}{3} + \frac{E}{5} = D.30$.

Let $\alpha = A$ and $B\text{-}\alpha = E$, therefore $\frac{\alpha}{3} + \frac{B\text{-}\alpha}{5} = D$, and $5\alpha + 3B - 3\alpha = 15D$ and $\alpha = \frac{15D - 3B}{2} = 75$.

Prop. 3. To find *A* and *E*, that $A = E + B$ (4) and $Aq = Eq + D$ (32) let α be put for *A*, then $\alpha - B = E$, and $\alpha q = \alpha q - 2B\alpha + Bq + D$, and $\alpha = \frac{Bq+D}{2B}$ and $\alpha = 6 = A$ and $E = 2$.

Prop. 4. To find *A* and *E*, that $Aq - Eq = B$ (6) and $A - E = X$ (5) let α be put for *A*, then $\alpha - X = E$. Then $\alpha q + \alpha q - 2\alpha X + Xq = B$. and $\alpha q - X\alpha = \frac{B - Xq}{2}$

Prop. 5. To find a number that being joyned with *B* (18) and taken from *C* (100) the Sum and Remain ſhall be as *R*. *S*. (1.3) let it be *A*. Then $B + A . C - A :: R . S$. Then $RC - RA = BS + SA$ and it is $A = \frac{RC - BS}{R+S} = 11\frac{1}{2}$.

Prop. 6. To divide *B* (30) into *A*, *M*, *N*, *E* ∺ that *A* may be = to *C* (2)

Put α for *M*, then $\frac{\alpha q}{C} = N$ &c. $\frac{\alpha C}{Cq} = E$.

Therefore $C . \alpha . \frac{\alpha q}{C} \; \frac{a^3}{Cq}$ ∺ $= B$ by Addition.

$\frac{Cc + Cq\alpha + C\alpha q + \alpha c}{Cq} = B$ and $\alpha c + c\alpha q + Cq\alpha = BCq - Cc$

Now $BCq - Cc = 112 = Ac + CAq + CqA$.

The

The Canon will be made thus:

$$\begin{array}{llll} & Ac. & +CAq. & +Cq:A \\ & \cdots & \cdots\cdots & \cdots\cdots \\ 3 & Aq:E & +\ C2A:E & +Cq:E \\ 3 & A:Eq & +\ \ C:Eq & ; \\ & :Ec & & \end{array}$$

$$\begin{array}{lr} & 112 \quad (4 = \alpha \\ & : \\ \hline Cq \ldots\ldots & 4 \\ C \ldots & 2 \\ \hline Dr \ldots & 6 \\ \hline CqA & +16 \\ CAq & +32 \\ Ac & +64 \\ \hline & 112 \end{array}$$

Therefore 2. 4. 8. 16 ∺ and $2+4+8+16=30$.

Prop. 7. To find a number from which if you take B (3) and to $\frac{1}{3}$ thereof add C (7:) the Sum being drawn in B, and from the Product take D (18) the remain shall be equal to F (21:) Let the number sought be A, and according to the Question, $A\text{-}B$ the $\frac{1}{3}$ of it is $\frac{A\text{-}B}{B}$ add C it is $\frac{A\text{-}B+BC}{B} \times B = A + BC\text{-}B, \text{-}D = F$. $A = F\text{-}BC + B + D$ and $A = 21$.

Prop. 8. To find a number from the Triple, whereof if I take B (30) and from the Double of that

that if I take C (140,) and if I draw the reſt in D (4) and from the Product take F (100) there remains nothing.

Let A be put for the ſought Quantity, then $3A\text{-}B$ and $6A\text{-}2B\text{-}C$ and $6DA\text{-}2BD\text{-}DC\text{-}F=0$, therefore $6DA=2BD+DC+F$ and $A=\frac{2BD+DC+F}{6D}$

Prop. 9. Two numbers are ſought in triple Proportion, and the leſs taken from the greater leaveth a number equal to the Quotient of the greater divided by the leſs. Let the greater be A, and the proportion $\frac{R}{S}$ then $\frac{SA}{R}$=the leſſer and $\frac{RA\text{-}SA}{R}=\frac{SA}{RA}$ and $RA\text{-}SA=S, A=\frac{S}{R\text{-}S}\,A=\frac{3}{2}E=\frac{9}{2}$.

Prop. 10. A number is ſought, from which take $\frac{R}{S}$ $(\frac{2}{3})$ it makes the number as much under B (100) as it was at firſt above B. Put A for the number ſought.

Then $A\text{-}\frac{R}{S}=\frac{SA\text{-}RA}{S}$ and $B\text{-}\frac{SA\text{-}RA}{S}=A\text{-}B$

and $2B=\frac{2SA\text{-}RA}{S}$ and $2BS=2SA\text{-}RA$

and $A=\frac{2BS}{2S\text{-}R}$

Prop. 11. The Reſolutions of the Queſtions, *Chap.* 9. *Sect.* 3. Of the firſt, $BA+C=DA$; $C=DA\text{-}BA$ and $A=\frac{C}{D\text{-}B}$

Prop. 12. Of the ſecond, for the greater I put A, the

the lesser $B-A$, then $CA=BD-DA$ and $CA+DA=BD$ and $A=\frac{BD}{C+D}$.

Prop. 13. Of the third, for the one put A, then $B-A$ is the other, and $CA+DB-DA=F$ and $CA-DA=F-DB$ and $A=\frac{F-DB}{C-D}$.

Prop. 14. Of the fourth, let the one be A, the other is $B-A$, then $\frac{A}{C}+\frac{B-A}{D}=F$ and $DA+BC-CA=DCF$, and $A=\frac{DCF-BC}{D-C}$.

Prop. 15. Of the fifth, let the greater be A, then $A-X$ is the other, therefore $BA+BA-.BX=C$, and $A=\frac{C+BX}{2B}$.

Prop. 16. Of the sixth, if you put the lesser Divisor A, the greater is $A+C$ and $\frac{B}{A}-\frac{B}{A+C}=D$ and $BA+BC-BA=DAq+DA+DC$ and $BC-DC=DAq-DA$ and $Aq-1A=\frac{BC-DC}{D}$.

Prop. 17. Of the seventh for the sought Mag. put A, then $BA-D=CA+F$, then $F-D=CA-BA$ and $A=\frac{F-D}{C-B}$.

Prop. 18. To find two Numbers in that proportion, that the $\frac{1}{2}$ of the second more by B (2) added to the former, is 9 times as much as the remainder

mainder of the ſecond: But the third part of the firſt more by C (3) added to the ſecond, is triple to the remainer of the firſt: I put for the firſt E, and for the ſecond $2A$, then $A+B+E=(A-B*9)$ $=9A-9B$, and $9A-9B-A-B=(E)=8A-10B$, and $\frac{8A-10B}{3C}+C+2A=\frac{14A-10B+3C}{3}=\frac{16A-20B-3C}{3}*\frac{3}{1}$ and $14A-10B+3C=48A-60B-9C$ and $60B+9C-10B+3C=48A\,14A$ and $50B+12C=34A$, and $A=\frac{50B+12C}{34}=4$ and $2A=8$. Then $64-20=2E=24=12$.

Prop. 19. Two men had ſeverally certain Sums of Crowns, the proportion was as $\frac{R}{S}$ $(\frac{4}{1})$ the Sum that both had wanted of B (100) but the Sum doubled and made leſs by D (29) there was twice as much above B as there wanted at firſt of B, I demand what either man had? Firſt, I find Z the Sum, and for the want which the Sum was ſhort of B I put E. Then $E=B-Z$ and $Z=B-E$, but $2Z-D=B+2E$, that is $2Z-D=B+2B-2Z$ and $4Z=3B+D$ and $Z=\frac{3B+D}{4}$ that is (80.) Now having Z and $\frac{R}{S}$ by the 18 Queſtion of this Chapter $A=\frac{ZR}{R+S}E=\frac{ZS}{R+S.}$ Therefore $A=64.\ E\,10.$

Prop. 20. A Merchant hath a Hogſhead of Wine of B (360) Gallons, he poured out three ſeveral times a certain number of Gallons, and filled them

up

up with Water, at the last there remained D ($108\frac{4}{5}$) of Wine, what was poured out at each time? Betwixt B and D find two mean proportionals thus, $B, \sqrt{c}BqD, \sqrt{c}BDq, D$: Then $A = B - \sqrt{c}BqD$:

The same Question may be wrought by prosecuting the tenor of the Question thus:

The first Draught, $B-A$.

The second Draught, $Bm-Am$. resteth in the Hogshead of mixt $\frac{Bq-2BA+Aq}{B}$

found thus, $B.B-A :: B.A. \frac{Bq-2BA+Aq}{B}$

The third Draught is $Bm-Am$. rests of Wine $\frac{Bc-3BqA+3BAq-Ac}{Bq}$ found by this Analogy.

$A.B-A :: \frac{Bq-2BA+Aq}{B} . \frac{Bc-3BqA+3BAq-Ac}{Bq}$

Wherefore $\frac{Bc-3BqA+3BAq-Ac}{Bq} = D$; and $Bc-3BqA+3BAq-Ac = BqD$.

And by Reduction $Ac-3BAq+3BqA = BqDBc$.

In Numbers $Ac-1080\ Aq+388800\ A = 19656000$
$A = 60$.

$$
\begin{array}{ll}
\dot{1}96\dot{5}60\dot{0}0 & (60 \\
388800. & +3Bq \\
1080.. & 3B \\
\hline
398003 & A \\
\hline
2332800 & +3BqA \\
.38880 & -3BAq \\
216 & +Ac \\
\hline
+2354400. & \\
\hline
1965600 & \text{Ab.} \\
000000 & \text{Resid.}
\end{array}
$$

For the invention of as many mean Proportionals as you please,(which may concern the former part of this Question) you may take notice they may all be found by the extracting of the square and cube Roots, for because in the fourth power it is,

$$
\left.\begin{array}{llll}
 & A & = & A \\
\sqrt{qq}. & AcE & =\sqrt{q} & AcE \\
\sqrt{qq}. & AqEq & =\sqrt{q} & AE \\
\sqrt{qq}. & AEqq & =\sqrt{q} & AEc \\
 & E & & E
\end{array}\right\} \text{All to be extracted by the square Root.}
$$

$$
\left.\begin{array}{llll}
 & A & & A \\
\sqrt{qc} & AqqE & \sqrt{q} & AqcE \\
\sqrt{qc} & AcEq & \sqrt{c} & AqE \\
\sqrt{qc} & AqEc & \sqrt{c} & AEq \\
\sqrt{qc} & AEqq & \sqrt{q} & AEqc \\
 & E & & E
\end{array}\right\} \text{All to be extracted by the square and cube Roots, \&c.}
$$

Prop. 21. To divide Z (100) into A and E, that $\frac{A}{4}$ may exceed $\frac{E}{6}$ by C (20) let $A=A$. $E=Z-A$. then $\frac{A}{4}=\frac{Z-A}{6}+C$ and $6A=4Z-4A+24C$ and $10A=4Z+24C$ and $A=\frac{4Z+24C}{10}=88$.

Prop. 22. To divide Z into A. E and O; that $A+E=3O$ and $O+E=4A$. Therefore $4O=Z$ and $O=\frac{Z}{4}$ and $5A=Z$. $A=\frac{Z}{5}$.

Prop. 23. To find A,E,O, that $A+E+\frac{O}{2}=Z$. $E+O+\frac{A}{3}=Z$ and $A+O\frac{+E}{4}=Z$.

Wherefore (1) $2A+2E+O=2Z$
(2) $3E+3O+A=3Z$
(3) $4A+4O+E=4Z$.

By substracting the (3) from (1,2) $4E-A=Z$ and $E=\frac{Z+A}{4}$

By substracting the (2 from 1, 3) $5A+2O=3Z$ and $O=\frac{3Z-5A}{2}$

By substracting the 3 from (1 and 2 in 2) $7E+3O=4Z$, therefore it is $7Z+7A+18Z-15A=16Z$, and $\frac{9Z}{23}=A$ &c.

Prop. 24. To find their numbers, *viz.* A, E, O, with this condition that $A+\frac{2}{3}E+\frac{2}{3}O=Z$ (100.) $E+\frac{3}{4}A+\frac{3}{4}O=Z$ and $O+\frac{4}{5}A+\frac{4}{5}E=Z$.

(1)

$$\left.\begin{array}{l}(1)\ 3A+2E+2O=3Z\\(2)\ 3A+4E+3O=4Z\\(3)\ 4A+4E+5O=5Z\end{array}\right\}\text{Reduced into Integers.}$$

By sub. (2, e, 3) it is $A+2O=Z$ and $O=\frac{Z-A}{2}$

By sub. (3, e, 1 and 2) it is $2A+2E=2Z$ and $E=Z-A$.

By sub. (1, e, 2) it is $2E+1O=Z$, therefore $4Z-4A+Z-A=2Z$ and $\frac{3Z}{5}=A=60.E40.O20$.

Prop. 25. To find four numbers, *viz.* A,E,I,O, that $A+\frac{1}{2}E+\frac{1}{2}I+\frac{1}{2}O=B$, and $E+\frac{1}{3}A+\frac{1}{3}I+\frac{1}{3}O=C$, and $I+\frac{1}{4}A+\frac{1}{4}E+\frac{1}{4}O=D$, and $O+\frac{A}{6}+\frac{E}{6}+\frac{I}{6}=D$.

$$\left.\begin{array}{l}(1)\ 2A+E+I+O=2B\\(2)\ A+3E+I+O=3C\\(3)\ A+E+4I+O=4D\\(4)\ A+E+I+6O=6D\end{array}\right\}\text{Reduced to Integers.}$$

Sub. 1 from 3 and 4 it is $E+4I+6O=(10D-2B)=F$
1 from 2 ———— ————$2E-A=(3C-2B)=G$
1 from 3 ———— ————$3I-A=(4D-2B)=H$
1 from 4 ———— ————$4O-A=6D-2B)=K$

Wherefore $E=\frac{G+A}{2}$ $I=\frac{H+A}{3}$ $O=\frac{K+A}{5}$

Wherefore $\frac{G+A}{2}+\frac{4H+4A}{3}+\frac{6K+6A}{5}=F$, and by Reduction $15G+15A+40H+36K+36A=30F$.

and $A=\frac{30F-15G-40H-36K}{91}$ wherefore $A6.E4.I8.O10$.

Prop. 26. To divide Z into A and E, that $A\times E=B+\sqrt{q}C$. $E=Z-A$ and $ZA-Aq=B+\sqrt{q}C$. and

$ZA\text{-}Aq\text{-}B = \sqrt{q}C$. Wherefore $ZqAq\text{-}2ZAc\text{-}2ZBA + Aqq + 2BAq + Bq = C$.
And $Aqq\text{-}2ZAc + \underline{Zq + 2B} \times Aq\text{-}2ZBA = C\text{-}Bq$.

Prop. 27. There is a Square, whose ſide drawn into the difference of the ſide and diameter produceth B, it is demanded to know the ſide and diameter?

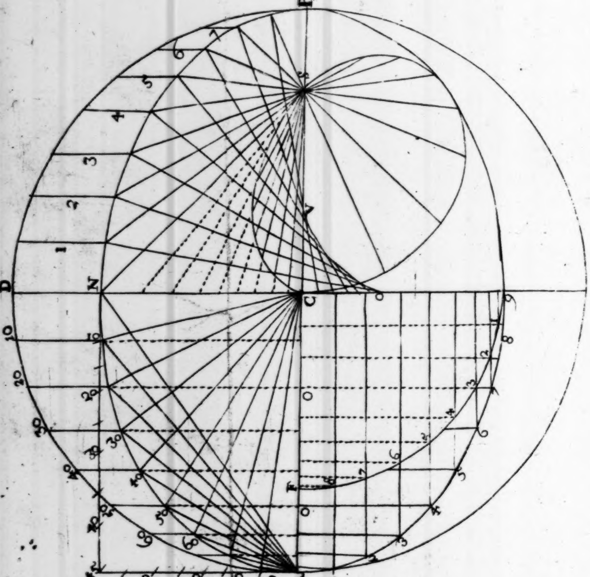
Let A be the ſide, then $\frac{B}{A}$ = Diff. of the ſide and diameter, and $\frac{B}{A} + A$ = Diam. $= \frac{B + Aq}{A}$ and by the (47 e 1) $\frac{Bq + 2BAq + Aqq}{Aq} = 2Aq$.

And $Bq + 2BAq + Aqq = 2Aqq$ and $Aqq\text{-}2BAq = Bq$ and $A = \sqrt{:}\sqrt{q}2Bq{:} + B$ and $Aq = \sqrt{q}2Bq{:} + B$. Wherefore Q: Diam $= \sqrt{q}8Bq{:} + 2B$ and the Diam. $= \sqrt{q}{:}\sqrt{q}8Bq{:} + 2B$ and the Diff. of L: and D: is $\sqrt{q}{:}\sqrt{q}8Bq{:} + 2B.\text{-}\sqrt{q}{:}\sqrt{q}2Bq{:} + B$, which multiplied together at the laſt leaveth B.

Prop. 28. To cut Z into A and E, that $Eq \times A$ ſhall be to X as $R.S.$ $E = Z\text{-}A$. $Eq = Zq\text{-}2ZA + Aq$: $\times A = ZqA\text{-}2ZAq + Ac$. $X :: R.S.$ that is $ZqA\text{-}2ZAq + Ac$. $2A\text{-}Z :: R.S.$ and $2RA\text{-}ZR = ZqSA\text{-}2ZSAq + SAc$ and $Ac\text{-}2ZAq + \frac{2R}{S} + Zq \times A = \frac{ZR}{S}$.

Theſe Examples ſhall put an end to this Part, leaving the reſolution of more Queſtions to the induſtry and practice of the Ingenious, to whom I wiſh all happineſs and good ſucceſs.

Soli Deo Gloria.



Contemplationes Geometricæ.

IN TWO

TREATISES.

1. A new *Contemplation Geometrical* upon the *Ellipsis*, plainly ſetting out the Nature of that *Figure.*

2. *Conical Sections*, or the ſeveral Sections of a *Cone* : Being an *Analysis* of the two firſt Books of *Mydorgius.*

By Sir JONAS MOORE.

LONDON,

Printed by *R. H.* for *Obadiah Blagrave* at the *Bear* in St. *Paul's* Church-Yard, 1688.

A new Contemplation Geometrical upon the Ellipſis.

CHAP. I.

Of the Inclination of a Circle to a Plain, and of the Curved Line made by the Orthographical projecture of the ſame at any Poſition unto a Quadrant.

IN the firſt Diagram: Let the Plane be *ANRP*, on which let *ADPA* be a Circle, whoſe Diameter *PA*, Semidiameters *CA*, *CP*, *CD*, let a Semicircle *PrnrA* of an equal Diameter to the Circle be ſo fitted upon the common Interſection and Diameter *AP*, that it may move to any part of a Quadrant from the Plane. Let the Semicircle be ſo inclined, that the point *n* of the Diameter *Cn* may be perpendicular over the point *N* of the Plane, let there be infinite Perpendiculars let down from the Semicircle to the Plane, which may make the Curved line *PRNRA*: *I ſay that that curved line is the half of an Ellipſis (or of an Elliptical line) whoſe Axis is* AP *and Con-* Fig. 1.

 jugate

Fig. 1. *jugate* CN, *and the lines* OR *being parallel to* CN, *are Ordinates rightly applied.* For the Semicircle *PnA* being elevated over the curved live *ANP*, upon all lines parallel to the Conjugate *CN*, and parallel likewiſe and equally diſtant from the Diameter *Cn*, the Angles are equal by the (18 *e* 11) and in the Scheme the Triangles *CNn*, *ORr* are ſimilar, therefore *Cn*. *CN* : : *or*.*OR*. but *Os* is in every place equal to *Or* being ſines of equal Radii, and *Cn* equal to *CD*, therefore *CD*.*CN* : : *OS*.*OR*. and the Squares *CD (q)*. *CN (q)* : : *OS (q)*. *OR(q)*. But every where the Rectangle *POA* is equal to *Os(q)*, equal to *Or(q)*, (*per* 17 *e* 6) and *AC*×*CP*=*CD(q)* becauſe *AC*, *CP*, *CD*, are equal, therefore *AC*×*CD*.*CD(q)* : : *Po*×*oA*. *Os(q)*. But it was proved before *CD(q)*.*CN(q)* : : *OS(q)*. *OR(q)*. therefore *AC*×*CD*.*CN(q)* : : *AO*×*OP*.*OR(q)*. Therefore the points *RR* are in the Ellipſis, by the 21 l° *Apollonii*, 8 l° *Mydorgii*, and 28 *Con. Walliſii.*

And the lines *CN*, *OR*, are Ordinates rightly applied becauſe they cut the Axis *PA* at right angles, and are but the halfs of other Ordinates, that would fall on the other ſide of the Ellipſis, which the Semicircle would make being inclined equally upon the common line *AP*, as well above as below.

CHAP.

CHAP. VIII.

Of the Division of the Line of the Ellipsis.

HAving in the Preface spoken of the simili- Fig. 2
tude or likeness betwixt a Circle and an Ellipsis, *I shall now here shew how the Ellipsis may be divided like to the Circle.*

1. In the second Figure let *Prnr* be the Semicircle, let the Axis *PA* and the Conjugate *CN* be given, and let the Semicircle be elevated so, that the point *n* may be perpendicular over *N*; then shall *Cn* of the Semicircle be Radius, being equal to *CD*=*CP*=*CA*. Therefore,

2. In the Triangle *CNn* right angled at *N*, *Cn* being Radius, the sides *nN* and *Nc* must one be the sine and the other the co-sine of the Angle *nCN*, by the ordinary Axiom of right angled Triangles, therefore *Cn* Radius and *CN* the Conjugate given, the angle of Inclination *nCN* may be had, and the perpendicular *Nn*, for in the Table of natural sines *Cn*=*CD* being 100000 finding out *CN* 76604 among the sines, the co-sine will give both the perpendicular *nN* 64279, and the Inclination 40.

3. The length of the Ordinates *OR* in the first Figure are had either by resolving the Triangle *ORr* right angled at *R* the side *Or*=*Os*= the sine complement of his distance from *D*, equal to the angle *DCS*, or by comparing the two like Triangles *CNn* and *ORr*, for it is *Cn*. *CN* : : *Os*.*OR*. so the Table *B* was made the Ta-

Fig. ble *A* being the natural sines of *OS*.

1. 4. The perpendiculars *Rr* will be gotten either in the Triangle *QRr* as before, or repeating the Analogy *CD*=*CN*.*Nn* : : so is *Os*—*Or*. *Rr*. by which the Column *C* in the Table is made.

2. 5. The Tangents upon the point *N* of the Conjugate are the same with those of the Semicircle or Circle, and pass through the like degrees of the Ellipsis, for *DT* is equal to *NG*. But the Tangent raised upon the point *A* of the Axis is shortned in proportion, as Radius is to the Conjugate, for *AG* is equal to the Conjugate. Therefore, as Radius to the Conjugate, so is any natural sine under 45°. to its proportional part of the line *AG*, which makes the Table *D*.

2. 6. The chords of the Ellipsis *AR* by help of the Perpendiculars *Rr* are gotten as before in the fourth Section of this Chapter; for in the Triangle *ARr* right angled at *R*, there are *Ar* the natural chord of the Correspondent angle *RCA* and the perpendicular *rR* given, to find the chord *AR* which makes the Table *E*. And upon this consideration I had hope (and not without just grounds) to believe that the length of any line from the Axis to the Ellipsis might be obtained, having the length of the line that should perpendicularly be over it in the Semicircle.

7. The Rayes from the Centre *C* to the Ellipsis *CR* are had very easily; for *Cr* being Radius equal to *CA*=*CP*=*CD*, and the perpendicular *rR* being given, the ray *CR* is the sine complement

plement of the perpendicular rR; or they are Fig. 1.
had thus, $Ar(q)$-$rR(q)$=$CR(q)$.

8. The Conjugate CN will be divided into a 3.
line of sines answering the Radius of Cn, if
from every part of the Radius divided into a
line of sines perpendiculars be let fall, or as Cn.
CN :: Cs.CS. so every sine on Cn to the like sine
on CN, the angle at C remaining the same.

9. The point r on the Semicircle falling down 4.
on the point R, R becomes the common intersection of the lines of the circles $CADP$ made upon the Axis, and CNO made upon the Conjugate, (that is) SR the sine of NCR on the lesser circle, shall meet OR in the point R. That at every R there is formed a parallelogram $CSRO$ made by the intersections of like sines of the circles.

10. Here we may consider by the way, that 3.
the Ordinates are nothing but the sines of the 2.
lesser circle, for every OR is equal to its answering Oe=CS the sine of the Arch Oe, and SR meeting OR in R=SRr the sine of the Arch $Drrr$, of the greater circle; which is the complement of the former to a Quadrant. And the like will be for the Tangents, for as the natural Tangents divide the Ellipsis from the point N on 2.
the Conjugate from the greater circle unto 45°: so doth the natural Tangents of 45°. of the little circle set off from the point A on the Axis towards Ai equal to CN the Tangent of 45°. divide that part of the Ellipsis towards A.

Lastly, as an addition to the first Section of 2.
this Chapter in the Triangle CNn it is $Cn(q)$ (=Rad. (q)) less by $CN(q)$ equal to the Square

of

Fig. of *nN* (*per* 47 *e* 1) for the Triangle *CNn* is right angled at *N*.

CHAP. III.

Of the Inclination of an Ellipsis to a Plane.

4 *IF an Ellipsis, or that half made from the Conjugate, be elevated to an Angle that the Semiax being Radius, the perpendicular shall be equal to the square Root of the square of the Semiax (equal to Rad. (q)) less by the square of the Conjugate, or if the Angle of Inclination be equal to the sine complement of the Conjugate, as before, then, I say, the Ellipsis shall be perpendicular over a circle on the Plane, which is equal to a circle made of the Semiconjugate.* In the fourth Scheme let the Semiellipsis *DNB*, upon the common intersection *DCB* equal to the Conjugate be inclined as before, so that *N* being perpendicular over *n* it may be *CN(q)-Cn(q)* =*Nn(q)* or let as before the Angle *NCn*=sine-complement of *Cn*, and let infinite perpendiculars *Rr* be let fall upon the Plane, then shall *CDrnBc* be a circle: For as before the Triangles *NnC*, *RrO* are alike, and *RO* is an Ordinate rightly applied being parallel to *NC*, therefore it will be every where as *CN.Cn* : : *RO.Or.* but every *or* was proved to be the Ordinates = to the sines of the circle made of the Conjugate, and *Cn*=*CD*=*CB*, and further *Cn*, *Or*, *or* are proportional, therefore *CDnBD* a circle; or let the circle made of the Semiconjugate be the Base of

of the Cylinder, and from setting of the perpendicular equal to *nN* upon the Cylinder, and cutting the Cylinder so that the Inclination may be equal as before, the Section will be an Ellipsis equal to the Ellipsis proposed; from hence it may appear how the lesser circle may be found. Fig. 4.

CHAP. IV.

Of the Tangents or Touch-lines of the Ellipsis.

1. LEt the Tangent *Tn* be drawn to any point of the Semicircle, or, which is all one in this case, to the Circle, by the (18 *e* 3) and let the Tangent line be continued until it intersect the Axis continued in *n*: *I say that a line drawn from the point* n *to* t *(which is the perpendicular under* T*) shall touch the Ellipsis in the point* t. The demonstration depends upon that which was said before; for in the perpendicular cadency of the point *T* upon the Plane, the Tangent is projected and falls upon the point *t*; or if *nt* should not touch, it would follow that no line would touch, but being projected into the Semicircle, it would cut the circle, and consequently the Ellipsis: but to make all clear, I shall shew that this new-found Touch-line will agree with the intent of those who have purposely writ of the Conical Sections. 5.

And first, by the 37 l° *Apollonii*, and the 23 l° *Mydorgii*, the square of the Radius (=*CAq*) is equal to the Rectangle made of the distance of the

Fig. 9 the Centre to the Section of the Tangent ($= Cn$) and of the Centre to the Ordinate, that is $CA(q) = nC \times CE$. For first, $nC \times CE = CEq + CE \times nE$ by the (3 e 2) and $CE \times nE = TEq$ by the (8 e 6), but the squares of CE and TE added together are equal to the square of CT (48 e 1) which is equal to the square of CA, for $CT = CA$ therefore $nC \times CE = CE(q) + TE(q) = CT(q) = CA(q)$ and $nC \times CE = CA(q)$ according to the intent of *Apollonius* and *Mydorgius*.

2. Again by the 25 *I. Mydorgii* and by the 30 Chapter of Dr. *Wallis* his Conicks, *the Rectangle made of the Segments of the Axis by the Ordinate from the Touch-point, is equal to the Rectangle made of the distance from the Centre to the Ordinate, and from the Ordinate to the place where the touch-line cuts the Axis produced, that is,* $PE \times EA = CE \times En$. which I thus demonstrate by the Figure, for as before $CE \times En = ET(q)$ for $CE . ET :: ET . EN$ by the (8 e 6). And $PE \times EA = ET(q)$ (*per* 14 e 2,) therefore both the Rectangles $CE \times En$ & $PE \times EA$ being equal to the square of TE, they will be equal between themselves.

3. By the 24 *I. Mydorgii* it is demonstrated, that *the rectangle made of the distance of the centre to the point where the touch-line cuts the Axis produced, and of the distance of that point of the touch-line to the Ordinate, is equal to the rectangle made of the distance of intersection of the touch-line to the circle, and of that and the diameter added together, that is*, $En \times nC = Pn \times nA$. which by my way I thus demonstrate, for first, $Tn(q) = nE(q) + ET(q)$ by the (48 e 1) and $nE(q) + ET(q) = nE$

(q)

$(q)+En*EC$, for $En*EC=ET(q)$ and $nE(q)+nE$ Fig.
$*EC=En*nC$ by the (3 e 2) but $Tn(q)=Pn*nA$ 5.
by the (36 e 3) therefore both the rectangles $Pn*nA$ and $En*nC$ being equal to $Tn(q)$ it will be $En*nC=Pn*nA$, which was to be demonstrated.

Therefore the point *n* was well found out, and agrees to the sense of *Apollonius*, *Mydorgius*, and Dr. *Wallis*, and a line drawn from *n* will touch the Ellipsis in *t*; which was the thing proposed.

2. Draw *tp* so that it may be *nE.Et* : : *Et*. 5.
Ep the Angle *ptn* shall be a right Angle, because *Et* is a mean Proportional, and is in the Semicircle.

CHAP. V.

Of the Nodes, Navels or Focusses of the Ellipsis.

1. THe Ellipsis hath its Centers as the Circle 5.
hath; but here is two, and the Circle but one, therefore in this place we shall take liberty a little more Physically than Mathematically to consider how, and after what manner, these new centres come, and how they come to be distant from the centre *C* of the Ellipsis: And

1. The Semicircle being at its elevation, and perpendicular over the Ellipsis, and the Semidiameter of the Semicircle being equal to the Radius of the circle, whilest the motion of this point

Fig. 5. point n of the Semicircle is falling down to N the point of the Conjugate, the Radius of the Semicircle will not be comprehended betwixt n and C, therefore in this motion let the centre C and the Radius Cn be split and divided into two points and two lines, and let them whilst the point n is coming down, slip on in the Axis, when n is come to N, the Radii will be equally distant from C, I mean the extream points, and will rest upon the centres or Nodes of the Ellipsis, which let be F and S.

Therefore $NF = NS =$ Radius will find out the Nodes, by setting of the Radius from N and intersecting the Axis.

NF or NS being Radius, and the angle C being a right angle $NFq - NCq = CFq$ or CF is equal to the sine complement of the Conjugate, and in the second and first Figure is always equal to nN the perpendicular or nearest distance of n to the Plane. Now that these points F and S are the Umbilici or Navels, as *Mydorgius* calls them, or the Foci or Focusses or burning Points, as others term them, according to what *Apollonius* intends and *Mydorgius*, that is, *that the rectangle made of the segments of the Axis, made by either of these Nodes, shall be equal to the square of the Conjugate*, that is, $PF * FA = CN(q) = PS * SA$. From the point F draw a parallel line to the Conjugate, which let be Fk drawn until it come to the circle at k, draw the Radius Ck, the two Triangles NcF, NkF are equal and alike, and $CN = Fk$. But $PF * FA = Fk(q)$ (*per* 14 *e* 2) $= CNq = AS + SP$.

Therefore

Therefore the Nodes agree to the intent of *A-pollonius* and *Mydorgius*. Fig. 5.

Having thus found out theſe two Centres, they will much agree, and have the ſame properties and natures that the centre *C* hath: For,

Firſt, *As the diſtance from the centre* C *to the circumference of the circle, and again to the centre, is equal to the diameter* AP, *ſo the diſtance from one of the Nodes to the Ellipſis, and from thence to the other Node, is every where equal to twice the Radius, or to* AP, *as before.*

Secondly, *As any line drawn from the centre of the circle to any point of the circle, will upon the touch-point reflect again to the centre, becauſe at right angles: ſo any ray or line from one Node to the Ellipſis will reflect to the other, becauſe the angles upon the Tangent are always equal, that is, the angles of Reflection will be equal.*

Thirdly, *As by tying twice the Radius about a fixed point in the centre, drawing another point in the extremity about, it will deſcribe a Circle; even ſo upon the Nodes, fixing twice the Radius, and carrying of it about with a point in the extremity, it will deſcribe the Ellipſis.* All which will appear as followeth.

2. In the Scheme let *PLDTA* be the circle made upon the Axis of the Ellipſis *PNtA*, let *tn* touch ehe Ellipſis in *t*, and *Tn* touch the circle in *T*, draw *TtE*, let the Tangent be continued to cut the circle in *L*, from *L* and *K* where the Tangent cuts the circle draw lines to the Nodes *S* and *F*, from the center *C* draw *Ce* at right angles to *Ln*: Firſt, I ſhall prove that the 5.

angles

Fig. angles SLn, FKn are right angles; for the tri-
2 angle nSL being cut proportionably by the lines Ce and FK, for $SC = CF$ and $Le = eK$ (by the Schol. 27 e 3) the lines LS, Ce, FK will be parallel (by the 2 e 6) and the triangles nSL, nCe, nFK are alike, and the angles SLn, Ccn, FKA will be right angles.

5. 3. In the Figure let tF be continued, and make $tF3 = t3$ equal to CA = Radius, from t draw tp perpendicular to In, then lengthning tp to P, it shall divide the line 33 into two equal parts, because the angles Ltp, Ktp are right angles; and therefore $3P = 3P$, and because the triangles ntE, tEp, CPp are alike, and $Cp + pE = CE$, the base of the triangle $3t3$ will pass by the centre C; these being thus premised, I say, that $tF + tS$ are in every place equal to twice the Radius, or unto $2CA = 2CP$.

5. For continuing, $C3$ to O, that CO may be equal unto $C3$, the triangles OSC, $CF3$ are equal, and the angles one to another answering equal sides by the (4 e 1) for $CO = C3$ and $CS = CF$ and the angles at C are equal by the (15 e 1) $SO = F$; and the angles O and 3 are equal. But the angles $t3C$ and $O3S$ are equal by the (15 e 1) therefore the angles $SO3$ and $S3O$ are equal, and the triangle $SO3$ having the angles at O and 3 equal, the sides OS and $3S$ will be equal by the (5 e 1) and $S3$ will be equal to $F3$, but $t3 + tF3$ being equal to FA, $(t3 + 3S) = tS + (t3 - 3F) = tF$ are equal to two Radii or unto AP. And therefore the lines from the Nodes meeting together at the Ellipsis are equal to the Axis

where-

wheresoever they be taken. There is likewise in this Scheme plentiful matter for finding very many conclusions in this kind, whosoever shall be at leisure to exercise himself in the Analyticks, for the triangles *nSL*, *nCc*, *nFK*, *ptE*, *cPp*, *ntF*, &c. are similar and alike, and the triangles *SLt*, *tKF*, *Cpp*, *Cpp*, *sr* 3 are alike. Fig. 5.

5. In the Scheme let the Ellipsis be *PTA*, the circle (upon $PC = CA$) *PDA*, let *S* and *F* be the Nodes, make *Aa* to be equal to $FA = SP$, upon $Sa = Pa = 2CA$ make a circle upon the centre *C*, draw *CD* parallel to *SH*, let perpendiculars fall from *H* and *D* upon the Axis *PS*, which let be *Hm*, *Dn*, first, $SH = 2CD$, for $SH = PA = 2CA = 2DC$, and because *CD* is parallel to *SH* the angles *HSC* and *DCF* are equal, and so the sine *Hm* is double to the sine *Dn* because the Radius *SH* is double to the Radius *CD*, and the sines are always proportional to the Radii which needs no demonstration: now the line *HF* will pass by *D*, for the triangles *HmF* and *nDF* are alike because *Hm* is parallel to *Dn*, and the angle *H* will be equal to *D*, draw *DO*, parallel to *mn*, it will be equal, and the angles *HOD* and *ODn* will be right angles, the triangles *OHD* and *DnF* will be alike, and the angles *OHD* and *oDH* will be equal to the angles *nDF* and *nFD*, and the angles *HDo*, *nDF*, *ODn* will be equal to two right angles; therefore *FD* and *DH* will be one streight line by the (13 *e* 1.) Again, I say *HF* is bisected in *D*, for $Hm . Dn :: HF . DF$, but *Hm* is equal to $2Dn$ therefore *HF* is equal to two *DF*. 6.

From the point *T*, where *TH* intersects the

Fig. Ellipsis, draw *TD* and *TF*; *TH* will be equal to *TF*, because $SH = 2CA = AP = ST + TF$ by the last Section of this Chapter, therefore the triangles *THD*, *TDF* having all their sides equal, the angles will be equal by the (8 *e* 1,) and the angle *SHD* equal to the angle *TFD*, which was before proved to be equal to the angle *FDC*.

6. The angles of Reflection and the distances of the Nodes to any point of the Ellipsis, and some other observable things, may be demonstrated another way, not without some pleasure because of the variety thereof.

7. In the Scheme let the Ellipsis be *PNTA*, the circle be *PLDA* upon the centre *C*, with the Radius *PA* draw two circles, one upon *S* (*S* and *F* being then Nodes) another upon the centre *C*, which let be the circles *SHA*, *ChB*, let the Arch *AH* be equal to the Arch *Bb*, from the points *Hh*, let fall the perpendiculars *Hm* and *hb*, draw *Hh* and *FH*, where *FH* cuts the circle *PLDA* draw *Dn* perpendicular, and from the point *T*, where *SH* cuts the Ellipsis, draw *TF*, and to the point *T* a touch-line *TN*, by the Rules aforegoing, and from the intersection of the Tangent and Circle *PLDA*, that is *L* and *D*, draw the line *LS* and *DF*, and the point *D* is the common intersection of the lines *LN*, *FH*, *CDH*; because of the perpendicular *Dn* the angle *FDN* is a right angle, and the angle *HDh* = to the angle *CDF*.

7. 1. I say *Hh* is equal to *mb*, for in the Parallellogram *Hmbh*, *Hm* is equal to *hb*, because sines of equal angles, and the angles *m* and *b* are right angles

angles by construction; therefore *Hm* is parallel to *hb*, and *Hb* is equal to *mb* by the (33 *e* 1) and in the Rhomboides *SHCb*, *Hb* is parallel to *SC* because it is so to *mb*, and *SH* is equal to *CH*, therefore *SC* is equal unto *Hb*. Fig.

2. I say that *DF* is the half of *FH*, for first *Dn* is the half of *Hm*, for it is the half of *bh*, it being as *Cb*, *bh* : : *CD*. *nD*, but *CH* is double to *DC* by construction, therefore *hb* is double to *Dn*, or *Dn* the half of *bb*, and *bb* is equal to *Hm*, therefore *Hm* is double to *Dn*, and because the triangle *FHm* is cut with *Dn* parallel to *Hm*, therefore *mH*.*HF* : : *nD*.*FD*, but *nD* is the half of *mH*, therefore *FD* the half of *FH*. 7.

3. It being proved in the second Section of this Chapter, that the angles *FDT* and *FDN* are right angles; I say that *ST*+*TF*=*PA*=2*CA*, for the two triangles *TFD*, *THD* will be equal (by 4 *e* 1) and *TH*—*TF* and *ST*+*TH*=*SH*=*PA*=2*CA*=*ST*+*TF*, because *TF*=*TH*, which proves that the lines from *S* and *F* the Nodes to *T* the Ellipsis are equal to the Axis *AP*=2*CA*. Further, *GD* will be equal to the half of *TF*, that is to *TG* or *GF*, for *GD* being parallel to *TH* it will be *FH*. *FT* : : *FD*. *FG*, but *FD* is the half of *FH*, therefore *FG* is the half of *FT*, and by the same Analogy *GD*=*FG*. 7.

4. Lastly, the triangles *LTS*, *TDF* are alike, for the angles *TLS*, *TDH*, *TDF* are right angles. The angle *LTS* is equal to *HTD* by (15 *e* 1) which is equal to the angle *DTF* because *HT*=*TF*, therefore the angle *LTS* is equal to the angle *FTD*, which angles are the angles of 7.

Fig. Reflection, which are here proved to be equal.
7. These demonstrations, which *Apollonius* and *Mydorgius* have performed in the 52 *Prop.* of the first of *Apollonius*, and in the 53 *Prop.* of the third Book, and in the 49 and 51 *Propositions* of the first Book of *Mydorgius*, and in many other places of both their Books; as likewise that mentioned in Dr. *Ward*'s Astronomy, I have here performed two several ways.

CHAP. VI.

Of the lines, and nature of such lines as may be drawn within the Ellipsis.

1. IN the Scheme let *PDA* be the circle, and
8. let *PRNtA* be the Ellipsis, let from any point in the Ellipsis *R*, an Ordinate rightly applied, and the sine from the circle *rRe*, from the points *N* of the Conjugate, and *R* taken in the Ellipsis, with the distance $CD = CA = \frac{1}{2}PA$ intersect the Conjugate in the points *O*, *o*: *I say, that every point on the Conjugate betwixt* C *and* O; *which the Semiax* = CD *shall intersect from any point of the Ellipsis* R *in* o *shall leave* vo *equal to* CO, *which is equal to* ND, *equal to the Semiax less by the Conjugate*, that is, $CD\text{-}CN = ND = CO$ and *vR* shall be equal to the semiconjugate *CN*, and measuring $cv = CO$ by the degrees of the Ellipsis or the circle, they shall be divided into like parts whether sines or otherwise; as the Ordinates or sine-lines shall divide *PC*; for because *rRE* is parallel to *DO*, and the lines *rC*, *RO* are

are equal, they ſhall be likewiſe parallel, and *Rr* Fig. 8.
equal to *Co* and the angle *ROC* will be equal to *rCD*, let *Ra* be drawn parallel to *ec*, becauſe in the parallelogram *Raec* all the ſides are parallel, they are equal, oppoſite ſide to oppoſite ſide; that is, *Ra*=*eC*, draw *rH* parallel to *eC* it ſhall be equal to *Ra*; but *rH* is the ſine of the angle *rCH* and *Ra*=*rH* is the ſine of the angle *ROa* and *ao* the coſine =*CH* for the triangles *rHC*, *Rao* are equal. It was proved in the firſt 8.
Chapter Sect. 1. that as Rad. was in proportion to the Conjugate, ſo any right ſine to its Ordinate, that is, *DC*. *CN* : : *re*. *Re*. and *re*. *Re* : : *DC*. *CN*. And the triangle *Rao* being cut proportionally by *vc* parallel to *Ra*, it will be as *ao* (which is equal to *HC* and to *re* the right ſine) is to *Ca* (equal to the Ordinate rightly applied *Re*) ſo is *Ro* (equal to Radius equal *CD*) unto *Rv*, that is in Symbols *aO*(=*re*).*Ca*.(=*Re* : : *Ro*(=Rad.= *DC*). *Rv* which muſt be equal to *Nc*, it having been proved before that *re*.*Re* : : *DC*.*CN*=*Rv*. and *vo* will be equal always to *CO* becauſe *NO*= *Ro* and *Rr* equal to *NC*, therefore the remains *Vo*=*CO* and *Dn* being equal to *CO* and *rR* before proved equal to *CN*, the diviſions on *CO* will be equal to the difference of the right ſines and Ordinates rightly applied, which are in the ſame proportion as Radius to the ſines, therefore the line *CO* will be divided like to a line of ſines, if the Ordinates *Re* divide the Radius *PC* according to a line of ſines.

Again, *OC* being ſtill the ſame, when the Radius in this ſuppoſed motion ſhall reſt upon *CP*,

Fig. then will $CV=CO$, and every vC will be the ſine-complement of Co, VO being Radius and the angle at C a right angle, and the angles remaining the one the complement of the other, and the line VC will be divided alike unto the line CO, that is, if CO will be a line of ſines ſo ſhall VC likewiſe.

8. 2. In the Figure let TtE be the ſine, and the anſwering Ordinate, let the touch-lines to the circle be Tn, to the Ellipſis tn, from any point p draw parallels Pih, plk to them both, *I ſay, that by a line from the centre C to either of the touch-points, the lines intercepted both in the circle and in the Ellipſis are biſected*: For let the Semicircle be now imagined at his inclination, and the ſame lines drawn upon it, that is, nT, pih, the line CT ſhall croſs the parallel ph in O, and make $hO=Oi$, (by the 3 and 28 *e* 3,) the point hOi ſhall fall perpendicularly in the lines hm, OS, ig upon the point KoL, and KoL will be parallel to tn, and the triangles made from thoſe points, from mSg, and p, and n, will be ſimilar and alike, and ms will be equal to sg, and ko will be equal to oK, which was the Propoſition.

3. It being formerly proved that the Semicircle having the ſame inclination either under or above the plane, will deſcribe another equal half of the Ellipſis, there will follow as ſo many conſectaries to what hath been ſaid before the following aſſertions, which either need no demonſtration, as being very plain, or if they do, may be by any indifferent judgment made out.

1. *By the laſt any two lines drawn parallel in the Ellipſis*

Ellipsis and bisected, the line that bisects those parallels shall pass by the centre C. Fig. 8.

2. *All these lines that pass by the centre being perpendicular under the several diameters of the circle, are likewise not improperly called diameters.*

3. *The greatest of these is the Axis, which is equal to the Semicircles diameter, and the least to the Conjugate rightly applied.*

4. *Which Axis being the common interjection of the two Planes, any two diameters that are equally distant from it in the Ellipsis, are equal.*

5. *And if any two diameters being equal be drawn; in the midst of these lies the Axis, crossing all in the centre.*

6. *Any line passing by the centre is a diameter.*

7. *All lines that are parallel to the tangent, are or may be ordinates to that diameter that shall pass by the tangent point* T.

These and many more consequences might be deduced from the premises, for my intent these are sufficient.

Upon consideration of what hath been said before, I could take most of the Propositions of the 3. Book of *Euclid*, and apply them to the Ellipsis with the like demonstrations: hereof sufficient.

4. It remains now to discourse of such means and ways, that will find out the length of any lines drawn from any point of the Axis, continued, if need be, (which is a far more general Proposition than that of Monsieur *Montford.)* I will first begin with the lines issuing from the Nodes or Foci, because they will be of some present use.

Fig. I suppose by this time very few but understand of Sir *Paul Neal*'s Prop. and of Dr. *Ward*'s, at the same time of resolving the Triangle in the Ellipsis; and withal Dr. *Ward*'s demonstrations of *Bullialdus* his failings in what he promised, and others (amongst whom I was one) too credulously believing: The Proposition is this, *Having the Axis of the Ellipsis given, and the distance of the Nodes from the centre, (which are equal to the Excentricity bisected) and the angle at one Node, which a line shall make with the Axis to the Ellipsis, to fi d out what angle that line returning to the other Node, shall make with the Axis?*
6 which I thus perform another way. In the Scheme let $CA = CP$, $CF = CS$, and the angle FST be given to find out the angle SFT, and the sides TF, TS: it was proved in the fifth Chapter that $ST + TF = 2CA$, and that $GD = GF$ angle at $D = F$. Dn is the sine of the angle $DCF =$ to the angle TSF, Cn is the sine-complement of the angle at C, and nF is the difference of the Excentricity and the co-sine of the angle aforesaid; therefore in the triangle DnF right angled at n, having nF and Dn the angle at F may be had, and $180 - F - C = D$, and $D - F$ or $F - D = TFS$, and having all the angles and one side, the rest of the sides are easily obtained. In the calculation of this kind there will be no difficulty, but there may be these admonitions given, which some who cannot understand the Scheme, and yet would use this way, may do it: For if the perpendicular Dn fall between F and S, then the Rule before holds good; for the angle being found,

found, the angle D is the complement of C and Fig.
F to 180°. and F less by D is the angle sought.

But if it fall betwixt F and A, as in the Scheme 7 it doth, then you find the complement of F to 180, which is all the difference.

This triangle may be resolved by the oblique 6.
angled triangled CDF, for CD is the given Radius, and CF the bisected excentricity, and the angle at C to find out the angle at D and F, and $D \cdot F =$ to the angle, but this in effect is that of Sir *P. N.*

To these two may be added the three following; for though they be general for any point of the Axis, or any point in it continued, and without any consideration had of the Nodes at all, yet are they appliable to find this angle. Now followeth

The threefold Resolution of the Question proposed by Monsieur Montfert, *because it comes in this place conveniently.*

The Proposition.

Having in Numbers the extreme Diameters of the Ellipsis, and the distance from the centre to any point in the transverse Axis, where let a line cut the same under an Angle given; to find the segments of that line produced, if need be, terminated betwixt the transverse Axis and Ellipsis in Numbers.

That is, having in the next Figure following AC 1.00000. NC :76604. CB :50000 and the angle CBD 70 degrees to find BD and BF.

The

Fig.

The Lemma premised was this :

9\. If a circle whose Diameter shall be equal to $PA=2CA$ shall upon the common intersection PCA and centre C, so incline to the Plane of the Ellipsis, that the point n of the Ray of the circle Cn ($Pn=90=nA$) be perpendicular over the point N on the Ellipsis, (CNn being a right angle :) Or, which is all one, If the perpendicular Nn equal to the square Root of $Cn(q)$-$CN(q)$. Or if the angle $nCN=40°=$ sq. complement of CN, then I say will the Semicircle $PndA$, in every part be perpendicular over the Ellipsis $PNDA$, that is, CNn, BDd are right angles. The demonstration and reason of this Lemma hath been made out in the first and precedent Chapters, therefore I shall need to say no more thereof.

The first way of resolving the Proposition.

In the Scheme let $PNAF$ be the Ellipsis, let $PndA$ be the Semicircle elevated above the Plane of the Ellipsis to an angle of 40 degrees, because CN:76604 is the sine of 50 degrees, then will the Semicircle be perpendicular over the Ellipsis, let CN be continued, and draw FBD out until it intersect the Conjugate in G, all the points P, N, G, H, D, C, I, E, B, A, F, are upon the Plane of the Ellipsis, all the rest upon the elevated Plane of the Semicircle, $Cnhg$ is the Ray of the Semicircle perpendicular over the

Conjugate

Conjugate produced, and *Bdg* over the sine Fig. BDG, *d* over *D*, and *g* over *G*: these premised 10. the calculation doth follow.

1. In the right angled Triangle *CBG* right angled at *C*, there are given the side *CB*:50000 and the angle *CBG* 70°. Therefore the side *CG* will be found to be 1:37374.

2. In the right angled Triangle *CGg* (*g* being upon the Semicircle and perpendicular over *G*) right angled at *G*. There are given *CG* found by the first Work 1:37374, and the angle of the inclination of the Semicircle to the Plane of the Ellipsis *gCG* 40°: the side *gC* will be found to be 1:79329.

3. In the right angled Triangle *CgB* right angled at *C*, there are given *Cg*, found by the second operation to be 1:79329, and the side *CB* :50000. Therefore the angle *CBg* upon the Semicircle to be 74°.25′.

4. In the oblique angled Triangle *CdB*, there are given the two sides *Cd*(=Rad.=*CA*)= 1.00000, and the side *CB*:50000, together with the angle *CBd* 74°.25′. Therefore the angle *dCB* will be found 76°.47′. which is *Kepler*'s anomaly of the Eccentrick, the sine-complement of which angle will be :22849, which taken out of *CB*:50000, leaves *EB*:27151.

Lastly, in the right angled Triangle *DEB* right angled at *E*, there are given the side *BE* :27151, and the angle at *B* 70, therefore the side *BD* will be found to be :79384:

The

Fig.

The second way.

10. In the Scheme let the lines be the ſame as before, only in this way let *BD* be drawn only to the circle upon the plane of the Ellipſis *H*, over which upon the Semicircle is *h* imagined to be perpendicular, and *d* over the point *D* on the Ellipſis. Then

In the Triangle *CHB* (upon the Plane) right angled there are given the two ſides *CB*:50000, and *CH* Rad. 1:00000, and the angle *HBC* 70 degrees, therefore the angle *HCB* will be found to be 81°. 58′, whoſe ſine is :49020, and the coſine =13984. The ſine is equal to *HI* and coſine *CI*, therefore *CB*:50000 leſs by *CI*:13984 will give *IB*:36016.

2. in the Triangle *IHh* right angled at *H*. There are given *IH*:99020 gotten before, and the angle of inclination 40°= to the angle *HIh*: Therefore there will be found *Ih* upon the Semicircle equal to 1.29262.

3. In the Triangle *IhB* right angled at *I*. There are given *Ih* gotten before 1.29262. *IB* :36016 gotten by the firſt part of this operation; therefore there may be found the angle *IBH* 74°. 25′.

Now having the angle at *B* 74°. 25′, which was found to be the ſame by the third operation of the laſt, then for the latter two operations, they will be the ſame as before, and to be taken out of the laſt, and ſhall not need to be here repeated: Therefore the ſide *BD* will be found

to

to be in length :79384 the thing sought. Fig.

The former of these two resolutions hath no great variety in the calculations, for the perpendicular may be taken as well on either end of the Axis as upon the Conjugate, if need be; as suppose if the angle *CBD* shall happen to be greater than a right angle, then will the line *BD* incline and meet with the Tangent set up upon the point *A*, and the same work will follow.

The latter will hold which way soever the line *BD* incline, the point *I* will fall sometimes betwixt *P* and *C*, sometimes betwixt *B* and *A*.

The third way.

In the Figure adjoyning in this way the Ellipsis is perpendicular over the circle made upon the Conjugate, the point *A* is perpendicular over *a*, *B* over *b*, *D* over *d*, so that *NcbadH* are in the subjected Plane, the rest upon the plane of the Ellipsis elevated over the other upon the common intersection *HNCn* to an angle of 40° equal to the inclination before of the circle; therefore having the angle *CBD*, having the side *BC*, the side *BD* will be found at five workings, as followeth: 11.

1. In the Triangle *bBC* right angled at *b*: There are given the side *CB*:50000 the angle at *C* 40°. therefore the side *Cb* will be found to be :38302.

2. In the Triangle *BcH* right angled at *C*: There are given *BC*:50000 the angle at *HBc* 70°. therefore the sides *CH* 1:37374, and the side

Fig. side *HB* 1:46190 may easily be obtained.
11. 3. In the Triangle *CHb* right angled at *C*: There are given the side *CH* 1:37374, and the side *Cb*:38302, therefore the side *Hb* will be found 1.42612, and the angle *HbC* 74.25. which falls exactly the same with the like operations of the last two ways.

4. In the oblique angle Triangle *Cdb*: There are given the two sides *Cb*:38302 *CD*:76604, and the angle at *b* 74°. 25′. gotten as before, therefore the side *db* will be found :77422.

Lastly, the Triangle *HBb* being cut by *Dd* which is parallel to *Bb*, and three of the four proportional parts given, *viz.* *Hb* 1:42612, *HB* 1:46120 and *db*:77422, all gotten as in the operation, the side *DB* will be gotten be the Golden Rule after this manner: As *Hb* 1:42612. *HB* 1:46190: : so is the part *db* :77422 to the side *DB* 79372. The same or like work will follow if perpendiculars were raised upon the points *A* and *a*, and the line *BD* should incline that way.

The difference of the two last Figures in the answer, ariseth from the small Canon of Sines and Tangents used by me; I mean, the Radius went but to five places, but the way being certain it matters not much for a guide.

For the resolution of any Triangle drawn within the Ellipsis, whose terms are known from the center and one angle, are by these ways resolved. The Anomaly of the Elliptical Eccentrick I have found out five several ways, and so many times resolved the Triangle in the Ellipsis,

lipsis, for having the three sides and one angle, Fig.
the rest of the angles may be had. 5.

CHAP. VII.

Of certain Conclusions arising from what hath been said before.

1. *The Axis and Conjugate of an Ellipsis being given, the inclination of a circle that shall be perpendicular over it*, will be had by the sine complement of the Semi-conjugate, if the circle be equal to the Semi-axis, which is always conceived to be Rad. with 1 and so many Cyphers. Or if the angle of inclination be given and the circle, the Conjugate will be had by the contrary work.

2. *Any Ellipsis being given the centre of it will be found out*, by crossing any two lines drawn parallel in the middle, for that line will pass by the centre, and is a diameter, which being bisected will find out the centre. Or if a portion of the Ellipsis be but given, there must be two pair of parallel lines bisected, which will cross one another in the centre: The rest of the lines may be had as followeth. 6.

3. The centre being found out, *to find out the Axis of any Ellipsis*, if the diameter drawn at any place be the longest that can be, it is the Axis; but if it be not, then upon the centre with a Radius equal to that half diameter drawn, make a circle which will cross the Ellipsis, and if

Fig. 5. if that part of the Ellipsis intercepted betwixt the diameter and the point of the Intersection be bisected, the Axis will pass by that bisected point, and this line being crossed again at right angles will give the Conjugate.

4. *The Tangents are drawn to any point of the Ellipsis*, by help of the circle; for if the Ordinate rightly applied be continued and made a sine at the point where it crosseth the circle, draw a Tangent-line, and from that point where that Tangent continued shall cross the Axis, a line drawn shall touch the Ellipsis in the point proposed. Or any of those proportions may be used which are set down in the 4 Chap. For

1. *CE. CA* : : *CA. Cn.*
2. *CE. PE* : : *EA. En.*

5. Therefore the point *n* upon the Axis will be found out.

And it is all one if the Tangent were to be found from any part of the Conjugate.

4. *The Nodes or centres of the Ellipsis are had*, by the length of the Conjugate, for Rad. (*q*) less by the square of the Conjugate; the difference is equal to the square of the distance of the Node from the centre.

Or the sine-complement of the Conjugate is the distance to the Node.

From any touch-line to the Ellipsis the Node may be found out several ways; one, by the drawing the circle upon the Axis: If from the points where the Tangent cuts the circle, perpendiculars

lars be let fall, they will point out the centres or Nodes upon the Axis: and there are two other ways from the 5 Figure, which I omit because too tedious. Fig. 5.

Or, lastly, if with the distance of the Radius or Semiax, you set one foot of the Compasses in the Extremity of the Conjugate, with the other you may intersect the Axis in both the Nodes.

For the length of the rest of the lines, or for any other business concerning the Ellipsis, they have been before declared, or may easily be made out.

CHAP. VIII.

Of drawing the Ellipsis, and the Division thereof.

1. H*Aving the Axis of any Ellipsis and thee. Conjugate, the inclination is found as before* As if the Axis be 200000, the Conjugate 76604, the inclination will be found to be 40 degrees: If it be required to draw an Ellipsis to those diameters, prepare a Semicircle of the same diameter with the Axis, either of metal or any thin board, work the edge of the Semicircle truly, and let it be fixed upon the Plane where you intend to describe the Ellipsis, and upon the Axis of the Ellipsis to the inclination gotten as before; then having made ready a perpendicular like unto a square, only the base must be of some considerable breadth, that the square

Fig. may not incline at all, and having in this ſquare a pointer in the rectange point: I ſay, if this ſquare be moved upon the Plane, ſo that the perpendicular may touch the edge of the Semicircle (inclined as before,) the pointer in the rectangle will upon ſuch motion deſcribe an Ellipſis by the firſt Chapter, and according to the intent of this Diſcourſe.

2. *Having the ſame things given*, viz. *the Axis and Conjugate (which hereafter I ſhall take for granted to be given)* make a circle upon the Axis, which divide into as many degrees and parts as the exactneſs of the Work requireth, (the ſmaller the more accurate) draw the chords or ſines 10.*r*.10. 20.*r*.20, *&c.* the Ordinates being proportionals to the ſines, as the Radius to the Conjugate; therefore finding by the (12 *e* 6) a fourth proportional to the three lines Rad. Conjugate and ſine, that will be the Ordinate: A ſhort way to do it, is to make a right angled triangle, whoſe Baſe ſhall be the Semiaxis or Radius, and the perpendicular the half of the Conjugate, then dividing the baſe like to a line of ſines, and raiſing perpendiculars to the Hypothenuſe, they ſhall be Ordinates ſought: And if the Sector be at hand from that inſtrument ſetting any ſine 10.0, 20. 0, *&c.* upon the Radius (or 10) upon the line equally divided, take out the length of the Conjugate and it will give the Ordinate, ſo altering the Radius every time the Ordinates may eaſily be taken out. Laſtly, by the Table following *B*, being the length of the Ordinates, and made as was taught in the ſecond

Chapter,

Chapter, the Ordinates may be pricked down from a scale of 100, equal to the Radius or Sine *CD*, and thereby the Ellipsis may evenly be drawn. Fig.

3. *By the Tangent-line.* One Tangent to be raised upon the point of the Axis *A*, another upon the Conjugate *N*, where these two meet, a line from the centre shall cut the sine of 45° in the Ellipsis; let those two Tangents be divided, the one of them from the greater circle made upon the Radius *CD*, the other from the circle made upon the lesser *CN*, then shall Secants from the center *C* cut the sines in the points of the Ellipsis; the Tangents may be divided from the Table of natural Tangents, having two scales equal to the two Radii, or by one scale and the Tangents taken out of the Table annexed *D* made as was taught in the second Chapter.

4. *By help of the Chords*, or the lengths of the subtending lines from the points *A* or [illegible], and these being set off from *A* to cross the sine-lines, will find out the several points to draw the Ellipsis by. The Table for the lengths of the 9.
Chords is marked *E*, and is made as in the second Chapter is set down.

5. *By the raies from the centre*, or the lengths of the lines from the centre *C* to the Ellipsis, the line of the Ellipsis may be drawn: The length of these lines you have in the Table under *F*, which are the sine-complements of the perpendiculars gotten as is shewed in the second Chapter. If these lengths be set off to cross the

Fig. the ſine-lines in the points *r*,*r*, the Ellipſis may from thence be evenly drawn.

6. *By the ninth Section of the ſecond Chapter*, it is apparent that the ſine-lines or chord-lines both of the great circle and leſſer meet in one point of the Ellipſis: Therefore if both the circles be divided into degrees, and ſine-lines drawn as is ſeen in the general Scheme, the ſine of 10 of the one ſhall croſs the ſine of 80 of the other in the Ellipſis, and by theſe interſections the Ellipſis might eaſily be drawn.

7. *But the eaſieſt way that can be* by numbers, is by the natural ſines taken out of the Canon, for having drawn the circle upon the Axis *PA* and the chords or ſine lines; and having a ſcale or the ſector ſet equal to the Radius of the leſſer circles *CN*, the natural ſine of 10 ſet off in the chord of 80, of 20 in 70, *&c.* will prick out the Ellipſis to be evenly drawn as aboveſaid.

8. According to what was laid down *Chap.6. Sect.* 1. the Ellipſis may be drawn ſeveral ways: Firſt, if the difference betwixt the Radius and Semi-conjugate *CO* from *C* be divided like to a line of ſines, and numbred as in the general
9. Scheme, and if from 8 to 1, 7 to 2, 6 to 3, *&c.* the Radius be ſet off, it will find the points *r*,*r*, whereby to deſcribe the Ellipſis. Or if the line *CV*=*CO* be divided into like parts, then if lines be drawn from 8,7,6, *&c.* of *CV*, and upon every one of thoſe lines Radius be ſet to *r*,*r*,*r*, *&c.* the Ellipſis may likewiſe be drawn.

From

From hence *Guydo Ubaldus* contrived a very fit Instrument to draw one quarter of an Ellipsis, by the flux of a point; for if a square made of Brass, Iron or Steel, be applied upon the semi axis or semi-conjugate, so that the rectangle may rest upon the point *C*, and if there be a Ruler of a sufficient length, with a Pointer or a place for a point or pen at the end, and two moveable Pins and Scrues which may move upon the Ruler from the Pointer; if the farthest pin be set from the pointer with a distance equal to *CA* or *CD*, and the middle pin be removed from the pointer with the distance of the semi-conjugate *CN*, then if the middle pin slide upon the Axis, and the furthest pin come close by the Conjugate towards *C*, the pointer will by the flux describe a quarter of the Ellipsis. The pointer being supposed to be *r*, the middle *p* in *v*, the furthest *p* in *o*, and *or* will be equal to Radius in every place; but this was sufficiently demonstrated before: In the doing hereof care must be had that the centre of the two pins may move by the fiducial edge of the square. Fig. 9.

Adrianus Metius, in the beginning of the second Tome of his Astronomy, composeth this Instrument with a double square joyned in the Ruler over *CO* the semi-conjugate, that the pointer *r* might describe the half of the Ellipsis at one motion, the cross *CO* having a Grove in it to receive the further pin *O*; but he was foully mistaken to make use of this Instrument to draw those lines in that projection which are

Fig. parts of circles, that Inſtrument being that
9. which *Gemma Friſius* writ upon, and the eye in this projection imagined to be placed contiguous upon the ſuperficies of the Globe, in the point of the Eaſt and Weſt where the Equinoctial cuts the hour circle of ſix a clock: They only ſhall be ſenſible of his error, who ſhall provide ſuch an Inſtrument to deſcribe the Ellipſis withal, as I did my ſelf when I was not able to diſcover his miſtake.

9. By the Nodes or Foci, and their diſtance from *C*, the Ellipſis may be drawn divers ways; for, if upon the one Focus many circles be drawn, the greateſt not having its ſemi-diameter longer than *FP*=*SP*, or ſhorter than *FA* or *SP*, and if with the remainder of theſe ſemi-diameters to the length of *AP*, or the Axis upon the other Node or Focus, thoſe former circles be interſected, thoſe points of interſection will be in the Ellipſis, and evenly drawn will deſcribe the ſame. Or if a thread or line being twice as long as is the diſtance *F* or *S* to *P* or *A* be tied at one end, and move upon two pins or points ſet in *F* or *S*, a pointer moving in the extremity of this thread, will deſcribe an Ellipſis; for every where in both theſe *Sr* and *Fr* are equal to *PA*, but this laſt is ſo well known, I ſhall need to ſay no more.

10. By what hath been ſaid, that is, by the 2, 3, 4 and 5 Sections, after the Ellipſis be drawn, it will be divided into 360 degrees and minutes (if occaſion were,) like unto the circle either by the Sines, Chords, Tangents or Raies,

and

and may be of good uſe to thoſe who ſhall pro- Fig.
ject the Sphere Orthographically. 9.

There might ſeveral other varieties be delivered for the deſcribing the Ellipſis, but I ſhall leave them to the Readers conſideration.

A Table of the length of the Sines, Ordinates, Perpendiculars, Tangents, Chords and Raies which have been ſpoken of before.

gr.	*Sines.*	*Ordin.*	*Perpe.*	*Tange.*	*Chords.*	*Raies.*	
10	:17365	:13283	:11146	:17632	:134..	:99376	
20	:34202	:26190	:21984	:36397	:269..	:97553	
30	:50000	:38885	:32611	:57735	:402..	:94533	
40	:64278	:49242	:41310	:83910	:551..	:91068	
50	:76604	:58684	:49240	:64278	:686..	:87035	64278.
60	:86602	:66022	:55387	:44228	:794..	:83243	
70	:93969	:71994	:60413	:27881	:981..	:79688	
80	:98481	:65430	:63293	:13507	1:085..	:77420	
90	1:00000	:76604	:64379	:00000	1:259..	:76604	
	A	*B*	*C*	*D*	*E*	*F*	

Soli Deo Gloria.

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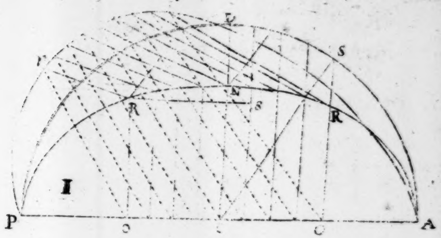
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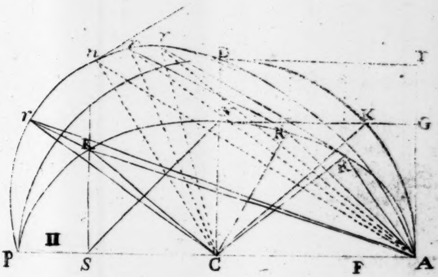
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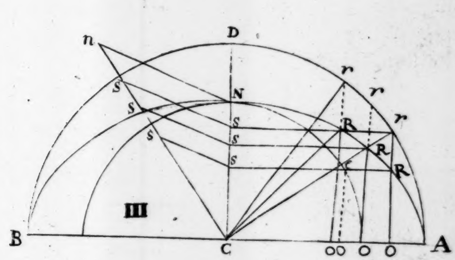
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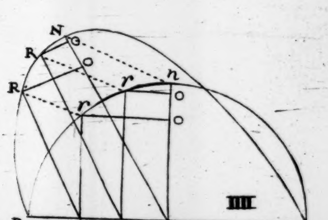
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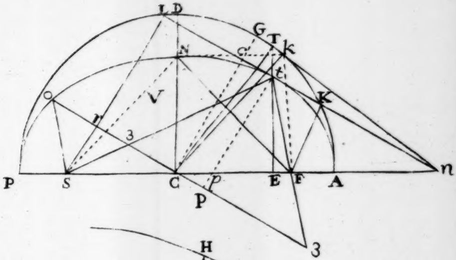
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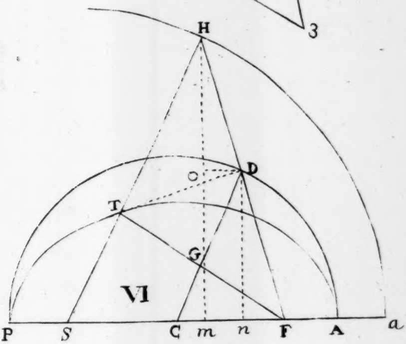
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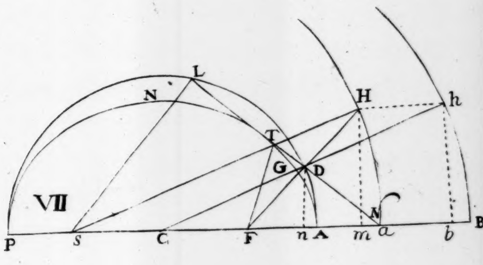
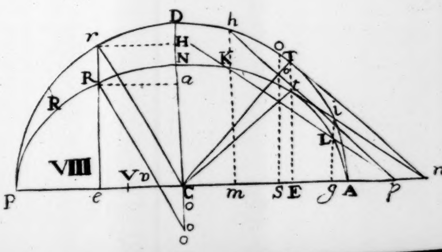

VII
L N H h T G D M P S C F n A m a b B


VIII
D h r H N K O T R a t R l L V v P e C m S E g A p n

CONICAL SECTIONS:

OR,

The ſeveral SECTIONS

OF A

CONE.

Being an ANALYSIS, or Methodical Contraction of the two firſt Books of *Mydorgius*.

And whereby the Nature of the *Parabola*, *Hyperbola* and *Ellipſis* are very plainly laid down.

Tranſlated and drawn from the Papers of the Learned Mr. *W. O.*

By Sir JONAS MOORE.

LONDON,

Printed by *R. H.* for *Obadiah Blagrave* at the *Bear* in St. *Paul*'s Church-Yard, 1688.

To the Reader.

About the beginning of these late Destractions and Troubles amongst us, I being then in *Lancashire*, had the favour from Mr. *Christopher Townley* of *Carr* (a geat promoter of Arts, and lover of all Ingenuity and ingenious men) not only the use of his *Library*, but Copies of such *Mathematical Manuscripts*, as he then had by him, the which with great care he had obtained from several Artists: Amongst which I had this of *Mydorgius*'s two first Books, though in many parts defective; a part of the beginning and many Propositions in several places were wanting. Afterwards I came to learn that these two Books were *analyzed*, and that my Copy was the work of that Reverend great Scholar, Mr. *William Oughtred*, my most kind

Friend

Friend and courteous Favourer in Mathematical Studies, I therefore with care finiſhed this Piece, and tranſlated it; and having with me Mr. *John Goddard, an excellent work-man*, at my Houſe in *Norfolk*, graving me Plates for *the great Level of the Fenns*, I cauſed him at ſpare hours to cut me the *Schemes* for this Book.

Now, Reader, if thou reapeſt either pleaſure or profit by this Piece, acknowledge it to Mr. *Oughtred*: what I have done I freely offer it as the poor Mite of my good will, being much aſſured that at ſome retired hours this Piece will be an amiable Companion, and will yield thee a delightful reward for the expence of that time thou wilt beſtow thereon.

Jonas Moore.

Of the Conical Sections.

DEFINITION I.

1. IF a line of ſuch a length as ſhall be needful, ſhall upon a point fixed above the plane of a Circle ſo move about the circle until it return to the point from whence the motion begun, the ſuperficies that is made by ſuch a line is called a *Conical Superficies*, and the ſolid Figure contained by that ſuperficies and the circle is called a *Cone*.

2. The point remaining ſtill is called the *Vertex* of the Cone, and in all the Figures is noted by the Letter *V*.

3. *The Axis* of the Cone or conical ſuperficies is that line which paſſeth from the vertex to the centre of the circle $=VC$.

4. *Oppoſite* Cones or ſuperficies are ſuch as the ſame right line that deſcribes the Cone, ſhall being lengthned make beyond the vertex, therefore ſuch Cones have a common vertex and a common axis.

5. *The circle* (to which the line that deſcribed the Cone was drawn about) is called the *Baſe*.

6. *A right Cone* is that whoſe axis ſtands at right angles with the baſe.

7. *A*

Fig. 7. *A scalonious Cone*, that whose axis standeth not at right angles with the base.

8. If a plane shall cut the Cone, the common intersection of that plane and the conical superficies, is called a *conical section* or a *conical line*.

9. That line which being drawn within the section shall cut other two parallel lines drawn within the conical section, and terminated in the same section, into equal parts, is called *the diameter of the conical section*.

10. And those lines cut into equal parts are called the *Ordinates to the diameter*.

11. The *Axis of any conical section* or portion of a section, is that diameter whose Ordinates are equally divided and stand at right angles.

12. The point of any diameter in a conical section is called the vertex. $=A$.

Prop. 1. *If a cone by a plane be cut by the vertex* V, *the section will be a triangle* =VDb. For *VD* is that streight line that describes the conical superficies, and so is *Vb*, and *Db* is the common intersection of two planes (which is a streight line by the 3 *e* 11) therefore *VDb* a triangle.

1. Prop. 2. *If the cone* VDb *be cut by a plane parallel to the base* FIHE *the section will be a circle.* For because the triangles *VCb*, *VCD*, *VCG* are all cut proportionally by the plane *FIHE*, it will be *Cb.VC* : : *HE.VH* and *DC.CV* : : *FH.HV*, therefore (by the 22 *e* 5) *ex æquo Cb..CD* : : *HE.*

$HE.FH$, but $Cb = CD$, therefore $HE = HF$, and Fig
by the very ſame argumentation IH will be proved equal to $FH = EH$, for $CG.VC :: IH.HV$ and $VC.CD :: VH.HF$, therefore *ex æquo* $CG. CD :: HI.HF$, but $CG = CD$, therefore $HI = HF$. And therefore the lines $HF = HE = HI$. FIE is a circle.

Prop. 3. *If a ſcalonious cone be ſo cut with a plane, that it cuts off a triangle ſimilar and alike to the whole, and yet not parallel to the baſe, viz.* DIHE, *then is the ſame ſection a circle.* For DIE being the diameter of the ſection, taking any point as I, let a plane parallel to the baſe cut the ſame in I, $FG \parallel BC$, the triangles ABC, AFG, AED, DIG, FIE are alike, it will be $DI.IG :: FI.IE$, therefore $DI \times IE = IG \times FI = HI(q)$ becauſe H is in the circle FHG, and it being the common interſection of two planes, it is likewiſe in the circle whoſe diameter is DE, for $DI \times IE = HI(q)$ and whereſoever I be taken, by the ſame reaſon H would be ſo placed that the ſame analogy would follow, therefore DHE is the Perifery of a circle. 2.

Definition 2.

1. The *parabola* is ſaid to be that ſection whoſe diameter is parallel to the one ſide of the Triangle cut by the axis of the cone.
2. The *hyperbola* whoſe diameter will cut one of the ſides of the triangle produced beyond the vertex. 1

3. The

Fig. 3. The *Ellipsis* whose diameter will cut both side of the Triangle under the vertex.

4. *Opposite sections* are two hyperboles in opposite superficies made by a plane cutting the cone not by the vertex.

5. The *transverse diameter* in the *hyperbole* is the distance of the terms of any two diameters in opposite sections, *viz*. *AE*, and in the Ellipsis it is the diameter continued to the sides of the Triangle.

5. *The centre* of any section is that point where the diameters all meet, and this point divides every transverse diameter into two parts: This in the Figures is noted *I*.

6. *The Parameter* of any section is a line, by which all the Ordinates applied to the diameter are estimated and valued, and is therefore called *Juxta quam possunt:* which if it be placed at right angles to the Axis, is called the *right Parameter*.

7. The *Node or Focus* of the Parabola is distant from the highest vertex of that Figure a fourth part of the *right Parameter*.

8. The Nodes, Focusses or burning points of the Hyperbole and Ellipsis are the points distant from the terms of the transverse by the space of a right line, that is, the square Root of the fourth part of the Figure produced by the said *transverse Axis* and the right Parameter; which applied to the transverse Axis is in the Hyperbole *exceedent*, in the Ellipsis *deficient*.

1. Note, that the Triangle *VDb* is conceived to be the section of the Cone by the Axis: *Db* is the

the base of the Triangle and diameter of the circle *DPb*, and that every parallel line to *Db* in the Triangle is the diameter of a circle. Fig

Note, that in all the Figures and Propositions the sections do cut the base of the Triangle at right angles, and all other parallel lines to the base; that is, the angles *DBP*, *PBb*, *BST* are right angles.

Note, that *A* signifieth the vertex of the section or axis, *T* the point of the Tangent upon the section, *I* the centre, *ST* the Ordinates rightly applied. These being understood will save many repetitions.

Prop. 4. *If the cone shall be cut by a plane that is parallel to the side of the triangle, the section will be a Parabola, and the common section of the plane and the triangle will be the diameter*: that is, let the section be *ptATP*,*pP* cutting *Db* at right angles, *BA* is parallel to *VD*; at any point in *BA* suppose *s*, draw *GSg* parallel to *DBb*, and *tST* parallel to *pBP*, because *DBb* cuts *pBP* at right angles, *GSg* will so cut *tST*; but *Db*, *Gg* are diameters of circles, and *Pp*, *tT* are inscripts bisected in *B* and *S*; therefore *AB* is the diameter, and because *AB* is parallel to *VD*, the section *pATP* is a Parabola. *ST* is an Ordinate rightly applied. 3.

Prop. 5. *If the cone be cut by a plane that will cut one of the sides of the triangles produced in* E, *and the common intersection of the plane and triangle be* AB, *the section will be an Hyperbole, and* 3.

Fig. AB *the Diameter.* Let the preparation be as in the last, *pP* and *tT* will be proved to be bisected as in the last, and *AB* the diameter of the section *ptATP*, which is an Hyperbole, because *AB* produced cutteth *DV* produced in *E*. *ST* is an Ordinate rightly applied, and *AE* is the transverse diameter.

Prop. 6. *If the cone be cut by a plane that shall*
3. *cut both sides of the triangle under the point* V, *and the common intersection of the plane and triangle be* AE, *the section* AtpPT *will be an Ellipsis, and the line* AB *or* AE *the diameter.* Let the plane making the section cut the base *Db* produced with *PR*, the angle *BPR* is a right angle equal to the angle *GST* and *RP* ∥ *tT*, and *tT*, *Pb* will be bisected by the same reasons given in Prop. 4. *AE* will be the diameter, and the section will be an Ellipsis by the definition.

Prop. 7. *In the parabola, if* ST *and* BP *be or-*
4. *dinately applied, it will be* AS.AB : : ST(q).BP(q).
5. For the triangle *ABb* is cut proportionally by *Sg*, Ergo *AS.SB* : : *Sg.Bb* and *GS=DB*, Ergo *AS.SB* : : *Sg×GS* (*=ST(q)* per 35 *e* 3). *Bb×DB* (*=BP(q)*). Ergo *AS.SB* : : *ST(q).BP(q)*. And if it be *ASq.STq* : : *AS. AR. AR* will be the *Parameter.*

Prop. 8. *In the Hyperbole, Ellipsis and circle if*
6. ST *and* BP *be Ordinates applied, it will be* AS×ES.
7. AB×EB : : STq. BPq. For the triangles *ABb*, *EDB* are cut proportionally by the line *GSg*, Ergo

Ergo it is $AS.AB :: Sg.Bb$ and $ES.EB :: GS.DB$, multiply like terms of both proportionals it will be $AS \times ES. AB \times EB :: Sg \times GS. Bb \times DB$, but $Sg \times GS = ST(q)$ and $Bb \times BD = BP(q)$. Ergo $AS \times ES. AB \times EB :: ST(q). BP(q)$. And if it be $AS \times ES. STq :: AE. AR$. AR will be the *Parameter*. Fig.

Prop. 10. *In the parabola it is* $AR \times AB = BPq$. For by the 7. $AS(q). ST(q) :: AS. AR$. and multiplying the latter pair of the proportionals by AS it will be $AS(q). ST(q) :: AS\,q). AS \times AR$. Ergo $AS \times AR = STq$. and by the 7. $ST(q).BP(q) :: AS.AB :: AS \times AR. AB \times AR$. AS and AB being multiplied by AR; but $AR \times AS = STq$. Ergo $AB \times AR = BPq$. and RA is the *Parameter*, for according to it all the powers of the Ordinates are proportioned: For let S be taken wheresoever, it will be $AS \times AR = STq$. 4, 5.

Prop. 12. *In the parabola it is* $VD \times Vb. Db(q) :: VA.AR$. For by the 10. $AR \times AB = BPq = DB \times Bb$. But because the triangle VDb is cut proportionally by AB it is $VD. Db :: AB. Bb$ and $Vb.Db :: VA.DB$ multiply the four former proportionals by the four latter respectively it will be $VD \times Vb.Db \times Db :: AB \times AV.Bb \times DB = BP(q) = AB \times AR$. Ergo $VD \times Vb. Db(q) :: AB \times AV. AB \times AR$. divide the latter pair by AB it will be $VD \times Vb. Db(q) :: AV. AR$ which was to be demonstrated. 4, 5.

Prop. 13. *In the Hyperbole, Ellipsis and circle, if from* E *a line be drawn to the end of the parame-*

Fig. *ter* AR, *gotten as in* Prop. 8. *it will be* $AB \times Br = BPq$. For $AR \parallel Sr \parallel Br$ it is $AE.AR :: ES.Sr :: AS \times ES.\ AS \times Sr$ multiplying the two latter proportionals ES and Sr by AS, and by the 8. it is
6, $AS \times ES.ST(q) :: AE.\ AR :: ES \times SA.\ AS \times Sr$ or
7. $AE.AR :: AS \times ES.STq = (AS \times Sr)$ STq will be equal to $AS \times Sr$. Alſo by the ſame argumentation becauſe it is $AB \times EB.BPq :: AE.\ AR :: EB.Br$. and ſo is $AB \times EB.AB \times Br$ it ſhall be likewiſe $AB \times Br = BPq$.

Corol. 1. *AR* is the parameter by which all the powers of the Ordinates are regulated, for whereſoever *S* be taken it is $AS \times Sr = ST(q)$.

Corol. 2. $BPq = AB \times Br.AB \times EB :: AR.\ ER :: Br.\ BE$.

6, Prop. 14. *In the Hyperbole and Ellipſis if* Vq
7. *be drawn parallel to the diameter* AB, *I ſay it will be* $Vq(q).\ Dq \times qb :: AE.\ AR$. For $Vq.\ Bq :: EB.\ DB$. and $Vq,\ qb :: AB.Bb$. multiply the former like proportionals by the latter, it will be $Vq(q).\ Dq \times qb :: AB \times EB.DB \times Bb$ which is equal to BPq. and by the 13. $AB \times EB.BPq :: AE.AR$. Ergo $Vq(q).Dq \times qb :: AE.AR$.

4, Prop. 16. *If the right line* Tk *touch the circle*
5. GTg, *and the point* N *be the interſection of the plane* VTk, *with the diameter* AE, NT *ſhall touch the ſection in* I. For the plane *VTk* toucheth the circle in the line *VT*. But it is to be underſtood that the right line *Tk* is parallel to *VN* if *S* be

not

not the centre of the circle GTg, becaufe the angle STk is a right angle. But if S be not the centre but C, let the right lines Tk and VN concur in the point k, and let the angle CTk be a right angle. Fig.

Prop. 17. *If* AR *be parallel to* BP *rightly applied to the diameter* AB, *I ſay, it toucheth the ſection in* A. For AR is perpendicular to the diameter of the circle, which is equidiſtant to the baſe cutting the Cone in A.

Prop. 18. *If the line* NT *touch the ſection of the parabola, then, I ſay,* AN *ſhall be equal to* AS. 4, 5.
For if S be the centre of the circle GTg, the right lines VN and Sg ſhall be equal and parallel to GS by the 16. Ergo $Sg.VN :: AS.AN$. But if C be the centre, then make the angle CTk a right angle, then it will be $Sk \times SC = STq$, becauſe CTk and TSk are right angles, and $Sk \times Sc = STq = GS \times Sg$. Wherefore

$$\left.\begin{array}{c} Sk.GS \\ :: \\ SG.SC \end{array}\right\} \& \left\{\begin{array}{l} Sk+GS \\ \\ Sk-GS \end{array}\right\} \begin{array}{l} = Gk.GS::(Sg+SC)=Cg.SC \\ \qquad :: \qquad\qquad\qquad = \;\; = \\ = gk.\ sg::(Sg-SC)=GC.SC \end{array}$$

And therefore it is $Vk.aS :: Gk.GS :: gk.Sg :: Vk.Sn.$
Therefore $Sn = aS = VN$ and $Sn.VN :: AS.AN$.
Therefore $Sn = VN$. $AS = AN$.

Prop. 19. *In the hyperbole, Ellipſis and the circle, if* NT *touch the ſection, I ſay, it will be* AN. 6, 7.

Fig. EN : : AS.ES. For if *S* be the centre it will be as followeth, *AS. ES* : : *SQ.* (*GS* =) *Sg* : : *NX. VN* : : *AN.EN.* because the triangles *EGS* and *AQS* are alike, and the triangles *ASG*, *VAN* are alike, and the triangles *NXA* and *VEN* are alike. But if *C* be the centre, the proof will be after the same manner as in the 18, and *aS* = *vS*, and it will be *AS. aS* : : *SQ. aS* : : *NX. VN* : : *AN EN*, because the triangles *ASQ ESa ANX*, *ENV* are similar.

But in the circle, because *NS×SC*=*ST(q)*= *AS×ES*, it will be *NS. ES* : : *AS. SC*. Therefore compounded (*NS+ES*=) *EN.ES* : : (*AS* +*SC*=) *AC. SC.* and again, (*NS-AS*=) *AN. AS* : : (*ES-SC*=) *AC.SC*. Therefore it is *EN. ES* : : (*AC. SC*) : : *AN.AS.* and *AN.EN* : : *AS. ES.*

4. 5. Prop. 20. *In the parabola if* AN=AS, *I say,* NT *will touch the section.* For if *S* be the centre, the right lines *VN* and *Sg* are parallel and equal, and to them *Tk* is parallel being perpendicular to *ST*, doth touch the circle *GTg* in *T*; and therefore by the 16 the plane *VTk* toucheth the Cone in the line *VT*. But if *C* be the centre, because *AN. AS* : : *VN. Sn* it will be *Sn* =*VN*=*vS*. But it is *Vk. aS* : : *Gk. GS* : : *gk. Sg* by the 18. Therefore

Comp. (*Gk+gk*=) 2*Ck.Gk* : : (*GS+Sg*=) 2*Cg.GS.*
Divid. (*Gk.Ck*—) *Cg.Gk* : : (*GS.CG*=) *SC.GS.*
and (*Gk.GS*=) *Sk.GS* : : (*Cg. SC*=) *Sg. SC.*

Wherefore

Wherefore $Sk \times SC = GS \times Sg = STq$: and therefore the angle CTk is a right angle, and the right line Tk doth touch the circle GTg in T. Therefore by the 16. the plane VTk toucheth the ſection in the point T, which was to be demonſtrated. Fig.

Prop. 21. *In the parabola*, I ſay, $At(q) = \frac{1}{4} AR \times AN$ for $AN = \frac{1}{2} NS$ and $At = \frac{1}{2} ST$, but by the 10. $4A(q)\,(= STq) = AR \times (AS =)\, AN$, Ergo $STq = AR \times AN$.

Prop. 22. *In the Hyperbola, Ellipſis and circle, if it be* AS.ES :: AN.EN. *I ſay* NT *ſhall touch the ſection.* For if S be the ſection the angle STk will be a right angle, wherefore Tk doth touch the circle GTg, therefore by the 16. NT will touch the ſection in T. But if C be the center, becauſe it is $SQ.aS :: AS.ES :: AN.EN :: NX.NV :: SQ.Sn$. It will be $Sn = aS$. Then the angle CTk will be proved a right angle, as in the 20, therefore the right line Tk will touch the circle, as in the 16. But in the circle becauſe it is $EN.AN :: ES.AS$. It will be 6, 7. 8, 9.

$(EN + AN = 2CN.EN :: (ES + AS =)\, 2AC.ES$ 10.
$(EN - CN =) = AC.EN :: (ES - AC =) = CS.ES$
$(AC - CS) = AS.CS :: (EN\ ES) = NS.ES$.
wherefore $NS \times CS = S \times ES = STq$, Ergo NT toucheth the circle.

Prop. 23. *In the Hyperbola, Ellipſis and circle, if* NT *touch the circle, I ſay it will be* IA(q) =

Fig. 10. $IS \times IN$. for because it is $ES.AS::EN.AN$. by the 19 Prop. It will be

$(ES \pm AS =) 2IS.AS :: (EN \pm AN =) 2IA.AN.$
$(IS \mp AS =) IA.IS :: (IA \mp AN =) IN.IA.$
Therefore $IA.IS :: IN.IA$ and $IAq = IN \times IS$.

6, 7. Prop. 24. *I say likewise* $NS \times IN = EN \times AN$. For because by the 23 Prop. $IS.IA :: IA.IN$. therefore it will be

$(IS + IA =) ES.IA :: (IA + IN =) EN.IN.$
$(ES - EN =) NS.EN :: (IA - IN) AN.IN.$
Therefore $NS.EN :: AN.IN$ and $NS \times IN = AN \times EN$.

Prop. 25. *I say thirdly* $AS \times ES = IS \times NS$. For by the 23. $IS.IA :: IA.IN$. it will be

$(IS - IA =) AS.IS :: (IA - IN =) AN.IA.$
$(IS + IA =) ES.IS :: (AS + AN =) NS.AS.$
Ergo $ES.IS :: NS.AS$. and $AS \times ES = NS \times IS$.

A Confect following from this and 13 $\left\} \begin{array}{l} \dfrac{IS \times NS.}{AS \times ES.} \quad \dfrac{STq}{AS \times Sr} :: \begin{array}{l} AE.AR. \\ ES.Sr. \end{array} \end{array} \right.$

12, 13 Prop. 26. *In the Hyperbole, Ellepsis and circle if the tangent* NT *be drawn, and likewise the Ordinate* Ih *and* EX, *and the parameter* AR *and* At, hT *then I say* $\frac{AE \times AR}{4} = At \times EX$. For by the 24 Prop. $NS.IN = AN \times EN$ and because $\left\{ \begin{array}{c} NS.IN :: NS.IN \\ \times \quad \times \quad\quad \times \quad \times \\ NS.NS :: IS.IS \end{array} \right\}$ Therefore NSq.

$NS \times$

$NS*IN :: NS*IS . IN . IS$. but $IN*IS = IAq$ Fig.
per 23. from hence it is $NS . IS . IA(q) :: NS(q) . AN*EN$ because $NS*IN = AN*EN$.

and it is $\left\{\begin{matrix} NS . AN :: ST . At \\ * \quad * \quad * \quad * \\ NS . EN :: ST . Ex \end{matrix}\right\}$ because the Triangles NTS, NtA and NTS and EAX are similar, and it being $NS*IS . IA(q) :: NS(q) . AN*EN :: STq . At*Ex$ alternly, it is $NS*IS . STq :: EAq . At*EX$. and by the Consectary to 25. it is $NS*IS . STq :: AE . AR :: (IAq =) \frac{AE(q)}{4}$.

$(At*EX =) \frac{AE*AR}{4}$ Ergo $At*EX = \frac{AE*AR}{4}$.

Prop. 27, *In the Hyperbole, Ellepsis and circle, I say* $ST*Ih = \frac{AE*AR}{4} = At*EX$. For because 12, 13.
the Triangles EXN *and* NTS are alike, $ST . EX :: NS . EN$. and *per* 24. $AN . IN :: At . Ih$. Ergo $ST . EX :: At . Ih$. Ergo $ST*Ih = EX*At = \frac{AE*AR}{4}$ by the last.

A general Consectary arising from the 23, 24, 25, 26, 27 Prop. If in the Hyperbole, Ellipsis and Circle it be either $IA(q) = IS*IN$ or $Ns*IN = AN*EN$ or $AS*ES = IS*NS$ or $AE . AR :: IS*NS . ST(q) :: IA(q) = IS*IN . At*EX$. or if it be $At*EX = \frac{AE*AR}{4} = ST*Ih$. the right line NT shall touch the section.

Prop. 28.

Fig. 14 Prop. 28. *In the Parabola, if* LT *be parallel to* AB, *I say* LT *shall bisect* Pp *and all other lines parallel to* NT *drawn within the section.* For [] *Sr*. [] *Br* and [] *br* : : *AS*. *AB* and *Ab*. and by the 7. *ST* (*q*). *BP* (*q*) and *bp* (*q*) : : △*NST*, △*kBP* and △*kbp*. but by the 18 [] *Sr*=△*NST* therefore [] *Br* = △*kBP* and [] *br* = △*kbp*. Wherefore the [] *Br*-[] *br*=△*kBP*-△*kbp* and the Remainder [] *Bg*=[] *BPpb*. Wherefore also △*PmL*=△*pmg* are alike, Ergo *Pm*=*mp*.

Corol. Every right line *TL* parallel to *AB* is a diameter.

15, 16. Prop. 29. *In the hyperbole, Ellipsis and circle. If from the centre* I *the right line* TI *be drawn at a sufficient length, it will bisect* Pp *and all other parallels to it drawn within the section.* For the △*IST*, △*INT* . : *IS*.*IN* : : *IS*(*q*). (*IS*×*IN* per 23) *IA*(*q*) : : △*IST*. △*IST* : : *IA*(*q*). △*IAh* : : *IB* (*q*). △*IBL* : : *Ib*(*q*) △*Ibg*. Wherefore and by the (6 e 2) it will be

IS(*q*). △*IST* : : (*IS*(*q*)-*IA*(*q*)). *AS*×*ES*. △(*IST*-△*IAr* [] *Sr*.

IB(*q*). △*IBL* : : (*IB*(*q*)-*IA*(*q*)). *AB*×*EB*. △(*IBL*-△*IAK*) [] *Br*.

Ib(*q*). △*IBg* : : (*Ib*(*q*)-*IA*(*q*)). *Ab*×*Eb*. △(*Ibg*-△*IAr*) [] *br*.

Wherefore *AS*×*ES*. *AB*×*EB*. *Ab*×*Eb* : : [] *Sr*. [] *Br*.*br*. But by the 8. *ST*(*q*).*BL*(*q*). *bg*(*q*) : : △*NST*.△*kBP*.△*kbp*. being similar and alike. But [] *Sr*=△*NST*. wherefore [] *Br*=△*kBP* and

[] *br*

[] $br = \triangle kbp$. and therefore ([] Br-br =) $BbgL$ Fig.
=($\triangle kBP$-$\triangle kbp$) $BbpP$. and taking away from both sides $BbpmL$, there remains the two similar Triangles $PmL = pmg$, Ergo $pm = Pm$.

But in the Ellipsis if B be further than the 17.
centre, the Triangle IBL will fall below the diameter; wherefore drawing bt parallel and equal to BL, and terminating at IT above the centre, let $\triangle Ibt = IBL$, it will be proved as before [] $br = \triangle kBP$, and adding to either side IBL it will be $kILP$ =($\triangle IAr$) $Ikph$. Take away from either part the triangle kIm, and there will remain the two similar and alike Triangles $PmL = pmh$, Ergo $Pm = pm$.

Corol. Every right line TI drawn from the centre is a diameter.

Prop. 30, 31. These Propositions are in a scalenous Cone, whose Triangle made by the Axis is inclined to the base, and all things are demonstrated as in the 7 and 10 Prop. of a right Cone.

Prop. 32, 33. These two Propositions are likewise found upon a scalenous Cone, whose Triangle made by the Axis is inclined to the base, and all things are demonstrated as in the 8 and 13 Prop. before, of a right Cone.

Prop. 34. *If the transverse diameter of an Ellipsis be* AE *and the parameter* AR, *and if unto the parameter* AR *by the centre* I *there be drawn a pa-* 18.
rallel

Fig. 18. *parallel* OV, *and unto the transverse* AE *another parallel* OS, *so that it may be* OI×IV. IA(q) : : OV. Os. I say,

1. That *OV* is also a transverse Diameter: For by the 29 Prop. it will be bisected by the diameter *AE*, and will also pass by the centre, therefore a transverse diameter.

2. That *Os* is a congruous Parameter to the transverse *OV*, by the first Corol. to the 13 Proposition.

3. That *AR*, *OV*, *AE*, *Os* are continual proportionals: For by the 13 Prop. *AR*. *AE* : : *OI(q)* (*AE*×*EI*=) *AI(q)* : : *OV(q)*. *AE(q)* wherefore *OV*, *AE*, *Os* are continual proportionals.

4. That the species of both *EAR* and *VOS* are alike, for the sides *OS*, *AE*, the sides *OV*, *AR*, and the sides *SV*, *RE* are proportional.

Prop. 36. *In the hyperbole: if there be two planes, one passing by the vertex of the cone* V *parallel to the section made upon the cone*, and cutting the Cone in the right line *VG*, and the base of the cone in the right line *GQG*, and the other plane touching the cone *VGm* in the right line *VG*; I say that the plane which toucheth in the line *VG* if produced shall intersect the diameter of the hyperbole *AE* in the centre *I*.

19. For first, if *mG* (the Intersection of the tangent plane with the base of the cone) be parallel to the diameter of the base *Db*, (that is, when *Q* is the centre of the base of the cone) the plane *mVI* (the intersection of the plane of the Tangent

Tangent, and of the Triangle by the vertex of the cone) shall be parallel to *mG*, and therefore it will be *DQ.QV*::*VI.IE*. And as (*Qb*)= *QD.QV*::*VI.IA*. Ergo the tangent plane passeth by *I* which is the centre, because *IA*=*IE*, Fig. 19.

But if the tangent plane *mGV* do not meet with the diameter of the base *Db* in the point *m* (that is, when the point *Q* is not the centre) let there be drawn by *Q* within the Triangle by the axis the right line *HQh* parallel to *mVI*, and cutting *VO* in *h*, and *Vb* in *r*. It shall be *Qr.mV*::*bQ.bm* because the Triangle *Vmb* is cut proportionally by *Qr*, and by the 19 Prop. it was, as *DQ.Dm*::*hQ.mV*. wherefore *hQ*=*Qr*, but it is *hQ.QV*::*VI.IE* because of the parallels within the Triangle *EHr*: and lastly, (*Qr*=) *hQ.QV*::*VI.IA* by the same reason aforesaid; but *VI* being proved to bear the same proportion unto *IE* and *IA*, therefore *IE*=*IA*, and *Ergo* the plane of the Tangent passeth by the centre *I* according to the Proposition. 20.

Prop. 37. *In the hyperbole: if* mG *do meet with the Ordinate* pBP *produced in the point* F, *and if from the centre* I *there be drawn the right line* IF; *I say that that line* IF *is an Asymptotos*, or such a line that being drawn how long soever, or how far soever, shall never meet with the section of the Hyperbole, drawn likewise how far or how long soever. For the plain *VGF*, in which always the right line *IF* is placed, cannot touch the cone in any place out of the line *VG*; but the Hyperbole being upon the cone, and being parallel by its diameter. 20.

Fig. diameter to VG, therefore that plane, or the line IF upon that plane, can never touch the Hyperbole. And after the same manner, if on the other part mg do concur with PBp in the point f, then If shall be the line called the *Asymptotos*.

20. Prop. 38. *If* m *be the intersection of the Asymptotos, with the Parameter, I say then* $Am(q) = \frac{AE*AR}{4}$ For by the 14 Prop. and because the triangles VQG, IBF, IAm are alike, it shall be $\frac{AE(q)}{4}. \frac{AE*AR}{4} :: AE. AR :: VQ(q). (DQ*Qb) = QG(q) :: IB(q). BF(q) :: IA(q). Am(q) :: \frac{AE(q)}{4}. Am(q)$. Ergo $Am(q) = \frac{AE*AR}{4}$

Corol. $IA(q) = \frac{AEq}{4}. Am(q) :: AE. AR :: EB. Br :: IB(q). BF(q)$.

20. Prop. 39. *I say* $\frac{AE*AR}{4} = Pf*Pf$. For by the 38 and 13 Prop. and by the (5 and 6 e 2) $AI(q). (Am(q) =) \frac{AE*AR}{4} :: IB(q). BF(q) :: AE. AR :: AB*EB. BP(q) :: (IB(q)-AI(q)) AI(q). (BF(q)-BP(q)) PF*pf$. Ergo $\frac{AE*AR}{4} = PF*pf$.

Corol. $PF = pf$. For the same proof is of f as of F.

Prop.

Prop. 40. *The Hyperbole and Asymptotos shall ever come nearer the longer they are produced, and shall come to a distance less than any distance given.* For if in the Hyperbole *IF* be an Asymptotos, *PF* will be greater than *pf*. For because *Eb* is greater than *EB*, and by the 8 Chap. $Eb \times bA$. $EB \times BA$: : $bp(q)$. $BP(q)$. but the rectangle $Eb \times bA$ is greater than $EB \times BA$ therefore *bp* is greater than *BP*, and *bf* is greater than *BF*, because *Ib* is greater than *IB*. But by the 39 Prop. concerning the Asymptotos it is $dF \times PF = \frac{AE \times Ar}{4} = Kf \times pf$. and as *Kf*. *dF* : : *PF*. *pf*. but *Kf* is greater than *dF*, Ergo *PF* is greater than *pf*, and therefore the Asymptotos is nearer to the Hyperbole in *f* than in *F*. Fig. 20.

Prop. 41. *Every right line drawn from the centre* I, *and within the Asymptotos is a diameter.* For it will cut the Hyperbole, and by the Corol. of the 19 Prop. is a Diameter.

Prop. 43. *Opposite sections of the Hyperbole have the same Parameter.* For because it is *BD*. *EB* : : *bd*. *Eb* and as *BN*. *AB* : : *bn*. *Ab*. let both the proportionals be multiplied it will be $DB \times BN (= BP(q))$. $BE \times BA$: : $bd \times bn (= bp(q))$. $Eb \times Ab$. and $BPq = BD \times BN$ and $bd \times b = bp(q)$ because *PB* is parallel to *bp*, therefore by this and by the 13 Prop. it will be BPq. $AB \times EB$: : $bp(q)$. $Ab \times Eb$: : *AE*. *AR*. Ergo *AR* is the Parameter to both the sections. Fig. 3. 4.

Prop.

Fig. 21, 22. Prop. 44. *In opposite Hyperboles as in the Ellipsis, if* NT *and* nD *concurr in* K; *and to the point in the middle of the right line* TD *which shall be parallel to* AE, *there be drawn* KM *cutting the diameter* AE *in* I: *I say, that* I *shall be the centre, and* IM *the conjugate Diameter to* AE. For by the 43 Prop. the lines *ST* and *ZD* are parallel and equal, and *AS* equal to *EZ*, and because by the 19 Prop. *EN. AN* : : *ES. AS* : : *AZ. EZ* : : *An. En.* it will therefore follow $(EN \mp AN)$ $AE.AN$: : $(An \mp En)$ $AE.En.$ wherefore $AN = EN$. And because it is $KM.\frac{1}{2}TD$: : $KI.IN$ and $KM.\frac{1}{2}TD$: : $KI.In.$ it will be $IN = In.$ Wherefore $IN \mp AN = In \mp En.$ that is $IA = IE.$ Ergo *I* is the centre.

14. Prop. 45. *In the Hyperbole, if* IM *be parallel to* AR *a tangent, then shall* IM *be a Conjugate diameter to* AE. For unto the diameter *AE* if *DT* be drawn parallel, it will make the Parallelogram *STDZ*, and *EZ* is equal to *AS*.

11. Prop. 47. *In the parabola, if the segment of the axis* $Av = \frac{AR}{4}$ *or if* $Av = \frac{1}{4}$ *of* AR *(for* v *is the node of the parabola according to the definition) it shall be* Nv = Tv. For let $Nu = Sv$ then will $Av = Au$. and because *ST* is rightly applied, it is $Tv(q) = STq + Sv(q)$. But because by the 18. $NA = AS$ and by the 21. it is $STq = AR \times AN = SA \times 4Av$ and $Tv(q) = (STq =)$ $SA \times Kv + Sv(q) = Su(q)$ by the 8 e 2. for $QZ + E = ZE + E(q)$. But $Su(q) = Nv(q)$ because $NA +$

$Av =$

$Av = SA + Au$. Ergo $Tv(q) = Nv(q)$ and $Nv = Tv$. Fig. 11.

Corol. 1. The angle $NTv =$ to the angle TNv.

Corol. 2. And if TF be parallel to the Axis AS, then the angles nTF and nTv are likewise equal.

Prop. 48. *But if* S *fall in* v, *it will be* ST $=$ 2AS. For $2AS = Nv$ by the 18 Prop. therefore $ST = Nv = 2AS$.

Prop. 49. *In the Hyperbole and Ellipsis, if* $\dfrac{AE \times AR}{4} = AI \times An \left\{ \begin{array}{l} + Au(q) \\ - Au(q) \end{array} \right.$ and therefore in the Hyperbole it will be $\sqrt{(q)} \dfrac{AEq + AE \times AR}{4} :- \dfrac{AE}{2} = Av$. But in the Ellipsis it will be $\dfrac{AE}{2} - \sqrt{\dfrac{AEq + AE \times AR}{4}} = Au$. 24, 25.

And let $Ed = Au$. I say that the angle $uTt =$ to the angle oTS. For because by the 26 Prop. and by the definition of the Nodes it will be $At \times ES = \dfrac{AE \times AR}{4} = AE \times Au \pm Au(q)$. And $ES.Eu :: Au.At$. and $ES.Eo :: Ao.At$. wherefore the right angled Triangles ESu, Aut are similar, and the right angled Triangles ESo, Aot are similar, and in them the angles $EuS = Atu$

 and

Fig. and the angle $ESo = Aot$, wherefore the angles
24. tuS and toS are right angles. Then upon the
25. line tT draw the perpendicular $T5$, meeting with ot in 5. Therefore the right angled Triangles tSo, tuA, $t5T$ are similar, and therefore $At.tu :: ot.tS :: Tt.t5$. and Alterned $At.Tt :: tu.t5$. and also the Triangles $S5o$, $t5u$ are similar, and therefore $tu.t5 :: So.S5$. wherefore $T5$, $t5$, $S5$ cut one another mutually in 5. Therefore by the 33 *e* 3 a circle upon the diameter $t5$ shall cut the points u, T: and a circle upon the diameter tS shall cut the points u and o. Lastly, by the 21 *e* 3 the angles $uTt = u5T = oTS$. Ergo $uTt = oTS$.

26, 27. Prop. 50. *In the Hyperbole and Ellipsis: if IH be drawn parallel to* uT *or* oT, *then I say* IH=IA. For drawing od parallel to uT, by the 49 Prop. the angles $otT = uTS = odT$, and therefore $od = oT$ and $HT = Hd$. wherefore the angles oHT and oHd are right angles, and by the 31 *e* 3, a circle upon the diameter oS will cut the points EH. And by the 21 and 32 *e* 3, the angle $EHo = ESo = Aot = uSt$: and by the demonstration in the 49 Prop. and because the circle upon the diameter ot, by the 31 *e* 3 doth cut the points AH, the angles AoT and AHt will be equal, wherefore also the angles AHE and oHt are equal and right angles. The circle therefore upon the diameter AE will cut the point H. Ergo $IH = IA$.

28, Prop. 51. *In the Hyperbole: I say it is* AE=
29 oT-uT, *but in the Ellipsis it is* AE=oT+uT.

For

For let HID be drawn parallel unto uT, and intercepted betwixt TN and To: by the demonstration in the 50 Prop. $HD = DT = Do$. and $ID = \frac{1}{2}uT$. and $IH = IA$. Wherefore in the Hyperbole $oT = 2ID + 2IH$. But in the Ellipsis $oT = 2IH - 2ID$. But $2ID = uT$. Ergo in the Hyperbole AE ($= 2IH$) $= oT$ ($= 2ID + 2IH$) $- uT$ ($= 2ID$). But in the Ellipsis $AE = oT + uT$. Fig. 28, 29.

Prop. 52. *To find the diameter of a section.* Let there be drawn any two streight lines however within the section, so that they be parallel, by the points in the middle of these lines the diameter of the section shall pass. 34.

Prop. 53. *To find the centre of a section.* The centre is the concourse of two diameters. But, note, that the Parabola hath no centre.

Prop. 54. *To find the axis of a section.* If it be of the Parabola, cross the diameter found with a perpendicular; now if the said diameter be in the middle of the perpendicular it is the axis sought; but if it be not, raise a perpendicular in the midst and it shall be the axis.

If it be the Hyperbole or Ellipsis: From the centre found describe an Arch which shall cut the section in two places or points. And by the point of the very middle of the Arch, betwixt the former points, draw a line from the centre, that shall be the Axis.

Fig. 34 Prop. 55. *From a point not within the section to draw a line that shall touch the section.* If it be in the Parabola, and if the point be in the section, describe the axis of the Parabola, and to it from the point given draw a perpendicular: then according to the 18 Prop. make $AN=AS$ a line drawn from N to the point given is a Tangent. But if the point from whence a Tangent is required to be drawn to the Parabola be without the Section, then describe an axis, and from the point given draw a parallel to the axis cutting the section, from the point of the intersection draw a Tangent, and taking with the Compasses the distance from the point given to the point of intersection, set that distance off from the point of intersection within the section; and by the end of that measure draw a parallel to the Tangent, which will cut the section in a point, to which if a line be drawn from the point given it shall be the Tangent required.

In the Hyperbole and Ellipsis if the point be in the section: draw however a right line within the section from the point given; and by the middle of it from the centre a semi-diameter: which produce beyond the section, until that the rectangle, made of the segments of the semi-diameter intercepted betwixt the centre and the line drawn within the section, and the line produced from the centre, be equal to the square of the semi-diameter, according to the 23 Prop. *&c.*

In the Ellipsis if the point be without the section, from the centre to the point given let a semi-

ſemi-diameter be continued cutting the ſection, Fig.
and having found the axis, let a right line be 34.
deſcribed touching the ſection in that very point of interſection. Then taking a point in the ſemi-diameter within the ſection, ſo that the rectangle under the ſegments of the ſemi-diameter from the centre to both the points, as well that which was given as that which was taken, may be equal to the ſquare of the ſemi-diameter: By that point thus taken let a right line be drawn parallel to the former Tangent: That ſhall cut the ſection in a point, to which if a right line be drawn from the point given, it ſhall touch the ſection.

In the Hyperbole if the point given from whence it is required to draw a Tangent, be within the Aſymptotos, then the ſame work being obſerved as in the Ellipſis, the Tangent may be drawn as before. If the point be in one of the Aſymptotoi, betwixt the point given and the centre, let a middle point be taken, from whence a parallel being drawn to the other Aſymptotos, it ſhall cut the ſection in the point of Contact; wherefore drawing a right line from the point given by the point of Contact to the other Aſymptotos, it ſhall touch the ſection and be biſected by the point of Contact.

If the point be *K* without the Aſymptotos. 30.
Let there be drawn by the centre *KI*, and let there be inſcribed any where within the ſection a parallel, to whoſe middle point from the centre let there be a diameter drawn cutting the ſection in the point *A*; let their be drawn to

Fig. 30. the Asymptotos Am parallel to KI, and let KI be extended to M, so that $KI \times IM = Am(q)$: Then drawing MT parallel to the diameter, it shall cut the section in the point of Contact T. For by the 27 Prop. it being $(SI =) IM \times KI = \frac{AE \times AR}{4} = Am(q)$ by the 38.

Corol. From any point of a section to any diameter a right line may be ordinately applied; if to the Tangent found at the head or vertex of the section a parallel be drawn to the point given.

Prop. 56. *To find the right diameter of a section, which shall agree to a transverse given is thus done:* By the Corol. to the 55, let the right line BP be ordinately drawn to the diameter wheresoever, and let it be $AB \times Br = BP(q)$ &c. according to the 10 and 13 Prop. of this Book. BP if produced will be the congruous diameter.

28. Prop. 57. *To find the Asymptotoi of an Hyperbole.* Let $Am(q) = \frac{AE \times AR}{4}$ by the 48 Prop.

Or taking the point C the middle of NS, let Ct be ordinately applied; and let Nt be drawn, and Im the Asymptotos parallel to it. For by the 6 e 2, $EC \times AC + IA(q) = IC(q) = (IS \times IN$ per 23$)$ $IA(q) + NC(q)$. And by the 8. $(EC \times AC)$ $NC(q).Ct(q) :: AE.AR :: IA(q).Am(q)$ per 38. wherefore $NC.Ct :: IA.Am$. Ergo Im is the Asymptotos.

Prop.

Prop. 58. *To find out the Nodes or Navels of a section, viz. u and o, or u in the Parabola alone.* Fig. 31.

In the Parabola let *Au* be made equal to the fourth part of *AR*. Otherwise let it be as $AB . \frac{1}{4} BP :: \frac{1}{4} BP . Au$.

In the Hyperbole let it be $Am(q) = \frac{AE \times AR}{4}$. 28.

Then upon the centre *I* and semi-diameter *Im* let the semi-circle *Omu* be described; therefore $Au = Eo$, and therefore $AE + Au \times Au = \frac{AE \times AR}{4}$ $= AE + Eo \times Eo$. and therefore by the definitions of the Nodes of an Hyperbole, there being applied to *AE* a rectangle equal to $\frac{AE \times AR}{4}$ and exceeding by the square of *Au* or *Eo*, for $Eu = AE + Au$, *u* is the Node within, and *o* the Node without.

In the Ellipsis upon *O* the term of the lesser axis, with the semi-diameter *IA*, let there be an Arch made that shall cut the greater axis in the points *u* and *o*, I say that those two points *u* and *o* are the Nodes of the Ellipsis: For by the 13 Prop. $IO(q) . (IA \times IE =) IA(q) :: AR . AE :: \frac{AE \times AR.}{4} \frac{AEq.}{4}$ wherefore $IO(q) = \frac{AE \times AR}{4}$. Also by the (5 e 2) $Eu \times Au + Iu(q) = IA(q) = Ou(q) = IO(q) + Iu(q)$. wherefore $Eu \times Au = IO(q) = \frac{AE \times AR.}{4}$ Therefore unto 29.

Fig. 29. *AE* is applied a Rectangle equal to $\frac{AE \times AR}{4}$ wanting the ſquare made of *Au* or *Eo*, for *Eu*= *AE*-*Au*. Therefore by the definition of the Nodes or Navels of an Ellipſis, *u* is the one and *o* the other.

31. Prop. 59. *To find the parameter of any ſection made upon the cone.* In the Parabola by the points *ADb* let a circle be deſcribed cutting the diameter produced in *R*, I ſay that *BR* is the parameter, for it is $BP(q) = DB \times Bb = AB \times BR$. Ergo *BR* the parameter.

31, 32. In the Hyperbole and Ellipſis let *EK* be drawn parallel to the ſide of the cone, *vb*, and *kAk* parallel to the baſe of the cone, cutting the ſide *VD* in *k* and the right line *EK* in *K*. By the three points *EKk* let a circle be deſcribed cutting *AE* in *R*. I ſay that *AR* is the Parameter; for if by *R* a circle be deſcribed parallel to the baſe of the cone, cutting the Triangle by the axis in *DRb*, and the ſection it ſelf in the right line *RP*. Then will *RP* be ordinately applied, and $RP(q) = DR \times Rb$; and by the 13 Prop.

$$\text{It is } AE.AR :: \left\{ \begin{array}{c} ER.DR :: AE.AK \\ \times \quad \times \\ AR.Rb :: AE.AK \end{array} \right\} = AE \times AR.$$

And by the 13. $AE.AR :: AR \times AR.RPq$. Therefore the *Parameter* is *AR*.

The End of the firſt Book.

LIB. II.

Fig.

Lib. II.

The second Book of Conical Sections: Wherein is taught the several ways of the describing the Conical Sections in Plano *by Points.*

PRop. 1, 4. *How to describe the Parabola by points about any diameter* AS, *and the Ordinate* TSt, *the said diameter and Ordinate being given by position.* 1.
Draw *AT* and taking the point *B* any where, betwixt *S* and *A* draw *BP* parallel to *ST*, cutting *AT* in *p*: and let it be *BPq*=*ST* in *Bp*. I say that the point *P* is in the Parabola; for because the Triangle *AST* is cut proportionally by *Bp* it is *AS.AB* : : *ST.Bp* : : *ST(q)*. (*ST* * *Bp*=) *BP(q)*. But by the 7. of the 1. it is *AS.AB* : : *STq.BPq*. Ergo the point *P* is in the Parabola. And after this manner taking as many points *B* as is requisite, and drawing the right lines *BP* the Parabola may be described.

Prop. 2, 5. *Having by position* AB *the curtated part of the diameter and the Ordinate* BP, *together with the species of the figure (which is the proportion* 2.

Fig. *proportion of* AE *to* AR) *to draw the hyperbole in*
2. *the plane by points given.* Let it be $AB \times Bn =$ BPq. and after placing Bn upon BP, let it be found $BE.Bn$ in the proportion as $AE.AR$ given. I say that the point P is in the hyperbole, whose transverse diameter is AE found, and congruous parameter AR. For it is $Bn(q).(AB \times Bn =) BP(q) :: Bn. AB :: Bn \times EB. AB \times EB.$ wherefore $BP(q). AB \times EB :: Bn. EB :: AR. AE.$ therefore by the 13 of the 1. the point P is in the Hyperbole: whose diameter is found E, and parameter AR. And if after this manner there be divers points B taken, and drawing Bn, meeting with ER in n, and making it $BPq = AB \times Bn$, the section may be described.

4. Prop. 3, 6. *Having any diameter* AE *of an Ellipsis, and the Ordinate* BP *given by position, and the species of the figure (which is the proportion of* AE *to* AR) *to describe the Ellipsis by points.* Make $AB \times Br = BP(q)$ and placing Br upon BP joyn Er: Or finding AR in the given proportion, let BP be set parallel to it, and draw a right line by E and R, let it meet with BP in r. I say that the point P is in the Ellipsis, whose transverse diameter is AE and parameter AR. for whereas it is $Br(q). (AB \times Br =) BP(q) :: Br. AB :: BR \times EB :: AB \times EB.$ wherefore $BP(q). AB \times EB :: Br. EB :: AR. AE.$ Ergo by the 13 of the 1. the point P is in the Ellipsis.

3. Prop. 7, 8, 9, 10. *Having the curtated part of the diameter* AS *of a section, and the Ordinate*

dinate TSt *given by position, together with the* Fig.
species of the figure (AE. AR) *to describe the se-* 3.
ction. The solution of this Problem depends upon the 7 and 8 Prop. of the first Book, by finding the triangle by the axis of the cone in which the section is: Therefore let it be *AS*× *Sr*=*STq*: and measuring from *S* on both sides *Sg*=*S*, and *ST*=*AS*, draw an infinite line *gA*.

Then for the parabola draw *GV* parallel to the diameter of the section or segment *AS* meeting with *gA* in *V*: But for the Hyperbole and Ellipsis, having found *SE* to *Sr* in the proportion of *AE* to *AR* given, draw the right line *GE* cutting *gA* in *V*. Then will *V* be the vertex of the cone, and *GVg* the triangle by the axis, and *AR* the parameter in the Hyperbole and Ellipsis: But in the Parabola the parameter is *sr*, having these the sections are drawn according to the 7 and 8 Prop. Lib. 1. and as in the last two.

Prop. 11, 12, 13, 14, 15, 16. *About any tri-* 1,
angle given to describe a conical section, whose spe- 2,
cies is given, that is, having ATt *and* AE. AR. 4.
Dividing the base of the triangle *Tt* into two parts in *S*, joyn *AS*, and make *AS*×*Sr*=*STq*. Draw the right line *rE* in the parabola parallel to *AS*, in the rest cutting it, in the hyperbole above *A*, but in the Ellipsis under *S*. The point of the intersection E is gotten by making *SE* to *Sr* in the proportion as *AE* to *AR* given. And so shall *AR* be the parameter in the Hyperbole and Ellipsis: But in the Parabola *Sr* will be

Fig. be the parameter; then taking any point B in
1, AS, draw the right line BP parallel to the base
2, Tt, and cutting AT in o, and At in v, and make
4. $BPq = Bo \times Bv$. I say the points $TPAt$ are in the same section: and the demonstration hereof depends upon the 7 and 8 Prop. of the first Book: For producing TA till it meet with Er in V, there is a triangle formed TVt, which is the triangle cut by the axis of the cone, and taking many B if it be $Bo \times Bu = BPq$. as many P as are desired may be found.

1. Prop. 17, 19. *Having by position the vertex of the Parabola* A, *and the Node* Au, *to draw the parabola by points.* Make $Av = Au$, and by A and v raise two perpendiculars infinitely to part Au, which let be vw and At. Then taking any point in the perpendicular from v, suppose w, joyn uw by a streight line, which shall cut the perpendicular by A in t; then drawing two streight lines, one from t perpendicular to uw, the other from w parallel to vu, meeting in the point T. I say that the point T is in the Parabola, and taking many points in vw, extended, if need be, find as many T as are needful to describe the Parabola by points. That the point T is in the Parabola is thus proved: For drawing TS perpendicular to the Axis, and lengthning Tt till it meet with the Axis in N, it will be $At = \frac{1}{2} ST$. for $At = \frac{1}{2} vw = ST$. and $AS = AN$. But in the right angled triangle NTS. $At(q) = (AN)\ AS \times Sv$. and therefore $(4At(q))$ $STq = AS \times AR$. Therefore by the 10 of the 1. T is in the Parabola. Prop.

Prop. 18, 20, 21. *Having given the axis and* Fig.
Nodes of the Hyperbole or Ellipsis by position, viz. 2,
in the Hyperbole uAEo *and in the Ellipsis* AuoE *to* 5.
describe the section by points. Upon the centre
o and the semi-diameter *AE*, let an Arch be described great enough, cutting the Axis in *y*; and in the Arch so described take any point *W*, and joyn *Wo* and *Wu*; in the right line *Wu* in the very middle at *D*, raise a perpendicular cutting *Wo* in *T*. I say the point *T* is in the Ellipsis and Hyperbole. For because *uT* is equal to *WT*, in the Hyperbole it will be $OT - uT = AE$. and in the Ellipsis it will be $oT + uT = AE$. Wherefore by the 51 of the 1. the point *T* is in the section.

Prop. 22, 25. *Having the vertex of a para-* 1.
bola and the node Au *given to describe a parabola by points very readily.* Producing out *Au* make $Av = Au$, and taking the point *S* any where in the axis, and from the point *S* drawing a perpendicular line to the axis, measure with the Compasses $uT = Sv$. I say the point *T* is in the Parabola. For by the 17. $uT = WT = Sv$. Otherwise $uT = Sv$. So by the 8 *e* 2 $Sv(q) - Su(q) = AS \times 4Au$: But by the 10 and 41 Prop. of the first Book $AS \times 4Au = STq = uTq - Suq$. by the 47 *e* 1. Ergo *T* is in the parabola. After this manner infinite of *T* may be found to draw the parabola by.

Prop. 23, 24, 26, 27. *Having by position the* 2,
axis and the nodes of the Hyperbole uAEo *and of* 5.
the Ellipsis AuoE *to describe the section very readi-*
ly

Fig. *ly by points.* Measure $Av = Au$ given within
2, AE in the hyperbole and without in the Ellip-
5. ſis ; and taking in the axis any point whatſoever b upon the centre o, and ſemi-diameter ob deſcribe an arch, likewiſe upon the centre u and ſemi-diameter vb deſcribe another arch cutting the former in T. I ſay the point T is in the ſection. For becauſe in the Hyperbole $oT - uT = ov = AE$, but in the Ellipſis $oT + uT = ov = AE$ there will be right lines drawn from either node to the ſection, and T will be the ſection ; and if many T be found, the ſection may readily be drawn.

A Compendium in the 26 and 27 may be ve-
2. ry well uſed. For let Ao 116 and Au 16, it will be $Ao - Au = AE$ 100 in the Hyperbole, and $Ao + Au = AE$ 132 in the Ellipſis, therefore taking as many numbers as pleaſeth N, and in deſcribing the Hyperbole $Au - N$ drawn from u, and $AE + Au + N$ drawn from o, ſhall meet in T, but in the Ellipſis deſcribing $Au + N$ drawn from u, and $AE - Au - N$ drawn from o will meet in T.

1. Prop. 28, 29, 30. *About any triangle, as* ATt, *to deſcribe a parabola by points.* Having biſected the baſe tT in S, joyn the diameter AS, in which taking any point b where ever (whether above or below S it is no matter) draw by the ſame two right lines, one tbP from the angle t, another bd parallel to the baſe, cutting the ſide AT in d, and by the point d draw the right line sdP parallel to AS, cutting the baſe in s, and the right line tbP in P: I ſay, the point P is in the

Parabola,

parabola, whose vertex is *A*: For drawing *PpB* Fig. 1.
parallel to the base, cutting the side *AT* in *p*, it will be (*tS*)=*ST*.*BP* : : (*bS*=) *ds*.*dP* : : *sT*. *Pp*. because of the triangles *sdT* and *dPp* which are alike, wherefore *ST*. (*ST*-*sT*=) *BP* : : *BP*. (*BP*-*Pp*=)*Bp*. therefore *ST*. *BP*. *Bp* are continual proportionals, wherefore *ST*(*q*). *BP*(*q*): : *ST*. *Bp* : : *AS*. *AB*. Therefore by the 7 of the first Book the point *P* is in the Ellipsis.

From hence a Compendium may be drawn 1.
thus: if the lines *AS* and *ST* be divided in many similar parts *Ab*, *b*, *b*, *b*, &c. and *Ss*, *s*, *s*, *s*, &c. Or if *AS* and *AT* be divided into similar parts, *viz*. *Ab*,*b*,*b*,*b*, &c. and *Ap*, *p*, *p*, *p*, &c. they may be divided into as many small parts, drawing *tbP* and *sP* neglecting the line *AT*. or *tbP* and *pP* neglecting the line *ST*. And note that the divisions *Abbbb* may be measured also in *AS*, and then the similar parts of those remains may be measured towards *t*.

Prop. 31, 32. The effection of these is very plain, from what hath been said and declared in the 28, 29, and 30 Prop. neither is it any thing material whether the parabola *TAt* be complete or no; for if the parabola be curtated it may be further produced; or if it be too long it may be completed.

Prop. 33, 34, 35, 36. *Having the axis of the hyperbole given by position, and the species of the figure, which is* AE *to* AR. Because *Mydorgius* hath effected these Problems not so closely, and with

Fig. with that clearness as might have been, and because they are done with much more labour than was needful, I have adventured to add my own solutions, in respect that their demonstration doth very little differ from that of *Mydorgius*. Let it be therefore $AE.AR :: IA(q).Am(q)$ and set Am at right angles upon the axis AE, and draw out Im the Asymptotos: Then either measuring in the axis of the space $Ib = Am$, and by b drawing bp perpendicular to the axis; or out of the centre I drawing Ib perpendicular to the axis, by the point m let the right line bmp be produced. Further, take the point B however or wheresoever under A, and draw BF parallel to Am, cutting Im in F: Then upon the centre I and the semi-diameter BF, describe an Arch cutting the right line bp in p: And lastly, make $BP = bp$. I say, that the point P is in the Hyperbole: For $AE.AR :: IAq.(Am(q) =) Ib(q) :: IB(q).(BF(q) =) Ip(q) :: (IB(q) - IA(q) =)$
26. $AB \times EB.(Ip(q) - Ib(q)) = bp(q)$. But $bp(q) = BPq$. Ergo by the 13 of the first Book the point B is in the Hyperbole.

Consect. In the Hyperbole $BF(q) = BPq + Am(q)$. Note also, that if AE given be not the axis, but the transverse diameter where-ever, then there ought to be given the angle of inclination, and then describing the Asymptotoi, and bisecting the angle betwixt them mIm: either take AE for the axis, and then by it find the Ordinates, and after inclining them to the diameter according to the angle given. Or:

Prop.

Prop. 37, 38, 39. *Having the axis of the Ellipsis* AE, AO, *cutting themselves mutually at right angles in the centre* I, *to describe the Ellipsis by points.* Upon the centre *I* and semi-diameters *IA*, *IO* let two circles be described, and taking any point in the outer circle where-ever *P*, joyn *IP* cutting the interior circle in *p*, then as well from *P* draw the right line *PB* cutting the axis *AE* at right angles in *B*, as another right line from *p*, that is *pT* parallel to *AE*, and cutting *PB* in *T*, I say that the point *T* is in the Ellipsis. For(*IP*=)*IA.PB* : : (*Ip*=) *IO.BP.* because the triangle *IPB* is cut proportionally by *pT*. Wherefore from the former it will be (*IA*(*q*)=) *IA***AE*. (*PB*(*q*)=) *AB***EB* : : *IO* (*q*). *BP* (*q*). Therefore by the 7. of the first Book *T* is in the Ellipsis. And so taking many *P* as many *T* may be found to draw the Ellipsis. Fig. 5.

Note also, that having the Conjugate diameters of the Ellipsis however, the Ellipsis may be drawn, for having found the length of the Ordinates, as was shewed by the last equal to 2*BT*, afterwards they are to be inclined to the angle given.

Prop. 40, 41, 42, 43. *These do altogether depend upon the* 28, 29, 30, 31 *and* 32 *Prop. aforegoing, neither do they any thing differ from those, except*, That in these not the diameter of the section it self is given, but a right line is divided into similar parts, which line is parallel to the diameter at the end of the Inscript, and therefore shall not need to be here set down.

Fig. 6. Prop. 44, 45. *Having the transverse diameter of the Hyperbole* AE, *together with the species of the figure* AE *to* AR *and the angle of inclination to draw the Hyperbole by points.* Let it be *AE. AR* : : *IA*(*q*). *Am*(*q*) incline *Am* to *AE* according to the angle given, and produce *Im* the right line, then taking the point *B* in *AE* under *A* where ever, draw *BF* parallel to *Am*, cutting *Am* in *F*, and let it be *BP*(*q*)=*BF*(*q*)-*Am*(*q*). I say the point *P* is in the Hyperbole. For *IA* (*q*). *Am*(*q*) : : *IB*(*q*). *BF*(*q*) : : (*BF*(*q*)-*Amq*=) *BP*(*q*). (*IB*(*q*)-*IA*(*q*) *per* 6 *e* 2) = *EB*×*EA*. Therefore by the 13. of the first *P* is in the Hyperbole.

Consect. In the Hyperbole it is *BP*(*q*)=*BF* (*q*)-*Am*(*q*).

6. Prop. 46, 47. *Having by position the Asymptotoi of the hyperbole given, and a point in the section, to describe the hyperbole by points.* Let the point however taken or given by *A*, draw from the centre (which is always given if the Asymptotoi be given) and from *A* draw *AH* to one of the Asymptotos, and parallel to the other; then measure the space *Hm*=*HI*, and drawing *mAm*, bisected in *A* by the 2 *e* 6. and so having found *Am* the rest are performed according to the last Proposition.

Or, if it shall seem more fit, to find out the axis, and having found it to describe the Hyperbole. Let the point given in the hyperbole be *P*, and draw the axis *Ib* by cutting the angle of the

the Asymptotos into two parts and halves; Fig. 6.
and by P draw the right line $FPBf$ perpendicular to the axis, cutting the Asymptotos in F and f, and the axis in B: and let it be $BQ(q) = BF(q) - BP(q)$. draw Qm parallel to the axis cutting the Asymptotos in m: and taking an equal space $Im = Im$ in the other Asymptotos, joyning mAm, cutting the axis in A, the remainder of the work followeth, as in the 44 Proposition.

Prop. 48, 49. *If the axis of the Ellipsis* AE *and* OV *be given, cutting one the other mutually, at right angles in the centre* I, *to describe the Ellipsis by points.* Let IX be measured equal to $IA - IO$, and within the distance IX take any point b however, then opening the Compasses to the distance of IX, and putting one foot of the Compasses in b, mark out in the other axis where the compasses will cross, and note that point with p: Lastly, by p and b draw the line $pbP = IA$, I say that the point P is in the Ellipsis. For, drawing PB perpendicular to the axis AE, because it is $bP = IO = XA$, it shall be $Ib.\ bB :: (pb =)\ IX.bP$. and therefore it is $(Ib + Bb =)\ IB.\ bB :: (IX + bP =)\ IA.\ bP$. and $IB(q).\ bB(q) :: IA(q).\ bP(q)$ wherefore also $IA(q).\ (IA(q) - IB(q))\ AB*EB :: bP(q).\ (bP(q) - bB(q)).\ BP(q)$. Therefore by the 8 of the first Book the point P is in the Ellipsis: and after this manner taking many b betwixt X and I, and as many p in IV, draw $pbP = IA$, and by that means the Ellipsis. 5.

Fig. 7. Prop. 52, 53, 54. *Having the transverse diameter of the Hyperbole* AE, *and the species of the figure which is* AE *to* AR, *together with the angle of inclination, to draw the Hyperbole by points.* Let it be $AE. AR :: IA(q). Am(q)$. let Am be inclined to AE according to the angle given; and let there be by I a line drawn parallel to Am, in which taking any point M at pleasure, draw MP parallel to IA, cutting the line Im in 8, so that it may be $MP(q) = IA(q) + M8(q)$. I say that the point P is in the Hyperbole. For let IB be measured equal to MP, and joyn $PB = IM$, because by the 6 e 2 $(IB(q))$ $MP(q) = IA(q) + AB \times EB$. it will be $AB \times EB = M8(q)$. but it is $AE. AR :: IA(q). Am(q) :: (M8(q) =) AB \times EB. IM(q) = BP(q)$. Therefore the point P is in the Ellipsis.

Consect. In the Hyperbole it is $IB = IAq + M8(q)$.

And the same note may be observed here as was upon the 33 Prop.

8. Prop. 55, 56. *Having the transverse diameter* AE *and the Ordinate* ST *given by position, to describe the Ellipsis by points.* Let it be $AS \times ES. STq :: IA(q). IO(q)$. and set IO parallel to ST, and joyn AO and EO, and taking in AE any point however B, draw by B a parallel to IO cutting AO in p, and EO in Pp: Let the line OH be drawn out parallel to AE, cutting the same $BpPp$ in H, which is the middle point of the segment pPp, because it is $IA\ IO :: OH.pH.$ and as

$IE.IO :: OH.HPp$. then let it be $BPq = BH(q) - pH(q)$ that is $Bp \times BPp$ by the 6 e 2. But it is $AB(q).AB \times EB :: AB.EB :: Bp.BPp :: BP(q).Bp \times BPp$. wherefore $AB \times EB.(BP \times BPp) BP(q) :: ABq.Bp(q) :: (IA(q) =) IA \times IE.IOq$. Therefore by the 8 Prop. of the first Book the point P is in the Ellipsis. Fig. 8.

And from hence it also appeareth, that if the angle AOE be a right angle, the section will be a circle, because it is $AB \times EB = BP(q)$.

Prop. 57, 58, 59, 60, 61, 62. *Because in the parabola (however the Ordinates be inclined to the diameter) by the 7 and 32 Prop. of the first Book it is* AS.AB :: ST(q). AB(q). If the right line AD parallel to the base TSt do touch the parabola in A; and if ST be divided into many equal parts, suppose 6, noted with the letters s, and by every part s there be drawn parallel lines to the diameter AS, dividing AD in the point 5 equally and alike unto ST, then divide AS equally to the square of the number of parts you divided ST into, *viz.* if ST be divided into 6, then AS must be divided into 36 parts; and making out the square numbers of those 36, *viz.* 1. 4. 9. 16. 36. from the first parallel $5s$ next the diameter let there be cut off $5P = 1 = Q.1$. from the second parallel $s5$ let $5P = 4 = Q.2$. be cut off, from the third parallel $5s$ cut off $5P = 9 = Q.3$. from the fourth $5P = 16 = Q.4$. and so forwards, *&c.* noting out the several points P, P, *&c.* the curved line drawn by the points P, P, P, P, *&c.* will be a 9.

Fig. 9. parabola. For of the parallels *ss* the interior segments *sP* betwixt the curved line and the base *sP*, every one of them are equal to the several rectangles made of the segments of the base, *viz.* $sP = ts \times sT$, which is thus proved: Because *AS* is equivalent to the quadrate of $TS \times St$, and *Ps* to the rectangle $Ts \times st$ by supposition. It will be $AS . TS \times ST :: Ps . TS \times st :: (AS - Ps) = AB . (TS \times ST - Ts \times st =) BP(q)$. Therefore by the 7 and 12 Prop. of the first book *P* is in the parabola. But because this last demonstration is something obscure and Analytical, here followeth a more Synthetical demonstration: For, because by the 7 Proposition of the first Book it is $AS . ST(q) :: AB . (BP(q) =) Ss(q) :: (AS - AB =) BS . (ST(q) - Ss(q)$ which by the 5 e 2) $= Ts \times st$. But *AS* is equivalent to STq, therefore $Ps = BS$ is equivalent to the rectangle $Ts \times sT$. The Mechanical performance of the description of these sections thus by points, may by a *sector* be easily done.

And from hence it followeth, that the defective parabola may be supplied, and the curtated continued; for if there be drawn a tangent to the point of the continuation from whence the section was defective, and likewise a right line parallel to the diameter betwixt the tangent and the point to which it is to be supplied or continued; and if the segment of the tangent betwixt that parallel and the point of contact be divided into equal parts, and the parallel it self into the squares of those parts as was done before: the outward

ward ſegments of the lines parallel to the diameter, equal to the ſeveral parts of the tangent, are thus had, to deſcribe the curved part or parts of the parabola withal. Fig. 9.

Prop. 65, 66, 67. *Having the tranſverſe diameter of the hyperbole* AE, *and the ſpecies of the figure* (AE *to* AR) *and the angle of inclination, to deſcribe the hyperbole by points.* Let it be *AE. AR* : : *IA(q). Am(q).* and let *Am* be inclined to *AE* according to the angle given, and draw out *Im*, *Im*, then having biſected *Im* in *H*, and drawing *AH*; let there be taken in *Im* a point *h* however, and draw *hP*, ſo that it may be *Ih*×*hP* =*IH*×*AH*. I ſay that the point *P* is in the Hyperbole. For lengthning *If* parallel to *AH*, and extending *Am* beyond *A* till it meet with *If* in *m*, and by *P* drawing *FPf* parallel to the ſaid *mAm*, cutting *IA* in *B*, and *If* in *f*, and laſtly, by the points *A* and *P* drawing the right line *LAPl*, cutting *If* in *L* and *IF* in *l*. For becauſe *IH*×*AH*=*Ih*×*hP*. it ſhall be *Ih.IH* : : *AH. HP* : : *Hl. hl.* and *Ih.* (*Ih-IH*=) *Hh* : : *Hl.* (*Hl-hl*=) *Hh.* wherefore *Hl*=*Ih.* and therefore *Pl* =*AL.* and becauſe *IH*=*Hm.* it ſhall be *mA* =*Am.* alſo becauſe *PF. Am* : : *Pl.ml* : : *AL.PL* : : *Am. Pf.* it ſhall be *Am(q)*= (*PF*×*Pf*=) *per* 6 e 2. =*BF(q)-BPq.* and after the manner in the 44 Prop. of this Book the Hyperbole may be drawn. 7.

Or otherwiſe it may be demonſtrated after the manner delivered in the 63 Prop. of this ſecond book, becauſe it is *Pl*=*AL*.

Fig. Consect. $Ih \times hP = IH \times AH$.

7. And from these it will be very easie to draw the Hyperbole by points, having the Asymptotos and the point in the Hyperbole given.

10. Prop. 68, 69. *If in the diameter of a section there be two points taken, one* b *within the section, another* a *beyond the vertex* A; *and if from the point* b *there be drawn many right lines cutting the section in the points* P, P, P, *&c. and if it be in every one* bA. ba : : bP. bp. *the curved line drawn by all the points* p *shall be a section of the same kind, name and nature with the section given.* For in the Parabola's it is $AB.BP :: ab.bp$. and in the Hyperbole's and Ellipsis it is

$$\left\{\begin{array}{l} AB.BP :: ab.bp \\ \times \quad \times \quad\quad \times \quad \times \\ EB.BP :: eb.bp \end{array}\right.$$

That is, $AB \times EB.BP(q) :: ab \times eb.bp(q)$.

Thus in this second Book *Mydorgius* hath shewed and performed very many ways and varieties for drawing of the Conical Sections by points, which are here contracted, (at least the very chief from whence the rest are drawn) into a little room: His design in this Book was, to prepare these Propositions in order to an Appendix, which certainly he intended, as appears by his *Monitum* to the end hereof, wherein he should treat of the Organical part of describing of these Sections, or of performing them by Instruments at one motion, and to use so many of these Propositions in this Book for the demonstrating of those Instruments, which otherwise in

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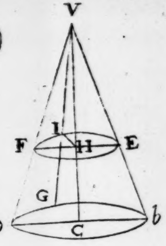

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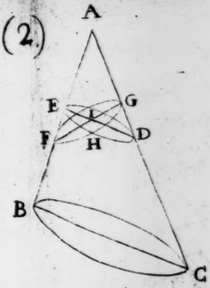

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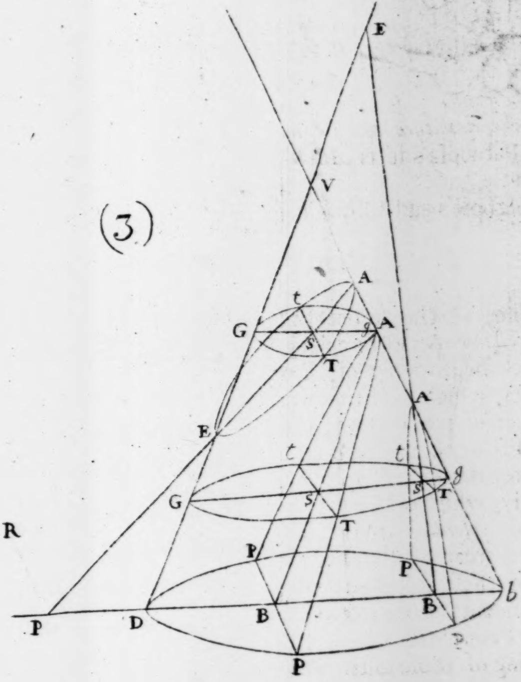

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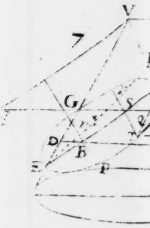

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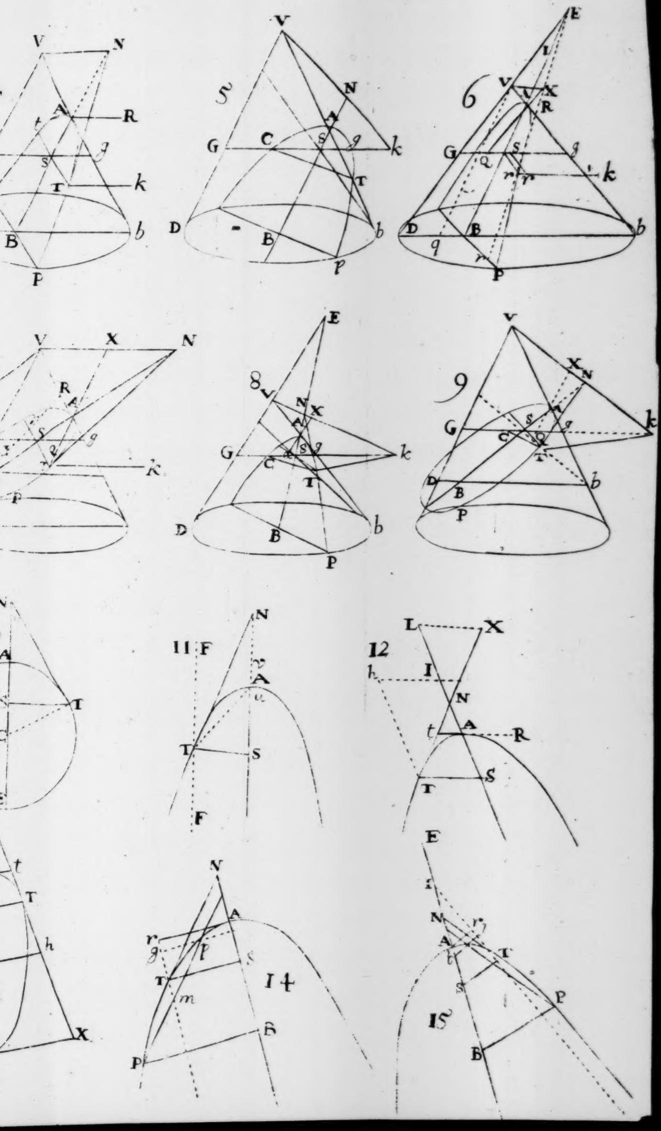

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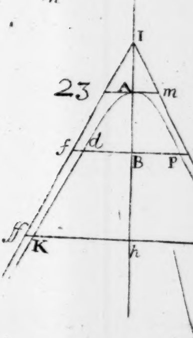

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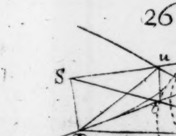

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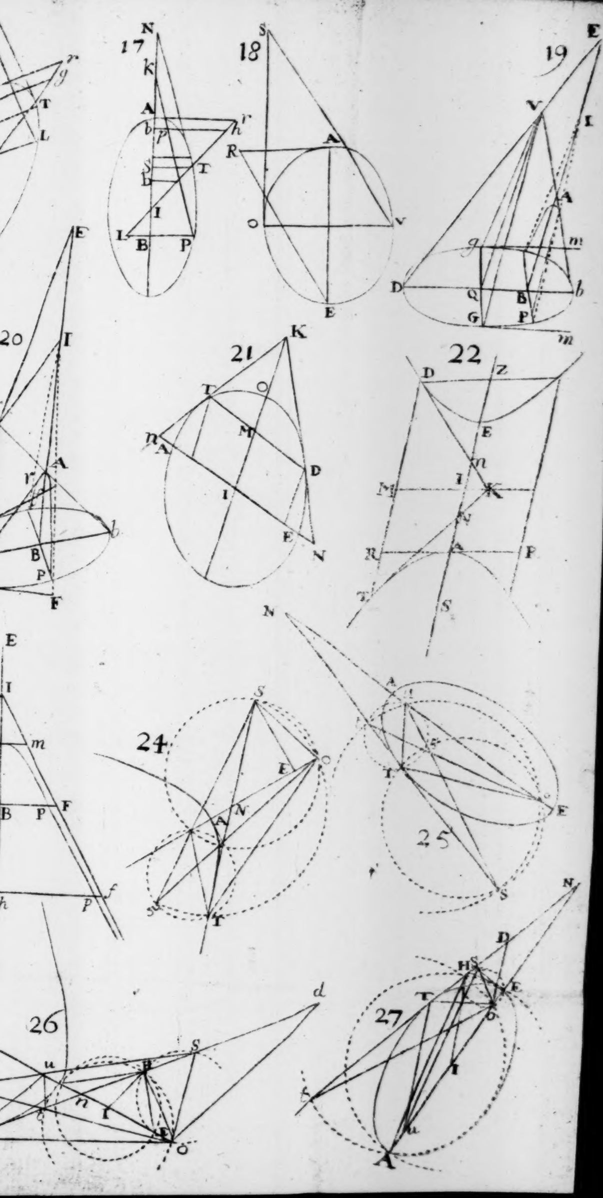

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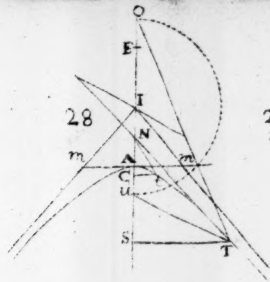

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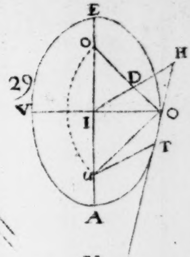

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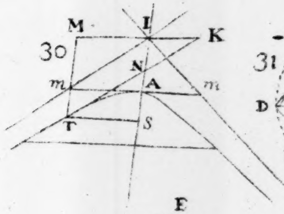

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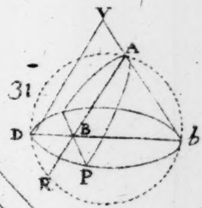

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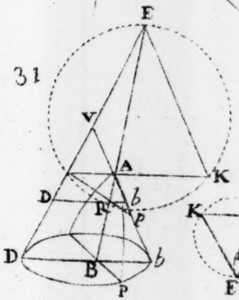

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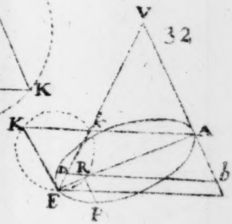

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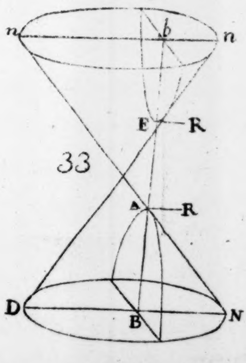

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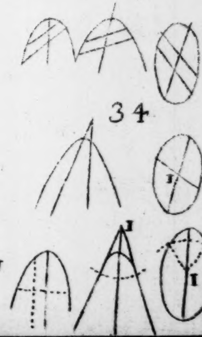

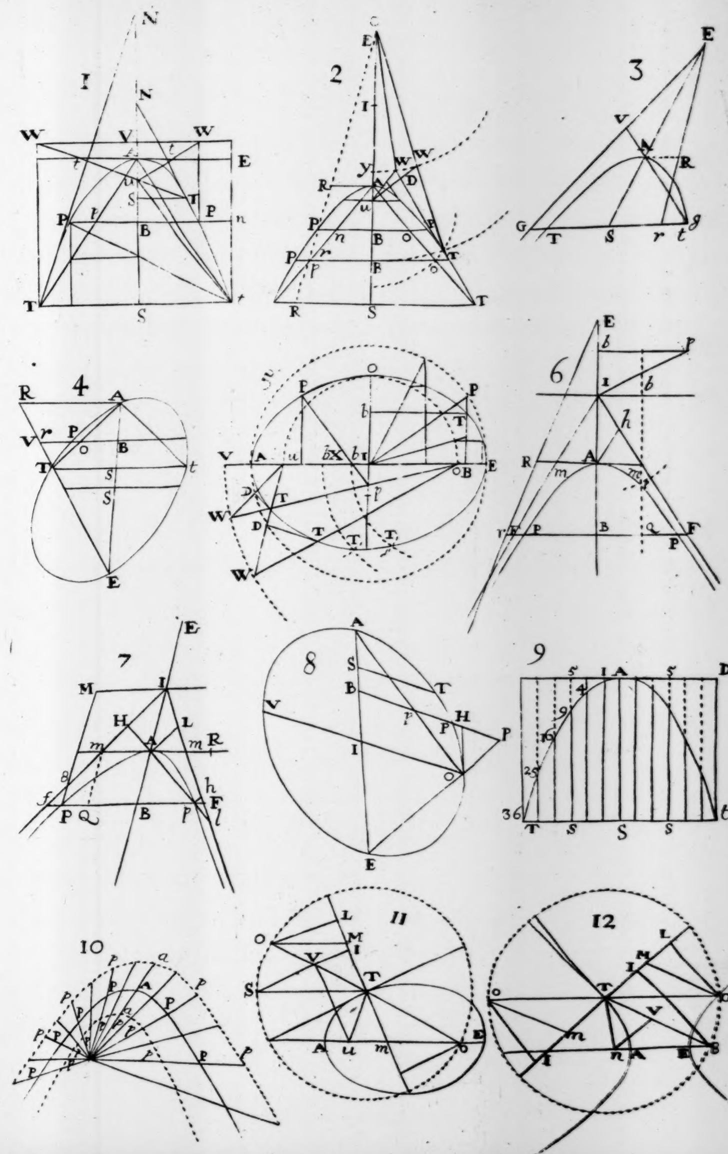
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in themſelves may ſeem tedious and trouble-ſome. This I thought good to adviſe the Reader of, that only thoſe Propoſitions which are moſt facile and eaſie, might be noted for the Mechanick part, the reſt being intended for another end. Fig. 10.

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